MA Advanced Macroeconomics:
11. The Smets-Wouters Model

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A Popular DSGE Model

- Now we will discuss a paper presenting a modern DSGE model that has a number of New-Keynesian features and which has been estimated with Bayesian methods.
- The paper is "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach" by Frank Smets and Raf Wouters which was published in the *American Economic Review* in 2007.
- Smets is an economist with the ECB and Wouters works for the National Bank of Belgium and the model was first developed for the euro area. Models like this have been used for policy analysis at the ECB and other central banks.
- This paper estimated the model for US data.
- Both the euro area and U.S. Smets-Wouters papers have been among the most cited papers in economics in recent years.
- We will first present the log-linearized version of the model. An appendix with the full model is available on the class website.
- We will then discuss the estimation process and the various applications of the model.

The Log-Linearized Model: The Supply Side

- The aggregate production function is

$$y_t = \phi_p \left(\alpha k_t^s + (1-\alpha)\, l_t + \epsilon_t^a\right)$$

where y_t is GDP, l_t is labour input, ϵ_t^a is total factor productivity and k_t^s is capital in use, which is determined by the amount of capital installed in the previous period and a capacity utilisation variable

$$k_t^s = k_{t-1} + z_t$$

- There are cost of adjusting the amount of capital in use so optimisation conditions for producers mean the rate of capacity utilisation is linked to the marginal productivity of capital

$$z_t = z_1 r_t^k$$

- The marginal productivity of capital is a function of the capital-labour ratio and the real wage

$$r_t^k = -(k_t - l_t) + w_t$$

- Total factor productivity evolves over time according to

$$\epsilon_t^a = \rho_a \epsilon_{t-1}^a + \eta_t^a$$

The Log-Linearized Model: The Demand Side

- The expenditure formulation of the aggregate resource constraint is

$$y_t = c_y c_t + i_y i_t + z_y z_t + \epsilon_t^g$$

where y_t is GDP, c_t is consumption, i_t is investment and ϵ_t^g is exogenous spending. (Terms like c_y and i_y are constant parameters here.)

- The variable z_t features here because we are assuming there are costs associated with having high rates of capacity utilisation.
- Exogenous spending is assumed to have two components: Government spending and element related to productivity because "net exports may be affected by domestic productivity developments."
- Taken together, exogenous spending changes over time according to

$$\epsilon_t^g = \rho \epsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a$$

The Log-Linearized Model: Consumption

- Consumption is determined by

$$c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 \left(r_t - E_t \pi_{t+1} + \epsilon_t^b \right)$$

  where c_1, c_2, c_3 are constant parameters, r_t is the interest rate on a one-period safe bond and ϵ_t^b evolves according to

$$\epsilon_t^b = \rho_b \epsilon_{t-1}^b + \eta_t^b$$

- There are a number of aspects to this equation

  1. It is a consumption Euler equation with a backward-looking element added to it. This represents "habit formation" so that a term of the form $C_t - \lambda C_{t-1}$ replaces C_t in the utility function.
  2. The term involving labour input allows for some substitution between consumption and labour input.
  3. The coefficients c_1, c_2, c_3 are themselves functions of deeper structural parameters.
  4. Smets-Wouters describe the ϵ_b term as a "risk premium" shock determining the willingness of households to hold the one-period bond. It can also be seen as a type of preference shock that influences the short-term consumption-saving decision.

The Log-Linearized Model: Investment

- Investment is determined by

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \epsilon_t^i$$

where

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) r_{t+1}^k - \left(r_t - E_t \pi_{t+1} + \epsilon_t^b\right)$$

and

$$k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \epsilon_t^i$$

- Again, there is quite a lot going on here
  1. Investment depends on lagged on investment because there is an adjustment cost function that limits that amount of new investment that can come "on line" immediately.
  2. The main driving force behind investment is q_t which itself is determined by a forward-looking stochastic difference equation.
  3. Solving the q_t equation would show that q_t depends positively on expected future marginal productivities of capital and negatively on future real interest rate (and "risk premia")
  4. The positive shock to investment also boosts the capital stock (representing "more productive" capital).

The Log-Linearized Model: Prices

- The mark-up of price over marginal cost is determined by

$$\mu_t^p = \alpha \left(k_t - l_t \right) + \epsilon_t^a - w_t$$

  which factors in diminishing marginal productivity of capital, the effects of the productivity shock on costs and the real wage.

