The Solow Model

We have discussed how economic growth can come from either capital deepening (increased amounts of capital per worker) or from improvements in total factor productivity (sometimes termed technological progress). This suggests that economic growth can come about from saving and investment (so that the economy accumulates more capital) or from improvements in productive efficiency. In these notes, we consider a model that explains the role these two elements play in generating sustained economic growth. The model is also due to Robert Solow, whose work on growth accounting we discussed in the last lecture, and was first presented in his 1956 paper "A Contribution to the Theory of Economic Growth."

The Solow Model's Assumptions

The Solow model assumes that output is produced using a production function in which output depends upon capital and labour inputs as well as a technological efficiency parameter, A.

$$Y_t = AF\left(K_t, L_t\right) \tag{1}$$

It is assumed that adding capital and labour raises output

$$\frac{\partial Y_t}{\partial K_t} > 0 \tag{2}$$

$$\frac{\partial Y_t}{\partial L_t} > 0 \tag{3}$$

However, the model also assumes there are diminishing marginal returns to capital accumulation. In other words, adding extra amounts of capital gives progressively smaller and smaller increases in output. This means the second derivative of output with respect to capital is negative.

$$\frac{\partial^2 Y_t}{\partial K_t} < 0 \tag{4}$$

See Figure 1 for an example of how output can depend on capital with diminishing returns. Think about why diminishing marginal returns is probably sensible: If a firm acquires an extra unit of capital, it will probably be able to increase its output. But if the firm keeps piling on extra capital without raising the number of workers available to use this capital, the increases in output will probably taper off. A firm with ten workers would probably like to have at least ten computers. It might even be helpful to have a few more; perhaps a few laptops for work from home or some spare computers in case others break down. But at some point, just adding more computers doesn't help so much.

We will use a very stylized description of the other parts of this economy: This helps us to focus in on the important role played by diminishing marginal returns to capital. We assume a closed economy with no government sector or international trade. This means all output takes the form of either consumption or investment

$$Y_t = C_t + I_t \tag{5}$$

and that savings equals investment

$$S_t = Y_t - C_t = I_t \tag{6}$$

The economy's stock of capital is assumed to change over time according to

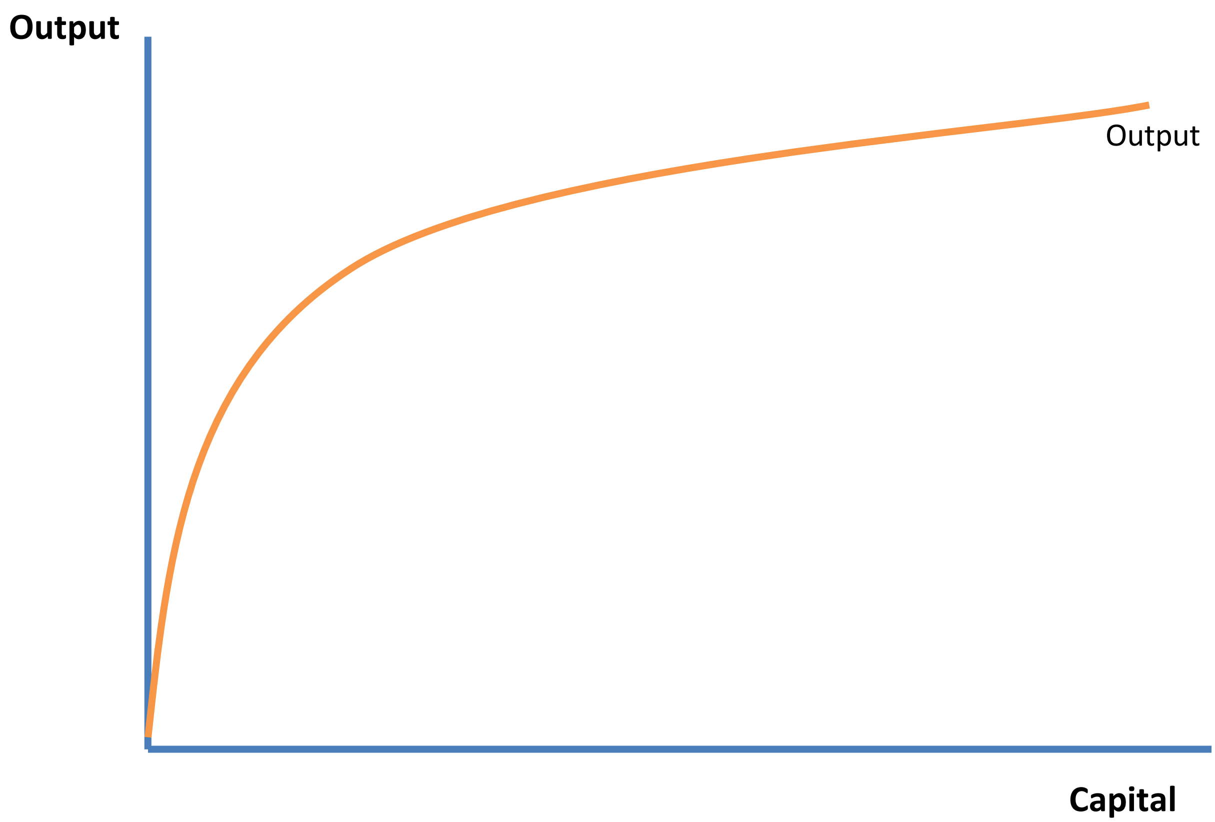
$$\frac{dK_t}{dt} = I_t - \delta K_t \tag{7}$$

In other words, the addition to the capital stock each period depends positively on investment and negatively on depreciation, which is assumed to take place at rate δ.

