

3.36pt

International Money and Banking:

12. The Term Structure of Interest Rates

Karl Whelan

School of Economics, UCD

Spring 2018

Beyond Interbank Interest Rates

- We have discussed how interest rates in short-term interbank markets are controlled by central banks.
- These interbank rates receive a huge amount of attention. But *economically* they are not very important: The sums borrowed in these markets are small when compared with the amounts borrowed by businesses and governments via issuance of bonds or borrowing by households in the form of mortgages, car loans, credit card debt and so on.
- In the euro area, where policy rates have been lowered to zero (and below) the most relevant interest rates are not zero.
- Check out the “Selected Interest Rates” release on the Fed’s website. As of March 20, the fed funds rate was 1.44%, one-year Treasury securities yielded 2.08%, the ten-year Treasury bond yielded 2.89%. Elsewhere, you can find out that AAA-rated corporate bonds had an interest rate of 3.82% last month and BAA-rated corporate bonds had an interest rate of 4.51%. The average 30-year mortgage had an interest rate of 4.44%.
- What accounts for the differences between all these interest rates? That’s what we will be focusing on in the next few lectures.

Maturity, Default and Liquidity Risk

- These debt instruments all have different interest rates for a reason. There are three principal factors that influence the interest rate on a particular type of debt:
 - ① **Maturity:** How much time does the borrower have to pay back? Interbank loans are often overnight but most business and consumer loans are for terms of years.
 - ② **Risk:** Some types of debt (e.g. government bonds in certain countries) have very little risk of default. Others (mortgages, credit cards, corporate bonds) differ in the probability of default and in how much the lender will lose if there is a default.
 - ③ **Liquidity:** Debt instruments differ in how easily they can be traded. A bond for which a liquid market does not exist will probably have to pay out a higher interest rate to attract investors.
- I will focus here on the first of these issues. We will look at how interest rates on assets with similar levels of risk and liquidity differ because of their varying maturity by focusing on the market for government bonds. I will discuss risk and liquidity later.

Part I

Bonds and Yields

What Is a Bond?

- The interest rates we will focus on in this lecture will be the interest rates associated with bonds.
- So what is a bond? Bonds are a form of IOU issued by governments or corporations to people who provide them with money.
- An investor agrees to provide the bond issuer (e.g. the government) with a sum of money, say \$100. This sum is called the *principal*.
- In return, the bond issuer provides the investor with a bond certificate that specifies how the money will be repaid.
- The terms of a bond contract specifies the length of time until the principal is repaid. This is known as the **maturity** of the bond. It may be that the final repayment exceeds the initial principal provided.
- It also specifies a schedule for potential interest payments to be made. These are sometimes called **coupon payments**
- Bond certificates can generally be sold by the holder to other investors at any time. Indeed, while the stock market gets a lot of attention, the global bond market is a far larger and more important one than the stock market.

Definition of Yield to Maturity

- Suppose you have \$100 to invest. You decide to purchase a bond with price \$100 that will pay you back \$116 in five years time. Investors will tell you that your bond has a **yield to maturity** of 3% because

$$\$100(1.03)^5 = \$116$$

- The investment turns out the same as investing at an interest rate of 3% and re-investing the proceeds until five years time.
- An alternative way of writing this relationship is:

$$\$100 = \frac{\$116}{1.03^5}$$

- More generally, the price of a bond that pays out \$1 in m periods time can always be expressed as

$$P_m = \frac{1}{(1 + y_m)^m}$$

- Alternatively, we figure the yield from the current price:

$$1 + y_m = \left(\frac{1}{P_m} \right)^{\frac{1}{m}}$$

Inverse Relationship Between Prices and Yields

- Let's still assume you're still the guy who bought the bond that pays \$116 in five years time.
- Now suppose something happens in financial markets, so that the yield on all bonds has to rise to 4%.
- If you had to sell your bond now, it's price would have to be

$$\frac{\$116}{1.04^5} = \$95.34.$$

- This increase in yields may have been good news for new investors who just arrived in the market looking for the best possible return.
- But if you wanted to sell your bond today, it was bad news for you. The terms of your bond haven't changed—it still only provides \$116 in five years time—so to satisfy the new market-driven yield of 4%, you would have to sell your bond today for less than the \$100 you spent to acquire it.
- Key Point: Bond prices and bond yields move in the opposite direction.

