MA Advanced Macroeconomics:
9. The Modern New-Keynesian Model

Karl Whelan

School of Economics, UCD

Spring 2016

Part I

Introducing The New-Keynesian Model

An Agenda for New-Keynesians

- Previously, we discussed critiques in the 1970s of Keynesian ideas from economists who favoured the use rational expectations as a modelling device.
- New Keynesians addressed the critiques by developing models in which people had behaved optimally and had rational expectations but monetary policy could still have systematic effects.
- Many different mechanisms invoked.
- But most common was sticky prices. If prices didn't jump in line with money, then central bank can control real money supply and real interest rates.
- 1980s New Keynesianism: Nice small "toy" models that made important theoretical points. Mankiw-Romer collection has most of the key articles.
- 1990s: Important breakthrough—NK models imply a version of the Phillips curve that looks quite like the traditional one.
- Also: Lots of evidence backing up the idea of sticky prices.
- An industry is born: Policy analysis with "realistic" and "micro-founded" New Keynesian models.

Starting Point for the Standard Model: Dixit-Stiglitz

- The model has no capital, only consumption goods.
- Consumers maximize utility function $U(Y_t)$ over an aggregate of a continuum of differentiated goods

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

- Implies demand functions for the differentiated goods of form

$$Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta}$$

- Where P_t is the aggregate price index defined by

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$$

Simple Price Rigidity: The Calvo Model

- Each period, only a random fraction $(1 - \alpha)$ of firms are able to reset their price; all other firms keep their prices unchanged.
- Apart from the different timing of when they set prices, the firms are completely symmetric. So all firms setting new prices today set the same price.
- This means the price level can be re-written as

$$P_t = \left[(1-\alpha) X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

  where X_t is the price that those resetting today have chosen.
- This can be re-written as

$$P_t^{1-\theta} = (1-\alpha) X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}$$

Optimal Pricing in the Calvo Model

- When firms do get to reset their price, they must take into account that the price may be fixed for many periods.
- Technically, they are picking their price to maximize

$$E_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k \left(Y_{t+k} P_{t+k}^{\theta-1} X_t^{1-\theta} - P_{t+k}^{-1} C \left(Y_{t+k} P_{t+k}^{\theta} X_t^{-\theta} \right) \right) \right]$$

  where $C(.)$ is the nominal cost function.
- Solution is

$$X_t = \frac{\theta}{\theta-1} \frac{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} MC_{t+k} \right)}{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} \right)}.$$

- Price as a markup over a weighted average of current and future marginal costs. (Without frictions, firms would set price as $X_t = \frac{\theta}{\theta-1} MC_t$.)

Log-Linearization

- We have two nonlinear equations:

$$P_t^{1-\theta} = (1-\alpha) X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}$$

$$X_t = \frac{\theta}{\theta - 1} \frac{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} MC_{t+k} \right)}{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} \right)}.$$

- Not easy to solve or simulate. Instead, we use log-linearized approximations (taken around a constant growth, zero inflation path—derivations provided in a separate handout):

$$p_t = (1-\alpha) x_t + \alpha p_{t-1}$$

$$x_t = (1-\alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^k E_t mc_{t+k}$$

Deriving the NKPC

- Remembering that this the solution to a first-order SDE, we can "reverse engineer" that the formula for the optimal reset price can also be written as

$$x_t = (1 - \alpha\beta)\, mc_t + (\alpha\beta)\, E_t x_{t+1}$$

- Combining this with

$$p_t = (1 - \alpha)\, x_t + \alpha p_{t-1}$$

- And doing a bunch of re-arranging (see the extra handout) we get

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} (mc_t - p_t)$$

- Inflation is a function of expected inflation and the ratio of marginal cost to price (i.e. real marginal cost). This relationship is known as the **New-Keynesian Phillips Curve**.

Output and the NKPC

- Assume standard diminishing returns to labour production function: Higher output reduces marginal productivity and raises marginal cost.
- This makes real marginal cost a function of the output gap

$$mc_t - p_t = \eta x_t$$

where

$$x_t = y_t - y_t^n$$

where y_t^n is the path of output that would have obtain in a zero inflation, no pricing frictions economy.