The Solow model does not attempt to model the consumption-savings decision. Instead it assumes that consumers save a constant fraction s of their income

$$S_t = sY_t \tag{8}$$

Figure 1: Diminishing Marginal Returns to Capital



Capital Dynamics in the Solow Model

Because savings equals investment in the Solow model, equation (8) means that investment is also a constant fraction of output

$$I_t = sY_t \tag{9}$$

which means we can re-state the equation for changes in the stock of capital

$$\frac{dK_t}{dt} = sY_t - \delta K_t \tag{10}$$

Whether the capital stock expands, contracts or stays the same depends on whether investment is greater than, equal to or less than depreciation.

$$\frac{dK_t}{dt} > 0 \quad \text{if} \quad \delta K_t < sY_t \tag{11}$$

$$\frac{dK_t}{dt} = 0 \quad \text{if} \quad \delta K_t = sY_t \tag{12}$$

$$\frac{dK_t}{dt} < 0 \quad \text{if} \quad \delta K_t > sY_t \tag{13}$$

In other words, if the ratio of capital to output is such that

$$\frac{K_t}{Y_t} = \frac{s}{\delta} \tag{14}$$

the the stock of capital will stay constant. If the capital-ouput ratio is lower than this level, then the capital stock will be increasing and if it is higher than this level, it will be decreasing.

Figure 2 provides a graphical illustration of this process. Depreciation is a simple straight-line function of the stock of capital while output is a curved function of capital, featuring diminishing marginal returns. When the level of capital is low sY_t is greater than δK. As the capital stock increases, the additional investment due to the extra output tails off but the additional depreciation does not, so at some point sY_t equals δK and the stock of capital stops increasing. Figure 2 labels the particular point at which the capital stock remains unchanged as K^*. At this point, we have $\frac{K_t}{Y_t} = \frac{s}{\delta}$.

In the same way, if we start out with a high stock of capital, then depreciation, δK, will tend to be greater than investment, sY_t. This means the stock of capital will decline. When it reaches K^* it will stop declining. This an example of what economists call *convergent dynamics*. For any fixed set of the model parameters (s and δ) and other inputs into the production (A_t and L_t) there will be a defined level of capital such that, no matter where the capital stock starts, it will converge over time towards this level.

Figure 3 provides an illustration of how the convergent dynamics determine the level of output in the Solow model. It shows output, investment and depreciation as a function of the capital stock. The gap between the green line (investment) and the orange line (output) shows the level of consumption. The economy converges towards the level of output associated with the capital stock K^*.

An Increase in the Savings Rate

Now consider what happens when the economy has settled down at an equilibrium unchanging level of capital K_1 and then there is an increase in the savings rate from s_1 to s_2.

Figure 4 shows what happens to the dynamics of the capital stock. The line for investment shifts upwards: For each level of capital, the level of output associated with it translates into more investment. So the investment curve shifts up from the green line to the red line. Starting at the initial level of capital, K_1, investment now exceeds depreciation. This means the capital stock starts to increase. This process continues until capital reaches its new equilibrium level of K_2 (where the red line for investment intersects with the black line for depreciation.) Figure 5 illustrates how output increases after this increase in the savings rate.