Coupon Bonds

- The example just given involves a bond that only pays out when it matures in five years time. It doesn't make interest coupon payments along the way. This is known as a *zero-coupon* bond.
- What about bonds that make interest coupon payments? One can calculate the yield for these bonds by viewing them simply as a series of zero-coupon bonds bundled together.
- For example, consider a bond that makes interest payments of C every period for ten years and then pays out F in the tenth year. We calculate the yield of the bond as the y that makes the following equation hold

$$P = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \frac{C}{(1+y)^3} + \dots + \frac{C}{(1+y)^{10}} + \frac{F}{(1+y)^{10}}$$

- For most bonds, the final payment at maturity and the amount of the coupons don't change. However, the yield does change as bond prices move up or down.
- The yield is thus a simple summary statistic that you can use to summarise the rate of return you can get from different types of bonds, whether they pay coupons or not.

Spreadsheet Examples

- On the website, I have linked to a spreadsheet that illustrates four different examples of bond yields, coupons and prices.
- Excerpts from the spreadsheet are shown in the next few pages.
 - ① The first excerpt shows prices for bonds with an annual interest payment of 3 and a final principle repayment of 100. The market yield is 3 percent – this means all the bonds are priced equal to 100.
 - ② Then we show what happens to these bonds when market yields fall to 2 percent. Bond prices rise, with the gains getting bigger as the maturity of the bonds increase.
 - ③ The NTMA publishes a daily report on Irish government bonds. The last two examples illustrate two of these bonds, as priced on March 23, 2018.
 - ★ The 2022 bond makes interest payments of 0.8%, has a yield of -0.12% and a price of 103.69.
 - ★ The 2024 bond makes interest payments of 3.9%, has a yield of 0.29% and a price of 118.42.

Example: Coupon Payments of 3%, Market Yield of 3%

Example: Bonds paying 3 percent coupon and market yields are 3 percent

Gross Yield	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03
Maturity of Bond in Years	1	2	3	4	5	6	7	8	9	10
Coupon Payment	3	3	3	3	3	3	3	3	3	3
Discount Rate	1.03	1.0609	1.092727	1.125509	1.159274	1.194052	1.229874	1.26677	1.304773	1.343916
Discounted Coupon Payments	2.912621	2.827788	2.745425	2.665461	2.587826	2.512453	2.439275	2.368228	2.29925	2.232282
Cumulating Discounted Coupon Payments	2.912621	5.740409	8.485834	11.1513	13.73912	16.25157	18.69085	21.05908	23.35833	25.59061
Price for Bond With Principal Repayment of €100	100	100	100	100	100	100	100	100	100	100

Example: Coupon Payments of 3%, Market Yield of 2%

Example: Bonds paying 3 percent coupon and market yields are 2 percent

Gross Yield	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02
Maturity of Bond in Years	1	2	3	4	5	6	7	8	9	10
Coupon Payment	3	3	3	3	3	3	3	3	3	3
Discount Rate	1.02	1.0404	1.061208	1.082432	1.104081	1.126162	1.148686	1.171659	1.195093	1.218994
Discounted Coupon Payments	2.941176	2.883506	2.826967	2.771536	2.717192	2.663914	2.611681	2.560471	2.510266	2.461045
Cumulating Discounted Coupon Payments	2.941176	5.824683	8.65165	11.42319	14.14038	16.80429	19.41597	21.97644	24.48671	26.94776
Price for Bond With Principal Repayment of €100	100.9804	101.9416	102.8839	103.8077	104.7135	105.6014	106.472	107.3255	108.1622	108.9826

Irish 2022 Bond Example

Example: Irish Bond 2022 bond (4 years), paying 0.8% coupon, current yield is -0.12 percent, price is 103.66

Gross Yield	0.9988	0.9988	0.9988	0.9988		
Maturity of Bond in Years	1	2	3	4		
Coupon Payments	0.8	0.8	0.8	0.8		
Discount Rate	0.9988	0.997601	0.996404	0.995209		
Discounted Coupon Payments	0.80	0.80	0.80	0.80		
Cumulating Discounted Coupon Payments	0.80	1.60	2.41	3.21		
Price for Bond With Principal Repayment of 100	100.92	101.84	102.77	103.69		

