MA Advanced Macroeconomics: 4. VARs With Long-Run Restrictions

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An Alternative Approach: Long-Run Restrictions

- The identifying assumptions in the recursive VAR approach require knowledge of how certain variables react in an instantaneous way to certain shocks.
- Sometimes, because certain variables are "sluggish" or because information about some variables is only available with a lag, we can be pretty confident about these restrictions. But often they are pure guesswork.
- And economic theory gives very little guidance.
- In fact, economic theory usually tells us a lot more about what will happen in the longer-run, rather than exactly what will happen today.
- For instance, theory tells us that whatever positive aggregate demand shocks do in the short-run, in the long-run they have no effect on output and a positive effect on the price level.
- This suggests an alternative approach: Use these theoretically-inspired long-run restrictions to identify shocks and impulse responses.
- I will explain one of these methods over the next four slides. The general idea is more important than the technical details.

Information in the Reduced-Form Covariance Matrix

- Consider the VAR model

$$Z_t = BZ_{t-1} + C\epsilon_t$$

where the covariance matrix of the structural shocks is

$$E(\epsilon_t \epsilon_t') = \begin{pmatrix} E(\epsilon_1^2) & E(\epsilon_1 \epsilon_2) \\ E(\epsilon_1 \epsilon_2) & E(\epsilon_2^2) \end{pmatrix} = I$$

so the structural shocks are uncorrelated and have unit variance (this is just a harmless normalization).

- Note that the covariance matrix of the observed reduced-form errors is

$$\Sigma = E(e_t e_t') = E\{(C\epsilon_t)(C\epsilon_t)'\} = CE(\epsilon_t \epsilon_t')C' = CC'$$

- Thus, the observed covariance structure of the reduced-form shocks tells us something about how they are related to the uncorrelated, unit-variance, structural shocks.

Calculating Long-Run Effects in an SVAR

- Suppose $Z_t = (\Delta y_t, \Delta x_t)'$
- Then the long-run effect of the shock on y_t is the *sum* of its effects of Δy_t, Δy_{t+1}, Δy_{t+2} and so on.
- The long-run effect is the sum of the impulse responses.
- The impulse responses for the model $Z_t = BZ_{t-1} + C\epsilon_t$ are
  1. C in impact period.
  2. BC after one period.
  3. B^2C after two periods, ... B^nC after n periods.
- Long-run level effects are $D = \left(I + B + B^2 + B^3 + ...\right) C$.
- If eigenvalues of B are inside unit circle then $I + B + B^2 + B^3 + ... = (I - B)^{-1}$.
- This is the matrix equivalent of the multiplier formula $1 + c + c^2 + c^3 + ... = \frac{1}{1-c}$.
- So the long-run responses are $D = (I - B)^{-1} C$.

The Blanchard-Quah Method: I

- Now note that $DD' = (I - B)^{-1} CC' \left((I - B)^{-1}\right)'$
- But two slides ago, we established that $CC' = \Sigma$, the covariance matrix of the reduced-form shocks, which can be estimated.
- So $DD' = (I - B)^{-1} \Sigma \left((I - B)^{-1}\right)'$
- Now make a restriction about the long-run effects described in D: Assume that D is lower triangular: Only the first shock has a long-run effect on the first variable, and only the first and second shocks have long-run effects on the second variable and so on.
- In the two variable case, this is just

$$D = \begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{22} \end{pmatrix}$$

The Blanchard-Quah Method: II

- $DD' = (I - B)^{-1} \Sigma \left((I - B)^{-1} \right)'$ is a symmetric matrix (the i, j entry is identical to the j, i entry)
- All symmetric matrices have a unique lower-diagonal matrix D so that DD' equals the symmetric matrix. This is known as the *Cholesky factor* of the symmetric matrix.
- D can be calculated directly in RATS and other software as the Cholesky factor of the known matrix $(I - B)^{-1} \Sigma \left((I - B)^{-1} \right)'$.
- Now remember that $D = (I - B)^{-1} C$.
- So the crucial matrix C defining the structural shocks can then be calculated as

$$C = (I - B)D$$

- Now, we can calculate the impulse response functions to the structural shocks.