Figure 2: Capital Dynamics in The Solow Model

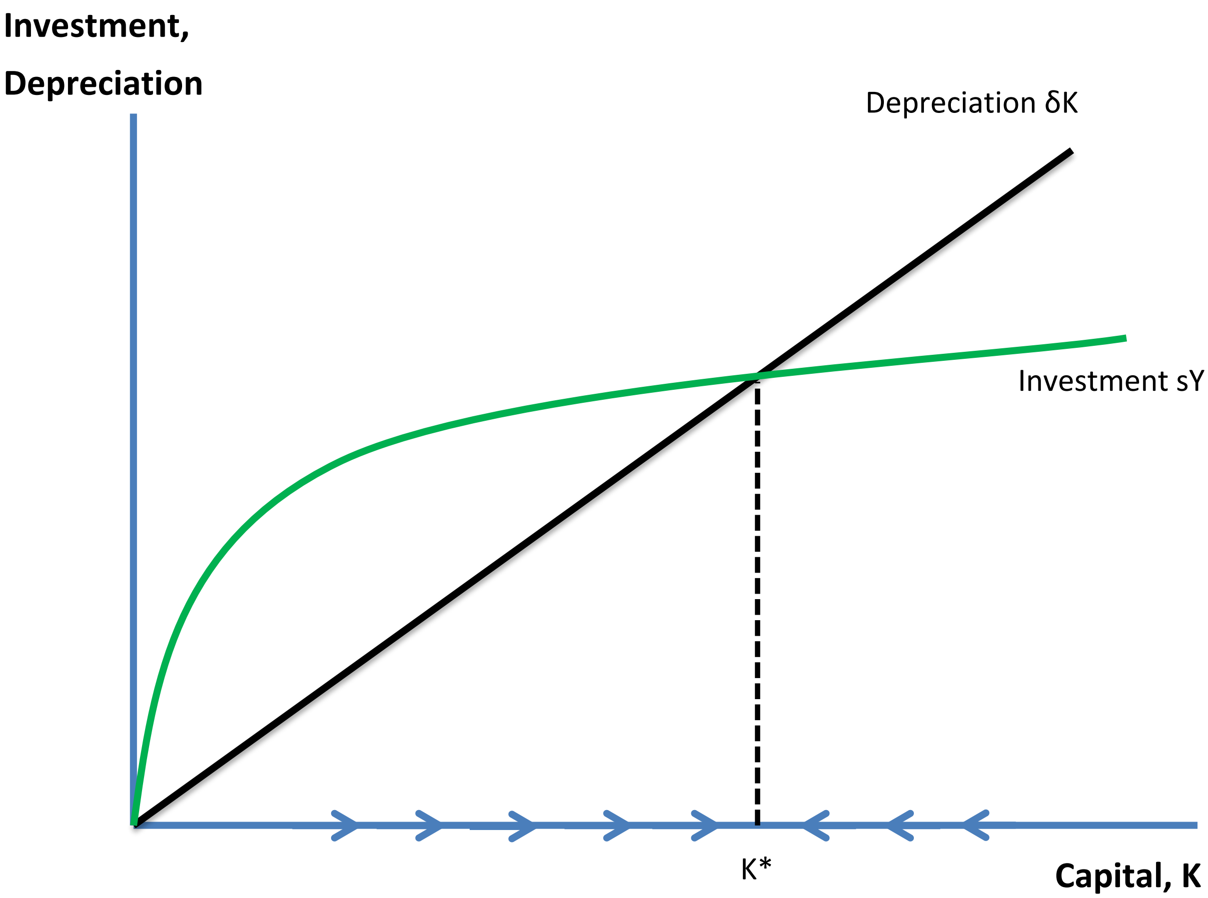