Irish 2024 Bond Example

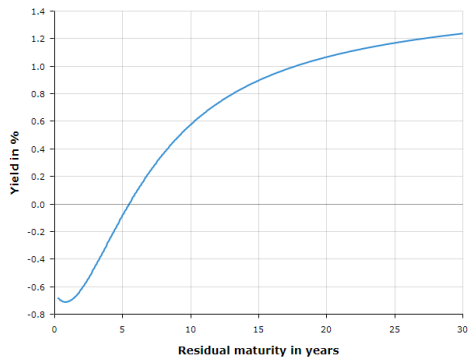
Example: Irish Bond 2024 bond (6 years), paying 3.4% coupon, current yield is 0.29 percent, price is 118.42

Gross Yield	1.0029	1.0029	1.0029	1.0029	1.0029	1.0029
Maturity of Bond in Years	1	2	3	4	5	6
Coupon Payments	3.4	3.4	3.4	3.4	3.4	3.4
Discount Rate	1.0029	1.005808	1.008725	1.011651	1.014584	1.017527
Discounted Coupon Payments	3.39	3.38	3.37	3.36	3.35	3.34
Cumulating Discounted Coupon Payments	3.39	6.77	10.14	13.50	16.85	20.19
Price for Bond With Principal Repayment of 100	103.10	106.19	109.28	112.35	115.42	118.47

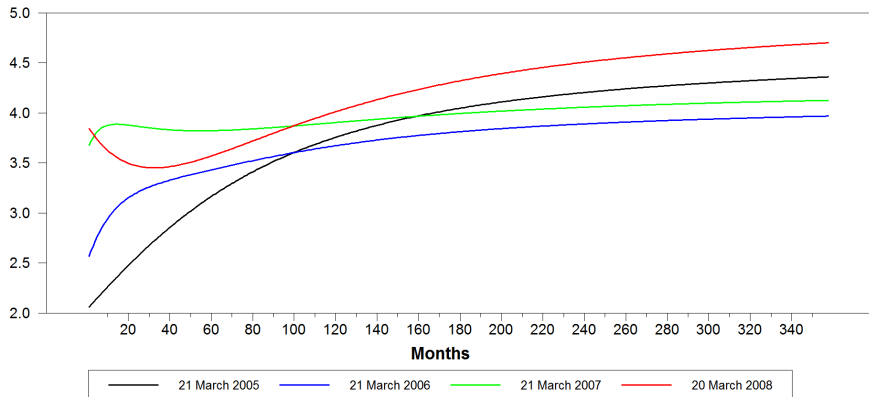
The Yield Curve

- A nice way to summarise information about the yields available on bonds that have different maturities (i.e. that differ according to when their final payout occurs) is to chart the different yields on the y -axis while charting the maturity on the x -axis.
- This line in this kind of chart is known as “The Yield Curve” and financial market participants pay close attention to it.
- Getting information on yields on government bonds is really easy.
- Type “Yield Curve” into Google and you’ll find information on the yields at various maturities for Europe from the ECB and for the US from the US Treasury.
- The next page shows a screencap of information from the ECB’s yield curve page which shows yields for AAA-rated euro area government bond.
- The pages after uses longer historical data to illustrate various yield curves over time. Notice how the pre-crisis 2007 curve looked so different to curves from recent years.

Example: Yield Curve for AAA-Rated Government Bonds



Pre-Crisis Euro Area AAA Yield Curves



Why Would Anyone Hold a Bond With a Negative Yield?

- A look at the current yield curve for AAA-rated sovereign bonds shows something strange: Yields on these bonds with maturities up to about 7 years have negative yields, meaning you will lose money by investing in them.
- Why would anyone ever purchase such a bond? After all, you could just keep your money in cash and get a return of zero.
- For some European countries (e.g. Sweden and Denmark) people may expect the currencies these bonds are denominated in to increase in value, so the return denominated in euros will not be negative.
- For bonds like German government bonds, a few factors determine the negative yields.
 - 1 Banks may see these bond purchases as an alternative to keeping money on deposit with ECB, which ECB now charges for. A slightly less negative yielding bond might be a better alternative.
 - 2 Financial institutions could keep large amounts of cash but there are storage and security costs associated with storing billions of euros. And criminal masterminds might figure out ways of stealing enormous amounts of money kept in storage, which may not be a risk worth taking.