- This implies an NKPC of the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

Implications of the NKPC

- NKPC *looks* a lot like a traditional expectations-augmented Phillips curve.
- It's got expectations, it's got output. And it's micro-founded. What's not to like?
- Be careful, however. It has some very different implications.
- It's a first-order stochastic difference equation. This means it has a solution of the form

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t x_{t+k}.$$

- Inflation has no backward-looking element: There is no "intrinsic" inertia in inflation. Lagged inflation effects in conventional models are a statistical artefact.

Cost-Push Shocks

- The NKPC has no "error" or "shock term.
- But one can think of many sources of price movements not consistent with this formulation.
- And there may be firm-specific shocks to marginal cost.
- For instance, firms may differ randomly in the markup they wish to charge.
- For this reason, the literature often adds a "cost-push" shock to the NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

- Makes the central bank's stabilization problem more interesting: Cannot stabilize inflation just by stabilizing the output gap.

Relating Output to Monetary Policy

- This is the first of 3 equations in the canonical New Keynesian model.
- It links inflation to output.
- The next step is to link output to monetary policy.
- The NK model does this via a link between output and interest rates.
- Recall that the basic model has no capital, so output = consumption.
- The consumption-interest rate relationship comes from a standard consumer intertemporal optimization problem.
- Here I'll sketch out that problem and how it gets used in the New Keynesian Framework.

Optimal Consumption Problem

- Consumer wants to maximize

$$\sum_{k=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{k} U\left(C_{t+k}\right)$$

- Subject to intertemporal budget constraint

$$\sum_{k=0}^{\infty}\frac{E_t C_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m}\right)} = A_t + \sum_{k=0}^{\infty}\frac{E_t Y_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m}\right)}$$

- Lagrangian:

$$\begin{aligned}\mathcal{L} &= \sum_{k=0}^{\infty}\left(\frac{1}{1+\beta}\right)^{k} U\left(C_{t+k}\right) + \\ &\quad +\lambda\left[A_t + \sum_{k=0}^{\infty}\frac{E_t Y_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m}\right)} - \sum_{k=0}^{\infty}\frac{E_t C_{t+k}}{\left(\prod_{m=1}^{k+1} R_{t+m}\right)}\right]\end{aligned}$$

Euler Equation

- Combine first-order conditions for C_t and C_{t+1} to get

$$U'\left(C_t\right) = E_t\left[\left(\frac{R_{t+1}}{1+\beta}\right)U'\left(C_{t+1}\right)\right]$$

- Set $U(C_t) = U(Y_t) = \frac{Y_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$ and this becomes

$$E_t\left[\left(\frac{R_{t+1}}{1+\beta}\right)\left(\frac{Y_t}{Y_{t+1}}\right)^{\frac{1}{\sigma}}\right] = 1$$

- Can be log-linearized as

$$y_t = E_t y_{t+1} - \sigma\left(i_t - E_t \pi_{t+1} - \rho\right)$$

where $\rho = -\log\beta$.

- Output today depends negatively on the real interest rate.

The Natural Rate of Interest

- Our inflation equation featured an output gap $x_t = y_t - y_t^n$.
- We can re-write the Euler equation as

$$x_t = E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} - \rho \right) + E_t y_{t+1}^n - y_t^n$$

- Or, more naturally, as

$$x_t = E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} - r_t^n \right)$$

where

$$r_t^n = \sigma^{-1} E_t \Delta y_{t+1}^n - \log \beta$$

- Defines a "natural' real interest rate, r_t^n (consistent with $x_t = E_t x_{t+1}$) determined by technology and preferences (would also depend on government spending if we had added government.)

Output is Also Forward-Looking

- Note that the output also follows a first-order stochastic difference equation.
- This has solution

$$x_t = \sigma \sum_{k=0}^{\infty} \left(i_{t+k} - E_t \pi_{t+k+1} - r^n_{t+k} \right)$$

- No backward-looking element: Output also has no intrinsic persistence.
- Lesson for monetary policy: What matters for output is not just policy today but the whole future stream of interest rates.
- Central bankers need to be very careful in managing expectations about future policy. In fact, this (and not today's interest rate) is their key tool.
- Interpreting i_t (correctly) as the short-term interest rate, and assuming that the expectations theory of the term structure holds, this model says that it is long-term interest rates that matter for spending.