Figure 3: Capital and Output in the Solow Model

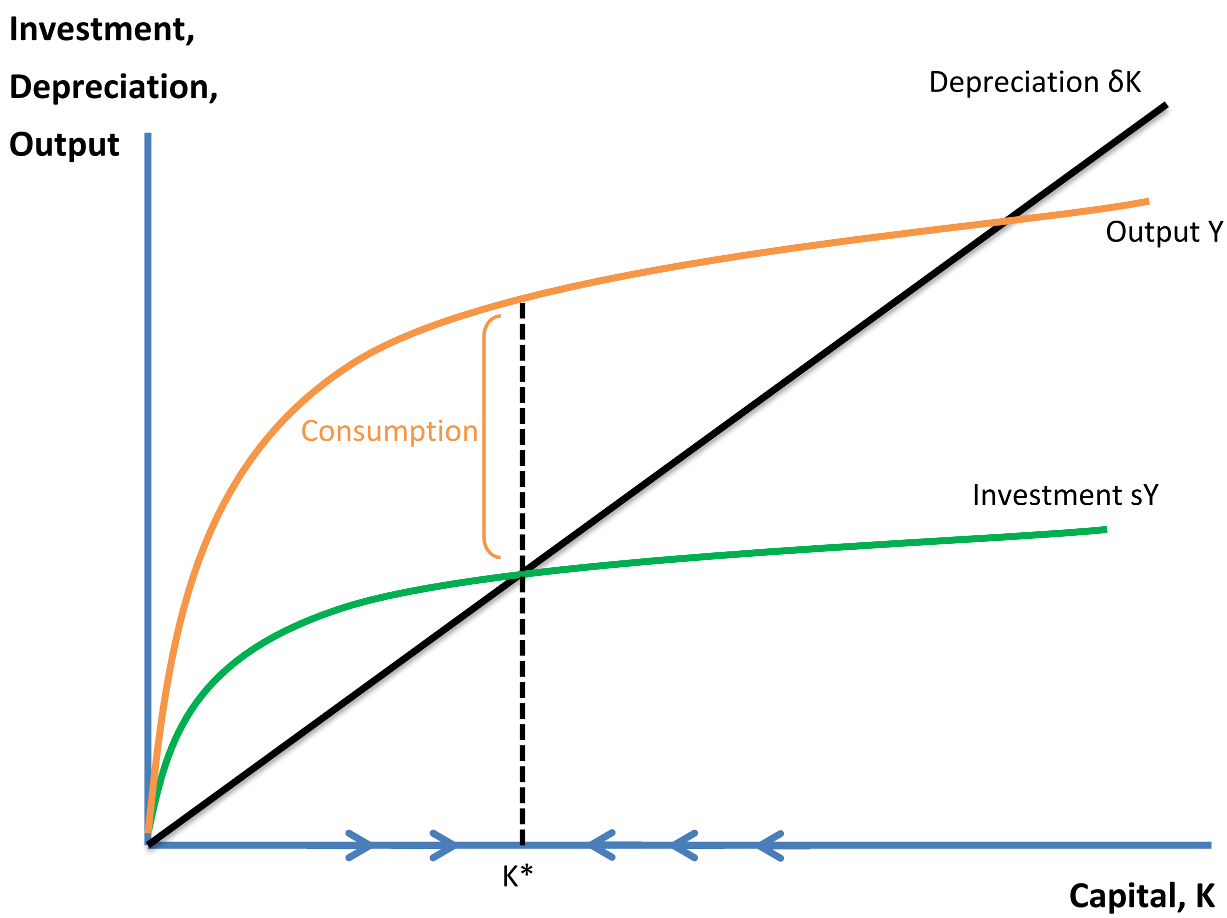


Figure 4: An Increase