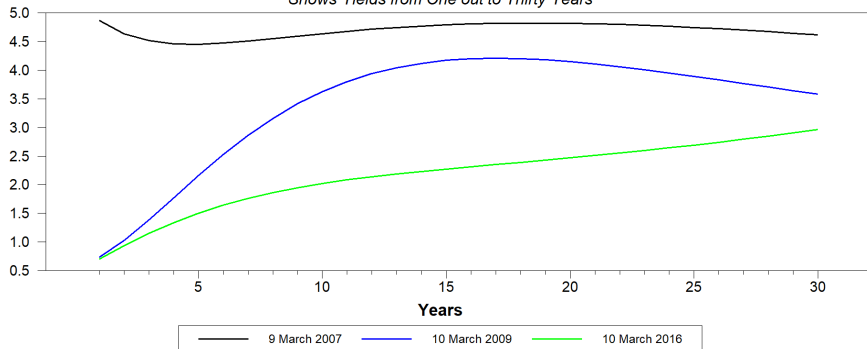
The Market for US Treasury Bonds

- Most of the research on the maturity or *term structure* of interest rates has focused on the market for US Treasury securities.
- The US government borrows from financial markets, issuing bonds that may off at lots of different maturities: Short-horizons such as one to three months and long horizons such as ten to thirty years.
- The market for these securities is enormous and highly liquid and because the default risk for the US government is considered to be low, you don't have to take this factor into account.
- This means that we can ignore the risk and liquidity factors and look purely at how maturity affects the interest rate on an asset.
- Treasuries are of interest beyond being a useful learning tool. They are a key *benchmark* for investments denominated in US dollars: They provide an alternative to lending over similar maturities to corporations or mortgage lenders and thus influence those rates.
- The next page shows US Treasury yield curves at various points in time. The page after shows a 3-D yield curve with yield curves from 1990 to 2014. The page after shows movements over time in the one-year and ten-year Treasury bonds

Yield Curves from 2007, 2009 and 2016

Three US Treasury Yield Curves From Different Points in Time

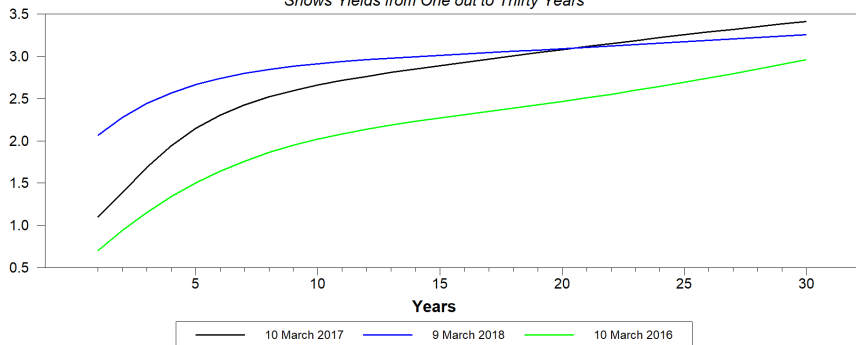
Shows Yields from One out to Thirty Years



Yield Curves from 2016, 2017 and 2018

Treasury Yield Curves From 2016 to 2018

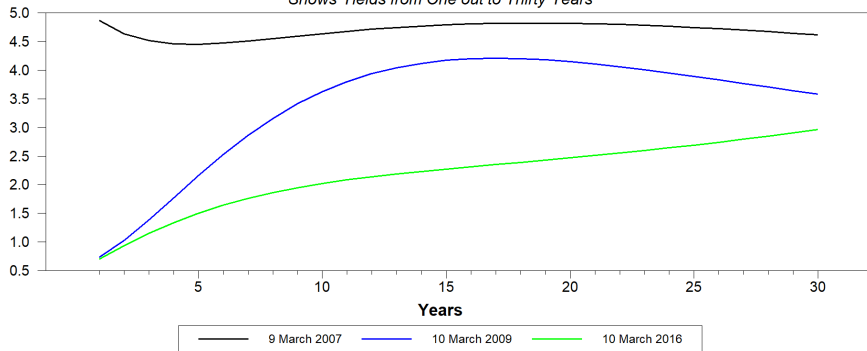
Shows Yields from One out to Thirty Years