The Canonical New-Keynesian Model

- Most New Keynesian macro takes as its starting point a three equation model.
  1. New Keynesian Phillips curve
  $$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$
  2. Euler equation for output
  $$x_t = E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} - r_t^n \right)$$
  3. And an equation describing how interest rate policy is set, usually described as an explicit interest rate rule.
- We now move on to looking at what form this interest rate rule might take.

Part II

Monetary Policy in The New-Keynesian Model

The Joint Behaviour of Inflation and Output

- Before discussing monetary policy rules, let's have a quick examination of the joint dynamics of output and inflation in this model.
- The output equation is

$$x_t = E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} - r_t^n \right)$$

- The inflation equation is

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

- This can be re-written as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa E_t x_{t+1} - \kappa \sigma \left(i_t - E_t \pi_{t+1} - r_t^n \right) + u_t$$

- We can gather together the inflation and output equations in vector form to write the NK model as

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & \sigma \\ \kappa & \beta + \kappa\sigma \end{pmatrix} \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} + \begin{pmatrix} \sigma \left(r_t^n - i_t \right) \\ \kappa\sigma \left(r_t^n - i_t \right) + u_t \end{pmatrix}$$

Eigenvalues of A

- Recall from earlier that for models of the form $Z_t = AE_t Z_{t+1} + BV_t$ to have a unique stable solution, we needed all the eigenvalues of A to be less than one.
- In this case, we have

$$A = \begin{pmatrix} 1 & \sigma \\ \kappa & \beta + \kappa\sigma \end{pmatrix}$$

- The eigenvalues satisfy

$$P(\lambda) = (1 - \lambda)(\beta + \kappa\sigma - \lambda) - \kappa\sigma = 0$$

- This can be re-arranged to read

$$P(\lambda) = \lambda^2 - (1 + \beta + \kappa\sigma)\lambda + \beta = 0$$

- $P(\lambda)$ is a U-shaped polynomial. We can show that $P(0) = \beta > 0$, $P(1) = -\kappa\sigma < 0$ and that $P(\lambda)$ greater than zero again as λ rises above one.
- Together, this means one eigenvalue is between zero and one and the other is greater than one.

No Unique Stable Solution

- This seems like a pretty serious problem for the model: In general, there is no unique stable solution. The model turns out to have multiple equilibria and there is nothing to determine which of the equilibria gets chosen?
- How to deal with this? One way is to accept that there are multiple equilibria and to analyse the impact of interest rate changes on output and inflation across a range of different possible equilibria.
- John Cochrane's recent paper "Do Higher Interest Rates Raise or Lower Inflation?" does this and reaches the conclusion (surprising to some) that higher interest rates lead to higher inflation in the NK model. This has lead to a debate about the so-called neo-Fisherite predictions of the New Keynesian model. We won't have time to get into detail on this.
- An alternative approach is to specify that monetary policy follows a particular rule and that the rule is designed to produce a unique stable equilibrium. This is the approach taken in the conventional New Keynesian literature.

A Taylor-Type Rule

- What might a good monetary policy look like?
- Let's start with a rule similar to the one proposed by John Taylor and which has received a huge amount of attention in the monetary policy literature:

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_x x_t$$

- Monetary policy "leans against" inflation and output gaps by raising the interest rate when these increase.
- "Similar" rather than identical because we are allowing the interest rate to move with the natural rate, whereas Taylor's rule has a constant intercept.
- Output equation becomes

$$x_t = E_t x_{t+1} + \sigma E_t \pi_{t+1} - \sigma \phi_\pi \pi_t - \sigma \phi_x x_t$$

- This can be combined with the NKPC to produce a system of first-order stochastic difference equations.

Dynamics under a Taylor Rule

- Let $Z_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}$ and $V_t = \begin{pmatrix} 0 \\ u_t \end{pmatrix}$
- Under this Taylor rule, the economy can be described by a system of the form

$$Z_t = AE_t Z_{t+1} + BV_t$$

where

$$\begin{aligned} A &= \frac{1}{1+\sigma\phi_x+\kappa\sigma\phi_\pi} \begin{pmatrix} 1 & \sigma\left(1-\beta\phi_\pi\right) \\ \kappa & \beta+\sigma\kappa+\beta\left(1+\sigma\phi_x\right) \end{pmatrix} \\ B &= \frac{1}{1+\sigma\phi_x+\kappa\sigma\phi_\pi} \begin{pmatrix} 1 & -\sigma\phi_\pi \\ \kappa & 1+\sigma\phi_x \end{pmatrix} \end{aligned}$$