in the Saving Rate

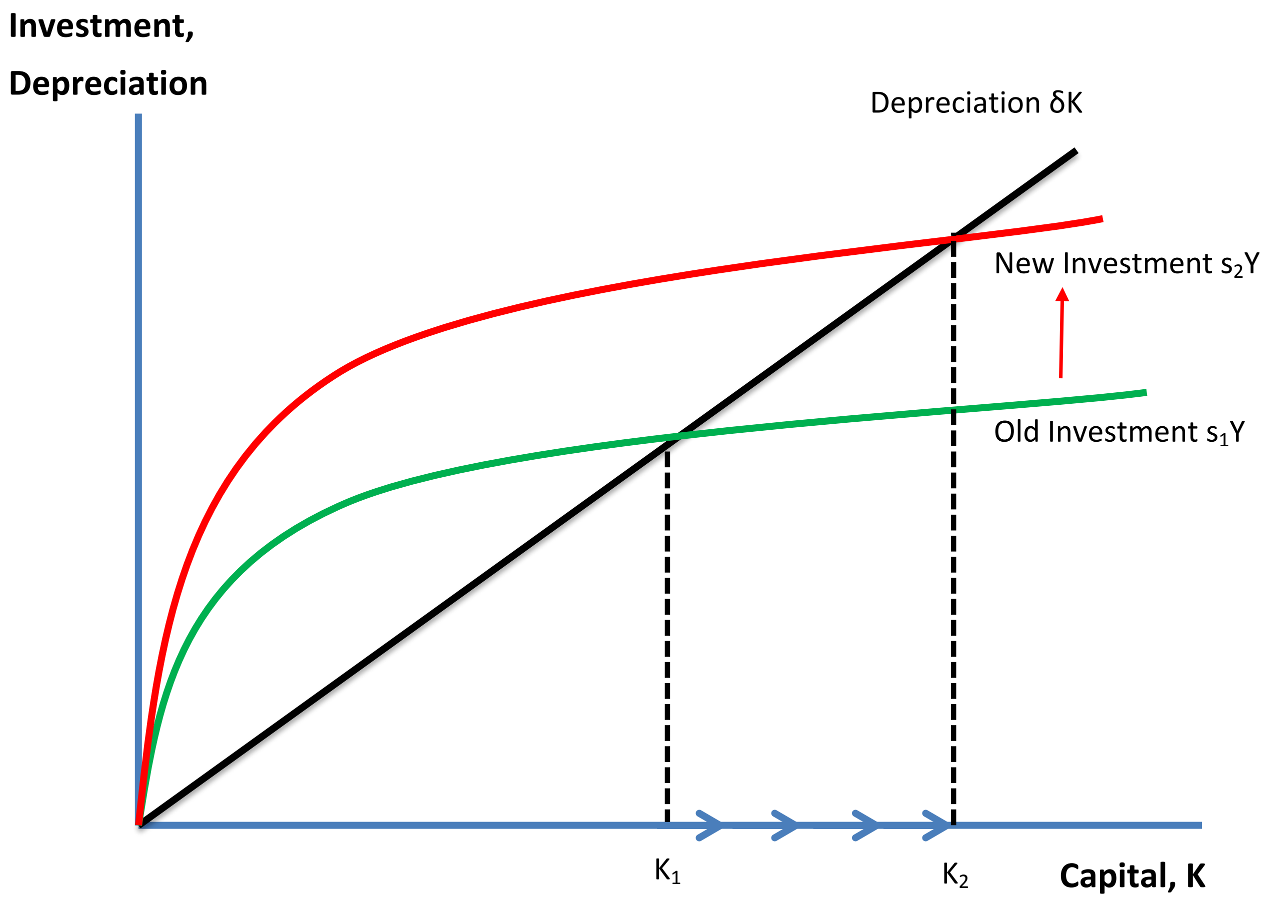
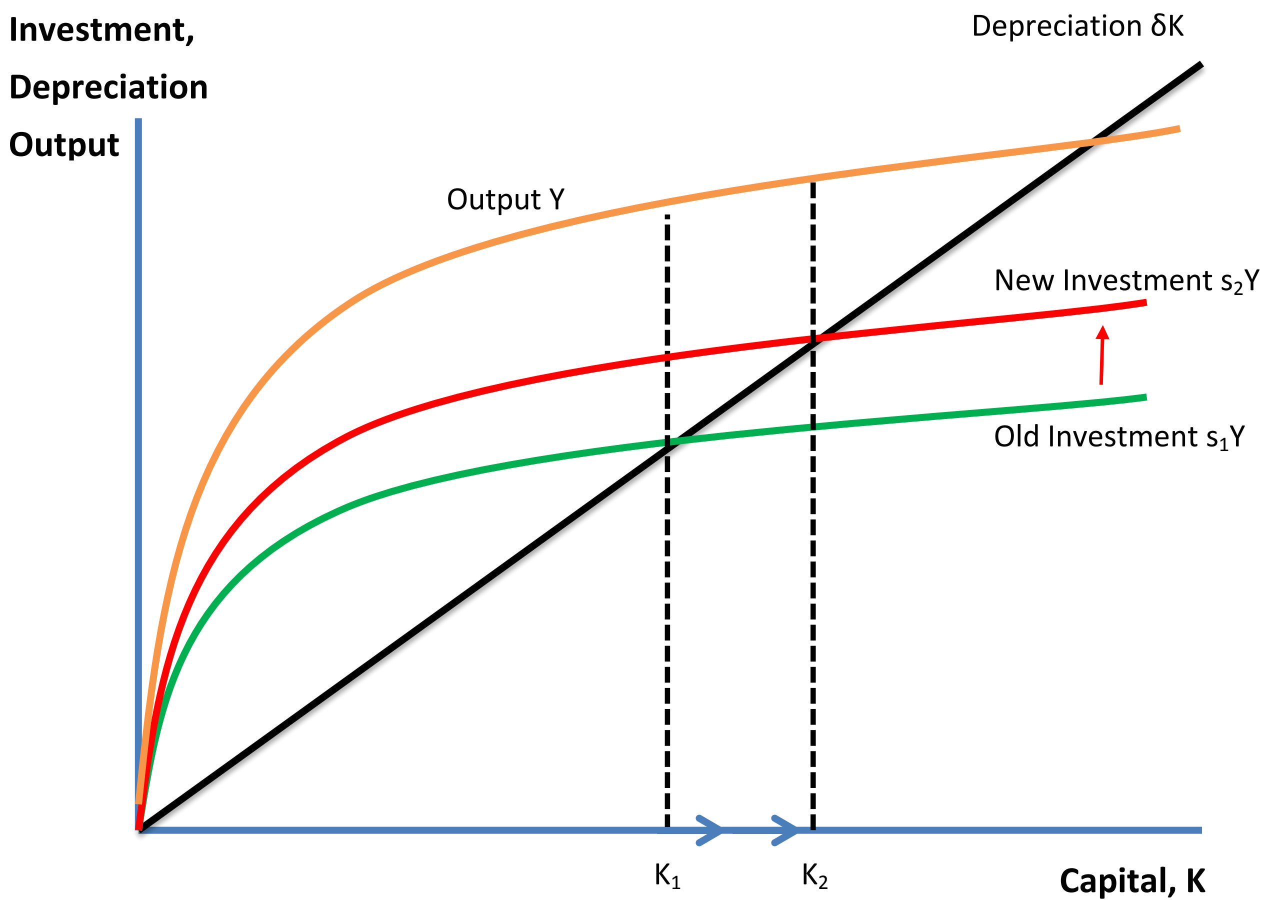