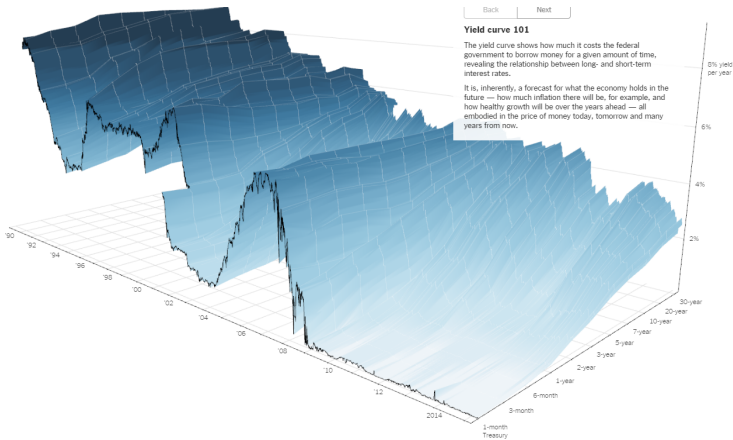
Movements in the U.S. Treasury Yield Curve

Three US Treasury Yield Curves From Different Points in Time

Shows Yields from One out to Thirty Years



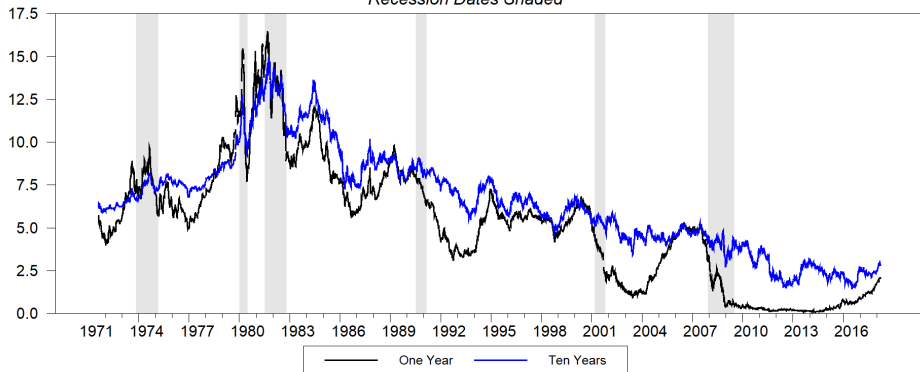
A 3-D Yield Curve from the New York Times



How Short- and Long-Term Bonds Move Together

Daily Yields on US Treasury Bonds

Recession Dates Shaded



Bond Market Stylized Facts

- ① Rates on longer-term bonds are usually higher than shorter-term bonds (i.e. yield curve usually slopes upwards.) Remember, the blue line for the ten-year interest rate was almost always higher than the black line for the one-year rate.
- ② Short and long rates usually tend to move together.
- ③ But short rates are more variable than long rates.
- ④ Short rates tend to draw close to long rates before and at the start of a recession and then the gap tends to widen again. In other words, the yield curve tends to flatten out just before recessions.
- ⑤ Interest rates on both short-term and long-term bonds are much lower today than in the late 1970s and early 1980s.

Part II

The Expectations Theory

Why Do Yields Differ by Maturity?