- This system is a matrix version of the first-order stochastic difference equations and, under certain conditions, it can be solved in a similar fashion to give

$$Z_t = \sum_{k=0}^{\infty} A^k B E_t V_{t+k}$$

Uniqueness and Stability Conditions

- For the model to have a unique stable equilibrium, we need both of the eigenvalues of A to be less than one in absolute value.
- I won't go through calculating the eigenvalues of the A matrix.
- However, it can be shown that both eigenvalues of A are inside unit circle if

$$\phi_\pi + \frac{(1-\beta)\,\phi_x}{\kappa} > 1$$

- Provided the policy rule satisifies this requirement, we get a unique stable equilibrium.

The Taylor Principle

- Interpretation of stability condition:

$$\phi_\pi + \frac{(1-\beta)\,\phi_x}{\kappa} > 1$$

- Quick interpretation: $\beta \approx 1$, so the condition is approximately $\phi_\pi > 1$.
- Nominal interest rates must rise by more than inflation, so real rates rise in response to an increase in inflation.
- Advocated by John Taylor: Now known as the Taylor Principle.
- Why is this needed for stability? Otherwise, inflationary shocks reduce real interest rates, stimulates the economy, and this further stimulates inflation.
- Full interpretation. NKPC implies that in the long-run

$$x_t = \frac{(1-\beta)}{\kappa}\pi_t$$

- Long-run response to inflationary shock

$$\Delta i = \phi_\pi \Delta\pi + \phi_x \Delta x = \left(\phi_\pi + \frac{(1-\beta)\,\phi_x}{\kappa}\right)\Delta\pi$$

Evidence on Monetary Policy Rules

- Clarida, Gali and Gertler (QJE, 2000) and others have argued that the Fed's monetary policy violated the Taylor Principle during the period prior to the appointment of Paul Volcker.
- Estimates from the Volcker-Greenspan era show estimates of θ_π well in excess of one.
- Thus, it has been argued that during the 1960s and 1970s, the Fed was not pursuing stabilizing monetary policy.
- This lack of stabilization may have contributed to macroeconomic instability and the Great Inflation.
- Some arguments about this: Former Fed economist, Athanasios Orphanides argued that if one uses real time data and real time estimates of the output gap, then the Fed thought it was pursuing a policy consistent with $\phi_\pi > 1$.

Part III

Optimal Monetary Policy

Quadratic Loss Function Framework

- How do we think about what is “optimal” for a central bank to do?
- Clearly, central banks don’t like inflation. They would also like to keep output on a steady path close to potential.
- For a long time, economists have formulated central banks as behaving in a way that minimizes a “loss function” something like

$$L_t = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t E_t \left(\pi_{t+k}^2 + \lambda x_{t+k}^2 \right)$$

  where, as before x_t is the output gap and λ indicates the weight put on output stabilization relative to inflation stabilization.
- Economists like quadratic loss functions: When you differentiate things to the power of 2, they give you equations with things to the power of one, i.e. linear relationships.
- Traditionally, though, the quadratic loss function was purely ad hoc.

Woodford's Rationale for the Quadratic Loss Function

- Michael Woodford has shown that one can use the formula

$$L_t = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t E_t\left(\pi_{t+k}^2 + \lambda x_{t+k}^2\right)$$

  as a quadratic approximation to consumer utility in the standard NK model.

- He shows that the correct value is $\lambda = \frac{\kappa}{\theta}$ (κ is coefficient on output gap in NKPC, θ is elasticity of demand for firms.)

- Rationale for the two terms:

  1. x_t^2 term: Risk-averse consumers prefer smooth consumption paths. Keeping output close to its natural rate achieves this.
  2. π_t^2 term: Consumers don't just care about the level of consumption but also its allocation. With inflation, sticky prices implies different prices for the symmetric goods and thus different consumption levels. Optimality requires equal consumption of all items in the bundle. Rationale for welfare effect of inflation, independent of its effect on output (though perhaps you can think of other, better, explanations for a negative effect of inflation on welfare.)