Figure 5: Effect on Output of Increased Saving



An Increase in the Depreciation Rate

Now consider what happens when the economy has settled down at an equilibrium unchanging level of capital K_1 and then there is an increase in the depreciation rate from δ_1 to δ_2.

Figure 6 shows what happens in this case. The depreciation schedule shifts up from the black line associated with the original depreciation rate, δ_1, to the new red line associated with the new depreciation rate, δ_2. Starting at the initial level of capital, K_1, depreciation now exceeds investment. This means the capital stock starts to decline. This process continues until capital falls to its new equilibrium level of K_2 (where the red line for depreciation intersects with the green line for investment.) So the increase in the depreciation rate leads to a decline in the capital stock and in the level of output.

An Increase in Technological Efficiency

Now consider what happens when technological efficiency A_t increases. Because investment is given by

$$I_t = sY_t = sAF\left(K_t, L_t\right) \tag{15}$$

a one-off increase in A thus has the same effect as a one-off increase in s. Capital and output gradually rise to a new higher level. Figure 7 shows the increase in capital due to an increase in technological efficiency.

Figure 6: An Increase in Depreciation

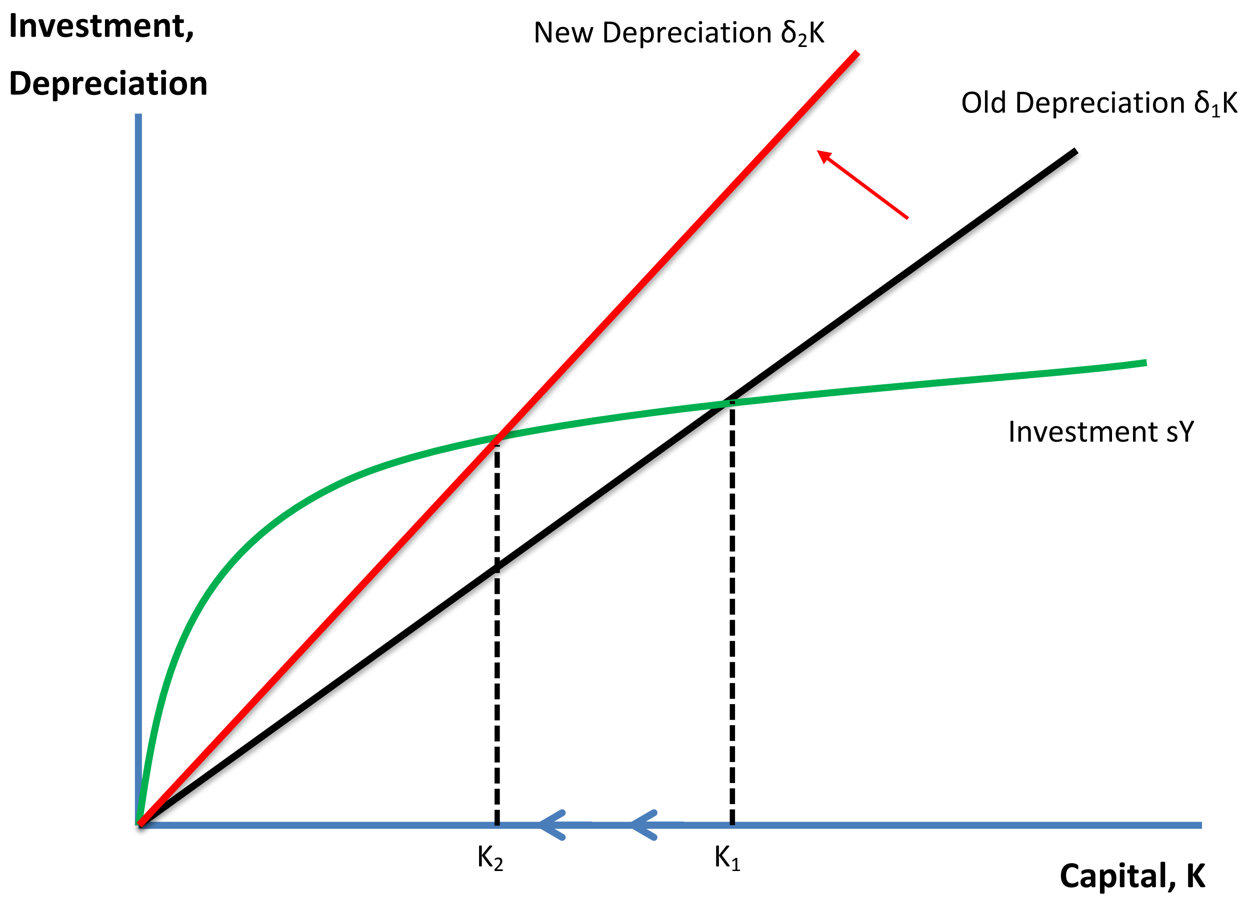
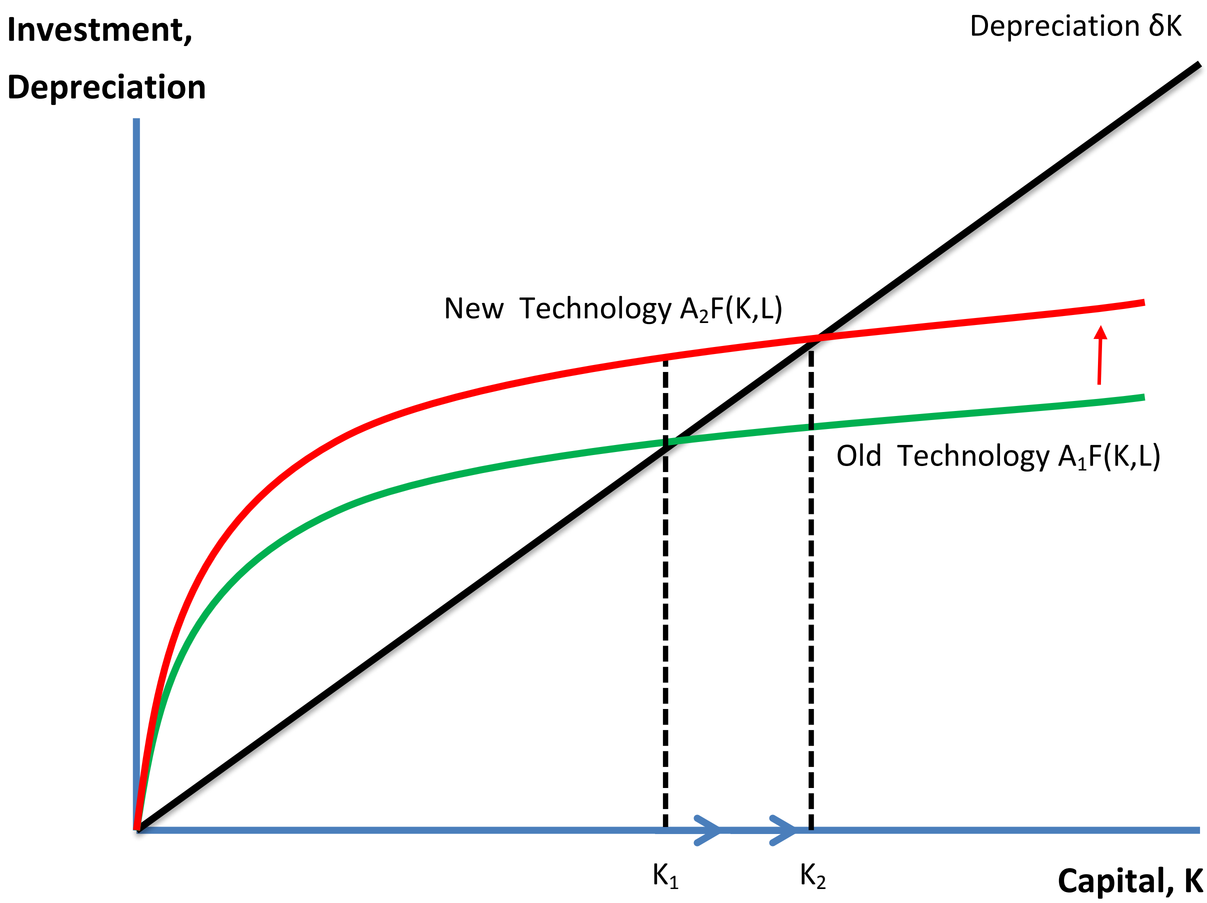


Figure 7: An Increase in Technological Efficiency



Solow and the Sources of Growth

In the last lecture, we described how capital deepening and technological progress were the two sources of growth in output per worker. Specifically, we derived an equation in which output growth was a function of growth in the capital stock, growth in the number of workers and growth in technological efficiency.

Our previous discussion had pointed out that a one-off increase in technological efficiency, A_t, had the same effects as a one-off increase in the savings rate, s. However, there are important differences between these two types of improvements. The Solow model predicts that economies can only achieve a temporary boost to economic growth due to a once-off increase in the savings rate. If they want to sustain economic growth through this approach, then they will need to keep raising the savings rate. However, there are likely to be limits in any economy to the fraction of output that can be allocated towards saving and investment, particularly if it is a capitalist economy in which savings decisions are made by private citizens.