- Going back to the data from the Selected Interest Rates page, the one-year US Treasury bond has a current yield of 2.08% while the ten-year bond has a yield of 2.89%. You might think, at first look, that this means that you should definitely choose to buy the ten-year bond instead of the one-year bond because it's a better deal.
- However, that's not necessarily the case. After the one-year bond has matured, you can buy another one-year bond next year, and then another the year after. And yields on one-year bonds may go up in the future.
- It might be that this strategy—buying ten one-year bonds in a row—is just as profitable as buying a single ten-year bond now.
- Indeed, if investors really all thought that one type of bond strategy was much more profitable than another, then they'd all sell bonds associated with the bad strategy to buy the bonds associated with the good strategy. This would drive down the price of the bad-strategy bonds until the point where investors were indifferent.
- This idea—that yields differ so as to make each investment strategy equally attractive—underlies what is called the *expectations* theory of the term structure.

Introducing The Expectations Theory

- Suppose you have a choice between two different strategies for investing over two periods:
 - ① Invest today at a fixed interest rate of i_{2t} per period.
 - ② Invest today for one period at an interest rate of i_{1t} , and then re-invest at next period's one-year rate of $i_{1,t+1}$.
- For the two-period rate to be worth taking up, these two strategies must have the same *expected* return.
- Two period bond has cumulative return $(1 + i_{2t})^2 = 1 + 2i_{2t} + i_{2t}^2$
- Rollover strategy has expected cumulative return $(1 + i_{1t})(1 + E_t i_{1,t+1}) = 1 + i_{1t} + E_t i_{1,t+1} + i_{1t}(E_t i_{1,t+1})$.
- For normal interest rates, both i_{2t}^2 and $i_{1t}(E_t i_{1,t+1})$ will be negligible.
- So, theory implies

$$i_{2t} = \frac{1}{2}(i_{1t} + E_t i_{1,t+1})$$

- The two-year rate is an average of the current one-year rate and the one-year rate expected a year from now.

General Formulation of the Expectations Theory

- One can apply this argument beyond two periods. So, the three period bond rate should be


$$i_{3t} = \frac{1}{3} (i_{1t} + E_t i_{1,t+1} + E_t i_{1,t+2})$$

and the four-period bond rate should be

$$i_{4t} = \frac{1}{4} (i_{1t} + E_t i_{1,t+1} + E_t i_{1,t+2} + E_t i_{1,t+3})$$

and the n -period bond rate should be

$$i_{nt} = \frac{1}{n} E_t \sum_{k=0}^{n-1} i_{1,t+k}$$

- The return on an n -period bond should equal the average of the return on one-period bonds over the n periods starting today.
- According to this theory, an upward-sloping yield curve must mean that investors expect future short-term rates to be higher than today's short-term rate.
- A downward-sloping yield curve would mean that investors think that short-term rates are going to decline in the future. 

Explaining Some Facts with the Expectations Theory

- The expectations theory helps to explain some of the facts we described earlier:
 - ① Long rates depend on current and expected future short rates, so if short rates are trending in one direction, we would expect long rates to also go in that direction: This explains why rates often move together.
 - ② But longer rates depend on expected short rates that may be far in the future. Today's developments may have little affect on these expectations: Explains why long rates move less than short rates.
 - ③ Central bank tightening leads to high short rates now but perhaps an expectation that rates will have to be cut if a recession arrives: This explains why the shape of the yield curve changes dramatically around recessions.
 - ④ When a central bank cuts rates sharply to deal with a recession, it is usually then expected to increase them later: This explains why the yield curve then becomes very upward sloping during a recession (as it is right now).
- But the theory does not explain why yield curve usually slopes up: We cannot always be expecting short rates to rise.

Implications for the Transmission of Monetary Policy

- Rates on longer-term government bonds have a key influence on mortgage rates, corporate debt, etc. For instance, the ten-year Treasury rate is used as a benchmark for US fixed-rate mortgages.
- We can now see how monetary policy can influence these rates.
 - 1 Central banks control overnight interest rates.
 - 2 One week interest rates will be an average of the next week's expected overnight rates.
 - 3 One-month interest rates will be an average of the expected one-week interest rates over the next few weeks.
 - 4 One-year interest rates will be an average of the expected one-month interest rates over the next twelve months.
 - 5 Ten-year interest rates will be an average of the expected one-year interest rates over the next ten years.
- Via this set of linkages, the expectations theory explains how the central bank influences all interest rates, not just the overnight interbank rate: All “risk-free” interest rates depend on investor expectations for average overnight rates over the maturity of the bond.