Optimal Policy Under Commitment: Solution

- Suppose that the central bank could commit today to a (state-contingent) strategy that it can adopt now and in the future.
- Lagrangian is

$$\mathcal{L} = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t E_t \left[\pi_{t+k}^2 + \lambda x_{t+k}^2 + 2\mu_{t+k} \left(\pi_{t+k} - \beta \pi_{t+k+1} - \kappa x_{t+k} \right) \right]$$

- First-order conditions:

$$\begin{aligned} \lambda E_t x_{t+k} - \kappa E_t \mu_{t+k} &= 0 \\ E_t \pi_{t+k} + E_t \mu_t - E_t \mu_{t+k-1} &= 0 \end{aligned}$$

  for $t = 0, 1, 2, ...$ where $\mu_{-1} = 0$ (The problem does not contain a time $t = -1$ constraint).
- We have $E_t x_{t+k} = \frac{\kappa}{\lambda} E_t \mu_{t+k} = \theta E_t \mu_{t+k}$.
- We also have $E_t \pi_{t+k} = E_t \mu_{t+k-1} - E_t \mu_{t+k} = -\frac{1}{\theta} E_t \Delta x_{t+k} \Rightarrow \Delta E_t x_{t+k} = -\theta E_t \pi_{t+k}$.

Optimal Policy Under Commitment: Characterization

- This means optimal policy will be characterized by

$$
\begin{aligned}
x_t &= -\theta \pi_t = \theta \left(p_{t-1} - p_t \right) \\
E_t \Delta x_{t+1} &= -\theta E_t \pi_{t+k} = \theta \left(p_{t+k-1} - p_{t+k} \right)
\end{aligned}
$$

- So, given some initial price level p_{-1}, we get

$$E_t x_{t+k} = \theta \left(p_{-1} - E_t p_{t+k} \right)$$

  because $\pi_t = p_t - p_{t-1}$.

- Optimal policy is set to "lean against the price level."
- Shocks temporarily affect the price level but have no cumulative effect. On average, inflation is zero.
- Note that this policy is history dependent: Policy today depends on the whole past sequence of shocks that have determined today's price level, not just today's shocks.

Optimal Policy Under Discretion

- Suppose that a central bank cannot commit to taking a particular course of action in the future. Instead, all they can do is adopt the optimal strategy for what to do today, and then tomorrow adopt the optimal strategy for what to do tomorrow when it arrives, and so on.
- What difference does this make?
- Recall that the optimality conditions for periods t and $t+1$ were

$$\begin{aligned} x_t &= -\theta \pi_t \\ E_t x_t - E_t x_{t+1} &= -\theta \pi_{t+1} \end{aligned}$$

- So the conditions for the first period are different from the rest. At time t, the previous period, time $t-1$, is gone and doesn't matter now. But we do take into account the effect that time t decisions have at time $t+1$.
- With discretion, the policy makers wake up every day and solve the optimal problem again with all the time subscripts pushed forward. So at time $t+1$ the optimal policy for x_{t+1} is the same as the optimal policy for previously implemented for x_t in the problem we have solved.
- So, under discretion, the policy-maker always sets $x_t = -\theta \pi_t$.

Inflation Under Optimal Discretionary Policy

- Policy implies "leaning against inflation": $x_t = -\theta \pi_t$.
- Inflation can be characterized as

$$\pi_t = \beta E_t \pi_{t+1} - \kappa \theta \pi_t + u_t$$

- New first-order difference equation

$$\pi_t = \left(\frac{1}{1+\theta\kappa} \right) \left(\beta E_t \pi_{t+1} + u_t \right)$$

- Repeated iteration solution:

$$\pi_t = \left(\frac{1}{1+\theta\kappa} \right) \sum_{k=0}^{\infty} \left(\frac{\beta}{1+\theta\kappa} \right)^k E_t u_{t+k}$$

Optimal Policy Under Discretion: AR(1) Shocks

- Often assumed that cost-push shocks are AR(1):

$$u_t = \rho u_{t-1} + v_t$$

where v_t are iid with mean zero.