- Price inflation is then determined by

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \epsilon_t^p$$

  where ϵ_t^p is a price mark-up disturbance that evolves according to

$$\epsilon_t^p = \rho^p \epsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$$

- Observations:
  - This is a New-Keynesian Phillips curve amended to provide a role for lagged inflation. This is modelled in the paper via the assumption that most firms index their price to past inflation and only occasionally get to set an optimal price.
  - The mark-up shock affects both current and lagged inflation in an attempt to get at temporary price level shocks.

The Log-Linearized Model: Wages

- The model treats wages similarly to prices, with sticky wages that gradually adjust so that real wages are move to equate the marginal costs and benefits of working.
- Specifically, wages move over time to equate real wages with the marginal rate of substitution between working and consuming. The gap between these is the "wage mark-up" defined as

$$\begin{aligned}\mu_t^w &= w_t - mrs_t \\ &= w_t - \left(\sigma l_t - \frac{1}{1-\lambda/\gamma}\left(c_t - \lambda c_{t-1}\right)\right)\end{aligned}$$

- Wages are then given by

$$w_t = w_1 w_{t-1} + (1 - w_1) E_t (w_{t+1} + \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_t \mu_t^w + \epsilon_t^w$$

where

$$\epsilon_t^w = \rho^w \epsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$$

The Log-Linearized Model: Monetary Policy

- The final element of the model is a rule for monetary policy. It is assumed that the central bank sets short-term interest rates according to

$$\begin{aligned} r_t &= \rho r_{t-1} + (1-\rho)\left(r_\pi \pi_t + r_y\left(y_t - y_t^p\right)\right) \\ &\quad + r_{\Delta y}\left[\left(y_t - y_t^p\right) - \left(y_{t-1} - y_{t-1}^p\right)\right] + \epsilon_t^r \end{aligned}$$

  where

$$\epsilon_t^r = \rho^r \epsilon_{t-1}^r + \eta_t^r$$

- Here the interest rate depends on last period's interest rate while gradually adjusting towards a target interest rate $\left(r_\pi \pi_t + r_y\left(y_t - y_t^p\right)\right)$ that depends on inflation and the gap between output and its potential level $\left(y_t - y_t^p\right)$. It also depends on the growth rate of this output gap.
- Potential output is defined as the level of output that would prevail if prices and wages were fully flexible. This means the model effectively needs to be "expanded" to add a "shadow" flexible-price economy.

Why So Many Bells And Whistles?

- Relative to the pure RBC or New Keynesian models we saw before, this model has lots of additional features:
  1. Adjustment costs for investment.
  2. Capacity utilisation costs.
  3. Habit persistence.
  4. Price indexation.
  5. Wage indexation.
  6. Lots of new autocorrelated distubance terms.
- These help the model to address the weaknesses of the previous models.
  1. Adjustment costs, utilisation costs and habit persistence all help to "throw sand in wheels" of the model, making variables more sluggish and giving random shocks a more long-lasting effect. This was a weakness of the RBC model.
  2. Indexation deals with the NK model's failure to match inflation persistence.
- Still, it is hard to argue these are really "micro-founded" mechanisms. In many ways, the model is quite ad hoc and hardly immune to the Lucas critique.

The Observable VAR System

$$
Y_t = \begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ lHOURS_t \\ dlP_t \\ FEDFUNDS_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \end{bmatrix}, \tag{15}
$$