Unlike the savings rate, which will tend to have an upward limit, there is no particular reason to believe that technological efficiency A_t has to have an upper limit. Indeed, growth accounting studies tend to show steady improvements over time in A_t in most countries. Going back to Young's paper on Hong Kong and Singapore discussed in the last lecture, you can see now why it matters whether an economy has grown due to capital deepening or TFP growth. The Solow model predicts that a policy of encouraging growth through more capital accumulation will tend to tail off over time producing a once-off increase in output per worker. In contrast, a policy that promotes the growth rate of TFP can lead to a sustained higher growth rate of output per worker.

The Capital-Output Ratio with Steady Growth

Up to now, we have only considered once-off changes in output. Here, however, we consider how the capital stock behaves when the economy grows at steady constant rate G^Y. Specifically, we can show in this case that the ratio of capital to output will tend to converge to a specific value. Recall from the last lecture that if we have something of the form

$$Z_t = U_t^{\alpha} W_t^{\beta} \tag{16}$$

then we have the following relationship between the various growth rates

$$G_t^Z = \alpha G_t^U + \beta G_t^W \tag{17}$$

The capital output ratio $\frac{K_t}{Y_t}$ can be written as $K_t Y_t^{-1}$. So the growth rate of the capital-output ratio can be written as

$$G_t^{\frac{K}{Y}} = G_t^K - G_t^Y \tag{18}$$

Adjusting equation 10, the growth rate of the capital stock can be written as

$$G_t^K = \frac{1}{K_t}\frac{dK_t}{dt} = s\frac{Y_t}{K_t} - \delta \tag{19}$$

so the growth rate of the capital-output ratio is

$$G_t^{\frac{K}{Y}} = s\frac{Y_t}{K_t} - \delta - G^Y \tag{20}$$

This gives a slightly different form of convergence dynamics from those we saw earlier. This equation shows that the growth rate of the capital-output ratio depends negatively on the level of this ratio. This means the capital-output ratio displays convergent dynamics. When it is above a specific equilibrium value it tends to fall and when it is below this equilibrium value it tends to increase. Thus, the ratio is constantly moving towards this equilibrium value.

We can express this formally as follows:

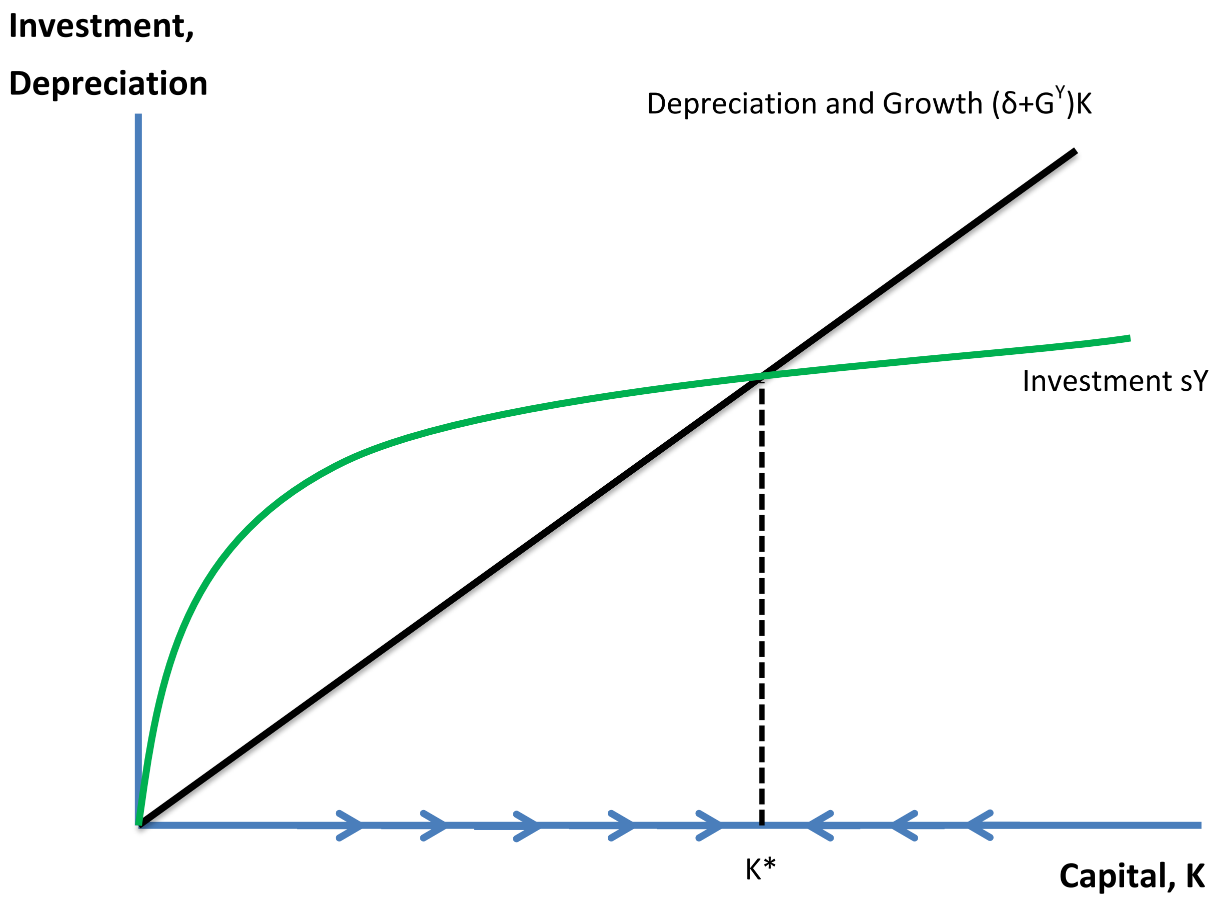
$$G_t^{\frac{K}{Y}} > 0 \quad \text{if} \quad \frac{K_t}{Y_t} < \frac{s}{\delta + G^Y} \tag{21}$$

$$G_t^{\frac{K}{Y}} = 0 \quad \text{if} \quad \frac{K_t}{Y_t} = \frac{s}{\delta + G^Y} \tag{22}$$

$$G_t^{\frac{K}{Y}} < 0 \quad \text{if} \quad \frac{K_t}{Y_t} > \frac{s}{\delta + G^Y} \tag{23}$$