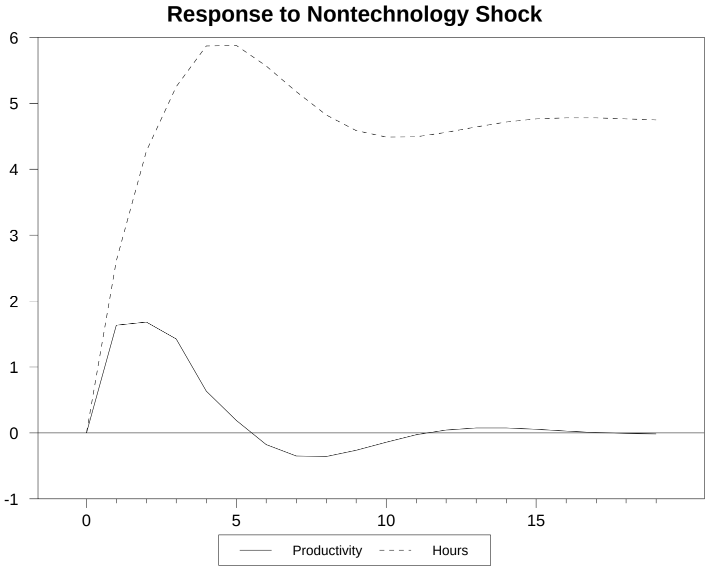
Blanchard-Quah: Identifying Supply and Demand Shocks

- Blanchard-Quah (1989) used a two-variable VAR in the log-difference in GDP Δy_t and the unemployment rate U_t.
- Unemployment was entered in *levels* form. Because the VAR is estimated to be stationary (eigenvalues inside unit circle) both structural shocks have zero long-run effect on the unemployment rate.
- The lower-diagonal assumption thus meant that of the two structural shocks only one of them could have a long-run effect on the level of output. BQ labelled this the "supply" shock while the shock that has no effect on long-run output was labelled the "demand" shock.
- The relative importance of supply versus demand shocks in determining output is a long-running theme in macroeconomics. Keynesians emphasize the importance of demand shocks while more classically-oriented economists, such as advocates of the Real Business Cycle approach, see supply shocks as being more important.
- BQ's results implied that demand shocks were responsible for the vast majority of short-run fluctuations.

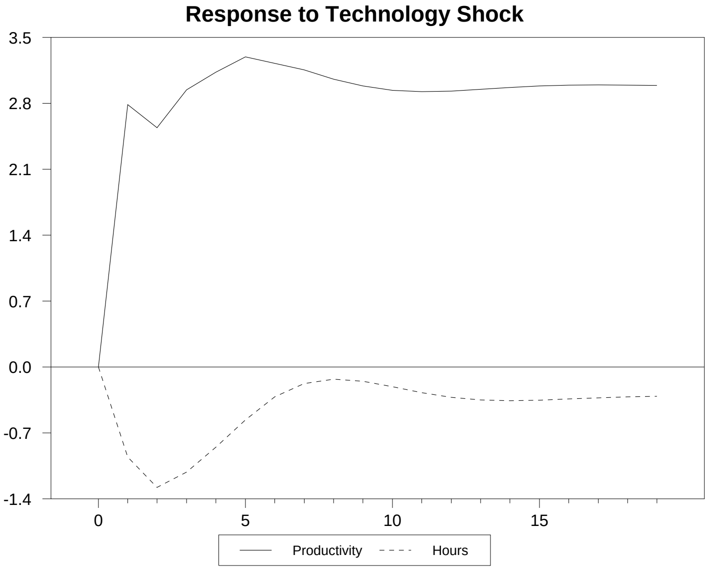
Galí (1999): Technology Shocks and Hours Worked

- BQ's formulation is a little bit restrictive: The assumption that neither supply or demand shocks can change the unemployment rate in the long-run may not be correct.
- Galí's paper applied a similar analysis to BQ, but for a formulation that moved a bit closer to the debate about RBC models and their predictions for the labour market,
- RBC models assume technology shocks drive the business cycle and explain why hours worked are higher in booms than in recessions: Better to work when you are productive than unproductive.
- Galí VAR: Log-difference of output per hour worked (labour productivity), Δz_t and the log-difference of hours worked, Δn_t.
- The lower-diagonal assumption about long-run responses now means that the supply shock (now called the "technology" shock) can affect productivity in the long-run, while the non-techology shock cannot.
- The model lets the data dictate the long-run effects of technology and non-technology shocks on hours worked.

Replicating Galí (1999)



Replicating Galí (1999)



Interpreting Galí's Results

- Nontechnology shocks cause both output and productivity to rise in the short-run.
- Evidence that short-run cyclical movements in productivity are not just due to technology shocks.
- Explanation? Costs of adjusting labour input. Rather than hire new labour in a boom, temporarily work existing labour a bit harder. In recession, employed labour is more likely to be under-utilized.
- Also shows nontechnology shocks raising labour input. Interpretation unclear. Some have specified this VAR with some stationary transformation of labour input, so shocks have no long-run effects.
- Technology shocks cause productivity to go up but hours to go down.
- Interpretation: Short-run output is demand-driven not supply-driven. More efficiency means that demanded output can be supplied with less labour.
- Bad news for technology-driven stories of the business cycle such as RBCs. These results have generated some controversy but I believe they are correct. See my own small contribution to this debate which is posted on the website.