Priors and Posteriors: Structural Parameters

TABLE 1A—PRIOR AND POSTERIOR DISTRIBUTION OF STRUCTURAL PARAMETERS

	Prior distribution			Posterior distribution			
	Distr.	Mean	St. Dev.	Mode	Mean	5 percent	95 percent
φ	Normal	4.00	1.50	5.48	5.74	3.97	7.42
σ_c	Normal	1.50	0.37	1.39	1.38	1.16	1.59
h	Beta	0.70	0.10	0.71	0.71	0.64	0.78
ξ_w	Beta	0.50	0.10	0.73	0.70	0.60	0.81
σ_l	Normal	2.00	0.75	1.92	1.83	0.91	2.78
ξ_p	Beta	0.50	0.10	0.65	0.66	0.56	0.74
ι_w	Beta	0.50	0.15	0.59	0.58	0.38	0.78
ι_p	Beta	0.50	0.15	0.22	0.24	0.10	0.38
ψ	Beta	0.50	0.15	0.54	0.54	0.36	0.72
Φ	Normal	1.25	0.12	1.61	1.60	1.48	1.73
r_π	Normal	1.50	0.25	2.03	2.04	1.74	2.33
ρ	Beta	0.75	0.10	0.81	0.81	0.77	0.85
r_y	Normal	0.12	0.05	0.08	0.08	0.05	0.12
$r_{\Delta y}$	Normal	0.12	0.05	0.22	0.22	0.18	0.27
$\bar{\pi}$	Gamma	0.62	0.10	0.81	0.78	0.61	0.96
$100(\beta^{-1} - 1)$	Gamma	0.25	0.10	0.16	0.16	0.07	0.26
$\bar{l}$	Normal	0.00	2.00	−0.1	0.53	−1.3	2.32
$\bar{\gamma}$	Normal	0.40	0.10	0.43	0.43	0.40	0.45
α	Normal	0.30	0.05	0.19	0.19	0.16	0.21

Priors and Posteriors: Shock Processes

TABLE 1B—PRIOR AND POSTERIOR DISTRIBUTION OF SHOCK PROCESSES

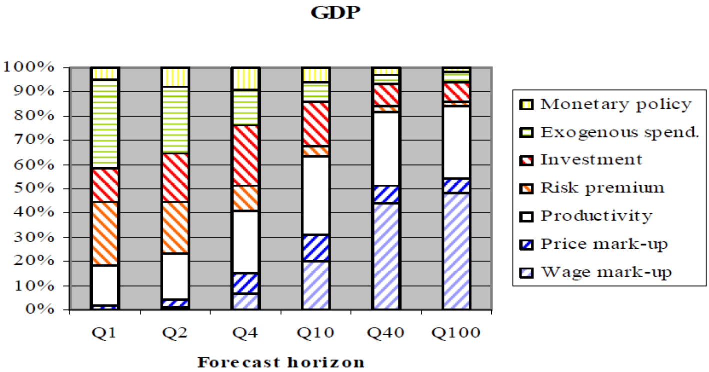
	Prior distribution			Posterior distribution			
	Distr.	Mean	St. Dev.	Mode	Mean	95 percent	5 percent
σ_a	Invgamma	0.10	2.00	0.45	0.45	0.41	0.50
σ_b	Invgamma	0.10	2.00	0.24	0.23	0.19	0.27
σ_g	Invgamma	0.10	2.00	0.52	0.53	0.48	0.58
σ_I	Invgamma	0.10	2.00	0.45	0.45	0.37	0.53
σ_r	Invgamma	0.10	2.00	0.24	0.24	0.22	0.27
σ_p	Invgamma	0.10	2.00	0.14	0.14	0.11	0.16
σ_w	Invgamma	0.10	2.00	0.24	0.24	0.20	0.28
ρ_a	Beta	0.50	0.20	0.95	0.95	0.94	0.97
ρ_b	Beta	0.50	0.20	0.18	0.22	0.07	0.36
ρ_g	Beta	0.50	0.20	0.97	0.97	0.96	0.99
ρ_I	Beta	0.50	0.20	0.71	0.71	0.61	0.80
ρ_r	Beta	0.50	0.20	0.12	0.15	0.04	0.24
ρ_p	Beta	0.50	0.20	0.90	0.89	0.80	0.96
ρ_w	Beta	0.50	0.20	0.97	0.96	0.94	0.99
μ_p	Beta	0.50	0.20	0.74	0.69	0.54	0.85
μ_w	Beta	0.50	0.20	0.88	0.84	0.75	0.93
ρ_{ga}	Beta	0.50	0.20	0.52	0.52	0.37	0.66