Monetary Policy and Expectations Management

- The expectations theory helps to explain why central banks are so careful in their communications with the public.
- They control a particular interest rate, but one that accounts for only a tiny fraction of total borrowing. And the linkage between this rate and many of the rates that matter depends on *expectations* of what the central bank will do in the future. Hence, central banks are very careful in their communications, so financial markets will set longer-term rates in line with where they want them to be.
- For instance, post-2008, most FOMC statements signalled the fed funds rate would be kept near zero for some time, with the aim being to keep longer term interest rates down. Now they are raising their fed funds target but continue to communicate about the likely pace of further increases. The latest statement includes the following: *“In determining the timing and size of future adjustments to the target range for the federal funds rate, the Committee will assess realized and expected economic conditions relative to its objectives of maximum employment and 2 percent inflation ... The Committee expects that economic conditions will evolve in a manner that will warrant further gradual increases in the federal funds rate”*

Part III

Term Premia

Term Premia

- Recall the expectations theory

$$i_{nt} = \frac{1}{n} E_t \sum_{k=0}^n i_{1,t+k}$$

- Yield curve should only slope upwards if markets expect that short-term interest rates will be increasing.
- But the yield curve almost always slopes up: Markets cannot always be expecting short rates to be increasing!
- Alternative: Adapt the model to allow for a positive *term premium*

$$i_{nt} = \mu_n + \frac{1}{n} E_t \sum_{k=0}^n i_{1,t+k}$$

where $\mu_n > \mu_{n-1}$.

- Term premia may vary over time. In this case, the model is

$$i_{nt} = \mu_{nt} + \frac{1}{n} E_t \sum_{k=0}^n i_{1,t+k}$$

Yields, Bond Prices, and Bond Returns

- For simplicity, consider a world in which the yield curve is flat, so the yield on all maturities is the same.
- Consider a two-year bond issued today, paying \$1000 in two years, selling today for $\$942 = \frac{\$1000}{(1.03)^2}$.
- The yield on this bond is 3%. If yields don't change, the bond will be worth $\$971 = \frac{\$1000}{1.03}$ next year, for a return of 3%.
- Suppose that at the end of one year, yields on all newly-issued bonds have jumped to 10%.
- The existing bond, with one year to run, can only be sold if it also provides a 10% return, so its price must change to $\$909 = \frac{\$1000}{1.10}$. The seller has incurred a one-year return of -3.4%.
- Think that's bad? A ten-year bond bought last year for $\$744 = \frac{\$1000}{(1.03)^{10}}$ is now worth only $\$424 = \frac{\$1000}{(1.1)^9}$, a one-year return of -43%.

Comparing the Riskiness of Short- and Long-Term Bonds

- Now let's compare the riskiness of long- and short-term bonds.
- In favour of long-term bonds:
 - ① Most investors have a long-term horizon and over that horizon these bonds provide a guaranteed return. Remember, in our example, the bond is guaranteed to pay \$1000 at the maturity.
 - ② They turn out to be a good investment if interest rates decline in the future.
- Against long-term bonds:
 - ① Their value is very sensitive to changes in interest rates. While risk-free if held to maturity, they are very risky investments for those with short horizons or those who may sometimes have to liquidate assets quickly.
 - ② The guaranteed payout is usually nominal, so they will turn out to be a bad investment if inflation is higher than expected. In contrast, interest rates on short-term bonds will tend to rise with inflation, so a “rollover” strategy has less inflation risk.
- On balance, these latter points appear to dominate most of the time, so the yield curve usually slopes upwards.

Recap: Key Points from Part 12

Things you need to understand from these notes:

- 1 The 3 characteristics that affect interest rates on debt instruments.
- 2 Definition of a bond and its associated terminology.
- 3 Definition of yield to maturity.
- 4 Relationship between bond prices and bond yields.
- 5 How coupon-paying bonds are priced.
- 6 What is meant by the term structure and what the yield curve is.
- 7 Basic facts about movements in short and long-term interest rates.
- 8 Definition of the expectations theory.
- 9 What the expectations theory explains and what it doesn't.
- 10 Definition of term premium.
- 11 Factors affecting the riskiness of short and long-term bonds.