- This implies that $E_t u_{t+k} = \rho^k u_t$.
- Inflation now becomes

$$\pi_t = \left(\frac{1}{1+\theta\kappa}\right)\left[\sum_{k=0}^{\infty}\left(\frac{\beta\rho}{1+\theta\kappa}\right)^k\right] u_t$$

- Use $\sum_{k=0}^{\infty} c^k = \frac{1}{1-c}$ for $|c| < 1$ to give

$$\pi_t = \left(\frac{1}{1+\theta\kappa}\right)\left(\frac{1}{1-\frac{\beta\rho}{1+\theta\kappa}}\right) u_t = \frac{u_t}{1+\theta\kappa-\beta\rho}$$

Optimal Policy Under Discretion: Interest Rate Rule

- AR(1) cost-push shock thus also implies that $E_t x_{t+1} = \rho x_t$ and $E_t \pi_{t+1} = \rho \pi_t$.
- Can substitute these and $x_t = -\theta \pi_t$ into the Euler equation

$$x_t = E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} - r_t^n \right)$$

  to back out what the optimal interest rate rule looks like.
- Get a rule of the form

$$i_t = r_t^n + \left(\rho + \frac{(1-\rho)\,\theta}{\sigma} \right) \pi_t$$

- Will be greater than one if $\frac{\theta}{\sigma} > 1$ which will hold for all reasonable parameterizations. Satisfies Taylor Principle.
- Note that inflation and thus interest rates do not depend at all on what happened in the past.

Comparing Policy Under Committment and Discretion

- It can show that committment policy produces a superior welfare outcome to discretionary policy.
- Woodford (2003): "Optimal policy is history dependent ... because the *anticipation* by the private sector that future policy will be different as a result of conditions at date t—even if those conditions no longer matter for the set of possible paths for the target variables at the later date—can improve stabilization outcomes at date t.
- About a transitory cost-push shock u_t: "If the transitory disturbance is expected to have no effect on the conduct of policy in later periods ... then the short-run trade-off between inflation and the output gap at period t is shifted vertically by u_t, requiring the central bank to choose between an increase in inflation, a negative output gap, or some of each. If instead, the central bank is expected to pursue a tighter policy in period $t+1$ and later ... then the short run tradeoff is shifted by the total change in $u_t + E_t\pi_{t+1}$, which is smaller. Hence greater stabilization is possible."
- But there may be problems with implementing this policy and sticking to it.

Other Topics in the Literature

- Unobservability of the natural rate.
- Arguments for interest rate smoothing.
- The zero-bound problem.
- Other (more realistic) types of price rigidity.
- Introducing capital.
- Wage as well as price rigidity.
- Robustness of conclusions to model uncertainty.
- More recently: Introducing financial frictions so there is a wedge between the policy rate and the interest rate in the Euler equation.

Part IV

Empirical Problems for the Model

The NKPC

- The NKPC is perhaps the central relationship in the modern approach to monetary policy analysis (The Euler equation was known long before the NKPC became popular in the 1990s).
- Despite this success, there are some well known problems with it as an *empirical* model of inflation.
- A practical problem is how to measure x_t, the gap between output and its natural rate.
- A reasonable first approach: Assume that, on average, output tends to return to its natural rate, so the natural rate can be proxied by a simple trend (as measured, for instance, by the HP Filter.) Proxy for $x = y_t - y_t^n$ with $\tilde{y}_t = y_t - y_t^{tr}$.
- Can now estimate the NKPC with data:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t$$

- Can't observe $E_t \pi_{t+1}$. Substitute realized π_{t+1} and use IV to deal with the fact that this is a noisy estimator of what we really want (classical measurement error).

The "Wrong" Sign!

- When we estimate

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t$$

  the sign of κ usually comes out negative.

- This is shocking to some but actually not so surprising once you work through it.
- We already know the "accelerationist" fact that $\Delta\pi_t$ is negatively correlated with the unemployment rate. This means it is positively correlated with the output gap.
- Because $\beta \approx 1$, we can proxy for $\pi_t - \beta E_t \pi_{t+1}$ with $\pi_t - \pi_{t+1} = -\Delta\pi_{t+1}$.
- Looked at this way, it's not too surprising that the estimated output gap coefficient is negative.
- Another reminder that, despite their apparent similarity, the new and older Phillips curves are very different.