Out-of-Sample Forecasting Beats VAR Models

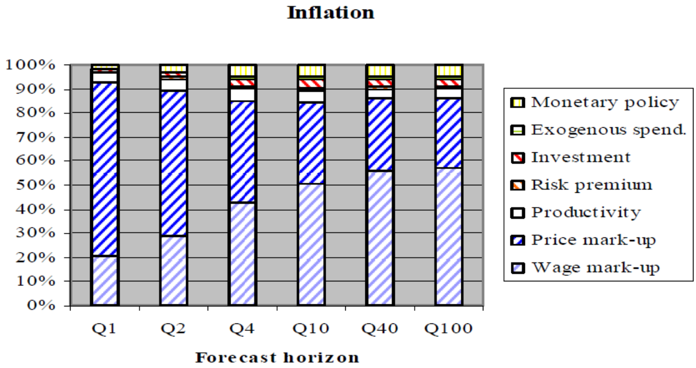
TABLE 3—OUT-OF-SAMPLE PREDICTION PERFORMANCE

	GDP	dP	Fedfunds	Hours	Wage	CONS	INV	Overall
VAR(1)	*RMSE-statistic for different forecast horizons*							
1q	0.60	0.25	0.10	0.46	0.64	0.60	1.62	−12.87
2q	0.94	0.27	0.18	0.78	1.02	0.95	2.96	−8.19
4q	1.64	0.34	0.36	1.45	1.67	1.54	5.67	−3.25
8q	2.40	0.53	0.64	2.13	2.88	2.27	8.91	1.47
12q	2.78	0.63	0.79	2.41	4.09	2.74	10.97	2.36
BVAR(4)	*Percentage gains (+) or losses (−) relative to VAR(1) model*							
1q	2.05	14.14	−1.37	−3.43	2.69	12.12	2.54	3.25
2q	−2.12	15.15	−16.38	−7.32	−0.29	10.07	2.42	0.17
4q	−7.21	31.42	−12.61	−8.58	−3.82	1.42	0.43	0.51
8q	−15.82	33.36	−13.26	−13.94	−8.98	−8.19	−11.58	−4.10
12q	−15.55	37.59	−13.56	−4.66	−15.87	−3.10	−23.49	−9.84
DSG	*Percentage gains (+) or losses (−) relative to VAR(1) model*							
1q	5.68	2.05	−8.24	0.68	5.99	20.16	9.22	3.06
2q	14.93	10.62	−17.22	10.34	6.20	25.85	16.79	2.82
4q	20.17	46.21	1.59	19.52	9.21	26.18	21.42	6.82
8q	22.55	68.15	28.33	22.34	15.72	21.82	25.95	11.50
12q	32.17	74.15	40.32	27.05	21.88	23.28	41.61	13.51

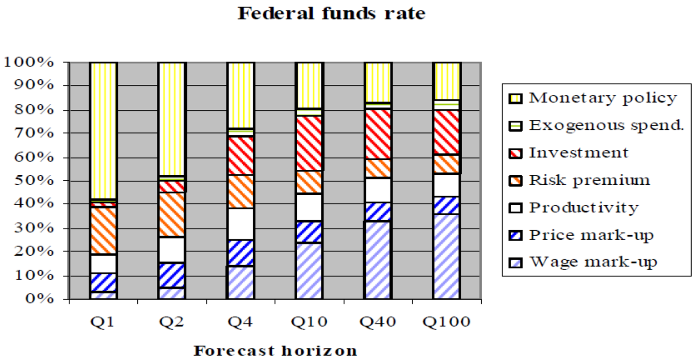
Explaining GDP Movements At Various Horizons



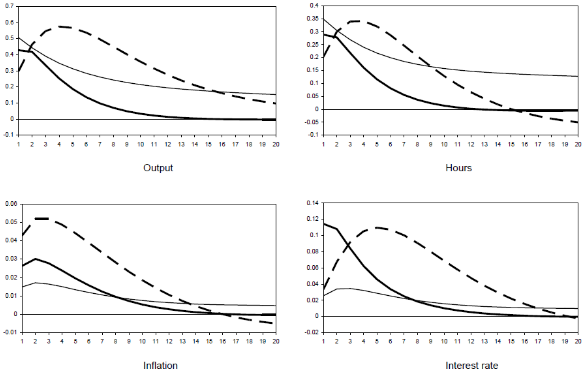
Explaining Inflation Movements At Various Horizons



Explaining Fed Funds Movements At Various Horizons

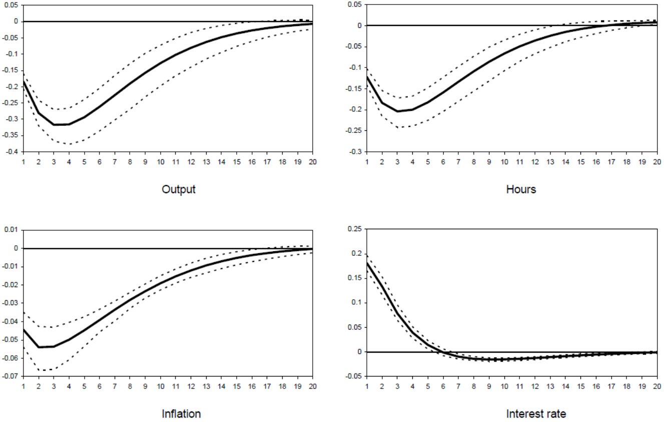


The Impact of Various "Demand" Shocks

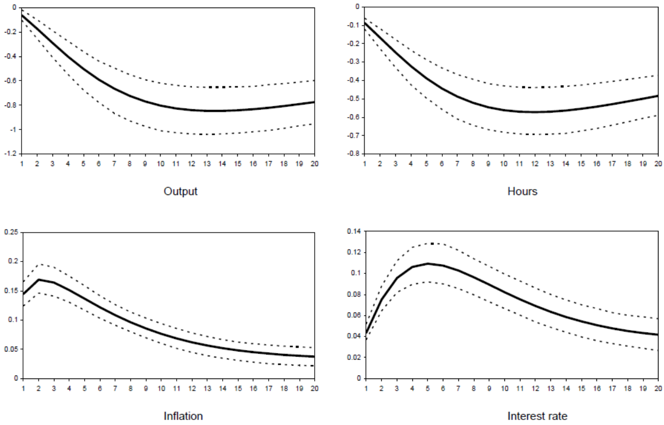


Notes: Bold solid line: risk premium shock; thin solid line: exogenous spending shock; dashed line: investment shock.

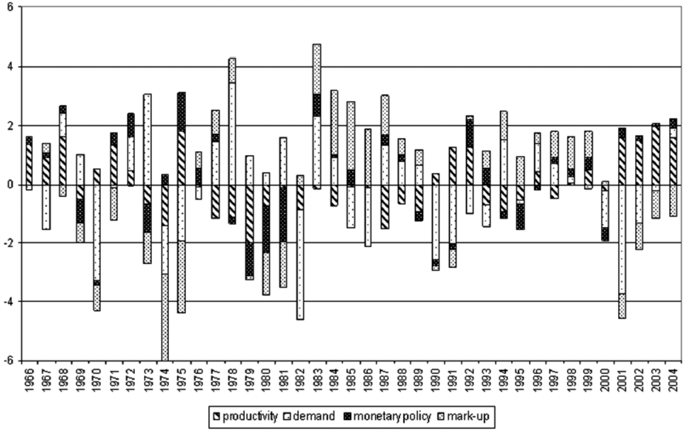
Impulse Response for a Monetary Policy Shock



Impulse Response for a Wage Mark-Up Shock



Decomposing the Growth Rate of GDP



Weaknesses and Strengths of DSGE Models

- You've seen enough now to have a good sense of what modern DSGE models look like and what they are used for.
- The following is a fair list of weaknesses of these models
  1. A large number of *ad hoc* economic mechanisms designed mainly to fit persistence properties of the data rather than because economists have a strong belief in these particular stories.
  2. A large amount of unexplained shocks which are often highly persistent.
  3. A minimal treatment of banking and financial markets (still true despite current ongoing work.)
  4. Very limited modelling of policy tools or details of national accounts.
  5. Plenty of evidence that pure rational expectations assumption is flawed.
  6. Claims that they are based on stable structural parameters and thus immune to the Lucas critique are silly.
- Still, there are a number of positive aspects that don't feature in VARs (imposition of budget constraints, a consistent story for how agents behave and a coherent handling of expectations) and these strengths may help DSGEs to be more useful for forecasting and "what if" analysis than VARs.

DSGE and the Crisis

- Some advocates of "new thinking" in economics have argued that DSGE models somehow played a key role in generating the global financial crisis. Is this fair?
- The idea that DSGE models represent a "laissez faire" approach to policy is sometimes put forward but is not really correct. New Keynesian models recommend systematic government intervention.
- For sure, the models did not feature banking or financial sectors but it is very unlikely that simple linearised models like these can capture the risk of low-probability and highly nonlinear disastrous events. Even the efforts being made to improve the financial sectors in these models are unlikely to make them useful as "crisis warning" tools.
- Economics will never be a "one tool for all tasks" business. All the major central banks had departments monitoring banking and financial market developments but failed to see the risks to the global economy. DSGE modellers cannot be held responsible for all failings.
- Chris Sims's INET lecture on DSGE models on the website (video and slides) is a "fair and balanced" assessement of DSGE.