Labour Share to the Rescue

- Two possible responses to this failure: Either the model is wrong or the output gap measure is wrong.
- In a famous paper, Gali and Gertler (1999) argued the latter. They suggest that deterministic trends do a bad job in capturing movements in the natural rate of output and suggest an alternative approach.
- Remember that the "correct" variable driving inflation is the ratio of marginal cost to the price level. GG argue for proxying marginal cost, with unit labour costs ($\frac{W_t L_t}{Y_t}$) so that the proxy for real marginal cost is the labour share of income ($S_t = \frac{W_t L_t}{P_t Y_t}$).
- Estimating

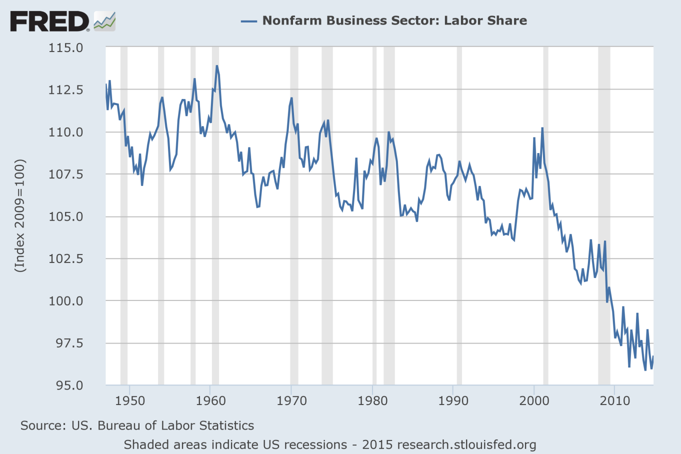
  $$\pi_t = \beta E_t \pi_{t+1} + \gamma s_t$$

  find a positive γ.
- Very popular widely-cited result: Seen as putting the NKPC back on a sound empirical footing.

Labour Share to the Rescue?

- Personally, I have never been convinced by this result.
- Rudd and Whelan (*JMCB*, 2007) show that updating GG's estimates, the estimated labour share coefficient is no longer statistically significant.
- Also, real marginal cost should be *procyclical*, rising when output is above potential. Lots of reasons for this: Overtime premia, bottlenecks.
- Labour's share, however, has generally moved in *countercyclical* fashion—it has generally spiked upwards in recessions
- Maybe output is actually above potential during recessions (negative technology shocks) but this seems unlikely.
- And in many countries there has been a downward trend in the labour share. This is now evident in the US data for the period after the GG study.
- Naive detrending methods may have problems but they seem to give a better proxy for output's deviation from potential than the labour share.

The US Labour Share of Income



The Persistence Problem

- If the NKPC does work well with the labour share or some other measure of real marginal cost, then it is completely forward-looking:

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t s_{t+k}.$$

- We can use VARs to forecast s_{t+k}, and use these forecasts to give a fitted value for the equation above. In fact, the fits are pretty terrible.
- Add lagged inflation

$$\pi_t = \rho \pi_{t-1} + \gamma \sum_{k=0}^{\infty} \beta^k E_t s_{t+k}.$$

and there is a big jump in fit.

- In fact, the lags seem to provide almost all the fit: The forward-looking terms are not significant (Rudd-Whelan, *AER* 2006).

The "Hybrid" New-Keynesian Phillips Curve

- Even proponents of NKPC concede that it fails to capture inflation's dependence on its own lags.
- Some have proposed a "hybrid" variant:

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \kappa x_t,$$

  where x_t is a measure of inflationary pressures.
- Popular in monetary policy papers. Many of the previous conclusions about optimal monetary policy still cary over with this hybrid NKPC.
- Despite its popularity in applications, the theoretical foundations for the various hybrid models are weak:
  1. Rule-of-thumb price-setters: Some people set backward-looking prices, some don't. (Gali-Gertler, *JME* 1999).
  2. Indexation: Each period, some set an optimal price and some don't. Non-optimizing price-setters index to past inflation. (Christiano, Eichenbaum, Evans, *JPE* 2005).
- These models are just as open to Lucas critique as traditional ones.

Problems with the Euler Equation

- Estimates of the relationship between output and real interest rates uncertain.
- This framework also makes predictions about asset returns and does not do well. For instance, it cannot explain why the equity premium is high or the risk-free rate is too low.
- Another persistence problem: Purely forward-looking equations for output gaps or consumption do not perform well.
- Another ad hoc patch to the model: Change the utility function to

$$U(C_t) = \frac{(C_t - \eta C_{t-1})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}$$

  so that consumers care, not only about the level of consumption, but it's level relative to last period.
- Modern Dynamic Stochastic General Equilibrium (DSGE) models (such as Smets-Wouters) tend to contain all of these devices to fix the persistence problems in the basic micro-founded model.