We can illustrate these dynamics using a slightly altered version of our earlier graph. Figure 8 amends the depreciation line to the amount of capital necessary not just to replace depreciation but also to have a percentage increase in the capital stock that matches the increase in output. The diagram shows that the economy will tend to move towards a capital stock such that $sY_t = \left(\delta + G^Y\right) K_t$ meaning the capital-output ratio is $\frac{K_t}{Y_t} = \frac{s}{\delta + G^Y}$.

Figure 8: The Equilibrium Capital Stock in a Growing Economy



Why Growth Accounting Can Be Misleading

Of the cases just considered in which output and capital both increase—an increase in the savings rate and an increase in the level of TFP—the evidence points to increases in TFP being more important as a generator of long-term growth. Rates of savings and investment tend for most countries tend to stay within certain ranges while large increases in TFP over time have been recorded for many countries. It's worth noting then that growth accounting studies can perhaps be a bit misleading when considering the ultimate sources of growth.

Consider a country that has a constant share of GDP allocated to investment but is experiencing steady growth in TFP. The Solow model predicts that this economy should experience steady increases in output per worker and increases in the capital stock. A growth accounting exercise may conclude that a certain percentage of growth stems from capital accumulation but ultimately, in this case, all growth (including the growth in the capital stock) actually stems from growth in TFP. The moral here is that pure accounting exercises may miss the ultimate cause of growth.

Krugman on "The Myth of Asia's Miracle"

I encourage you to read Paul Krugman's 1994 article "The Myth of Asia's Miracle." It discusses a number of examples of cases where economies where growth was based on largely on capital accumulation. In addition to the various Asian countries covered in Alwyn Young's research, Krugman (correctly) predicted a slowdown in growth in Japan, even though at the time many US commentators were focused on the idea that Japan was going to overtake US levels of GDP per capita.

Perhaps most interesting is his discussion of growth in the Soviet Union. Krugman notes that the Soviet economy grew strongly after World War 2 and many in the West believed they would become more prosperous than capitalist economies. The Soviet Union's achievement in placing the first man in space provoked Kennedy's acceleration in the space programme, mainly to show the U.S. was not falling behind communist systems. However, some economists that had examined the Soviet economy were less impressed. Here's an extended quote from Krugman's article:

> When economists began to study the growth of the Soviet economy, they did so using the tools of growth accounting. Of course, Soviet data posed some problems. Not only was it hard to piece together usable estimates of output and input (Raymond Powell, a Yale professor, wrote that the job "in many ways resembled an archaeological dig"), but there were philosophical difficulties as well. In a socialist economy one could hardly measure capital input using market returns, so researchers were forced to impute returns based on those in market economies at similar levels of development. Still, when the efforts began, researchers were pretty sure about what: they would find. Just as capitalist growth had been based on

growth in both inputs and efficiency, with efficiency the main source of rising per capita income, they expected to find that rapid Soviet growth reflected both rapid input growth and rapid growth in efficiency.

But what they actually found was that Soviet growth was based on rapid–growth in inputs–end of story. The rate of efficiency growth was not only unspectacular, it was well below the rates achieved in Western economies. Indeed, by some estimates, it was virtually nonexistent.

The immense Soviet efforts to mobilize economic resources were hardly news. Stalinist planners had moved millions of workers from farms to cities, pushed millions of women into the labor force and millions of men into longer hours, pursued massive programs of education, and above all plowed an ever-growing proportion of the country's industrial output back into the construction of new factories.

Still, the big surprise was that once one had taken the effects of these more or less measurable inputs into account, there was nothing left to explain. The most shocking thing about Soviet growth was its comprehensibility.

This comprehensibility implied two crucial conclusions. First, claims about the superiority of planned over market economies turned out to be based on a misapprehension. If the Soviet economy had a special strength, it was its ability to mobilize resources, not its ability to use them efficiently. It was obvious to everyone that the Soviet Union in 1960 was much less efficient than the United States. The surprise was that it showed no signs of closing the gap.

Second, because input-driven growth is an inherently limited process, Soviet growth was virtually certain to slow down. Long before the slowing of Soviet growth became obvious, it was predicted on the basis of growth accounting.

The Soviet leadership did a good job for a long time of hiding from the world that their economy had stopped growing but ultimately the economic failures of the centrally planning model (combined with its many political and ethnic tensions) ended in a dramatic implosion of the communist system in Russia and the rest of Eastern Europe.

A Formula for Steady Growth

All of the results so far apply for any production function with diminishing marginal returns to capital. However, we can also derive some useful results by making specific assumptions about the form of the production function. Specifically, we will consider the constant returns to scale Cobb-Douglas production function

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{24}$$

This means output growth is determined by

$$G_t^Y = G_t^A + \alpha G_t^K + (1-\alpha) G_t^L \tag{25}$$

Now consider the case in which the growth rate of labour input is fixed at n

$$G_t^L = n \tag{26}$$

and the growth rate of total factor productivity is fixed at g.

$$G_t^A = g \tag{27}$$

The formula for output growth becomes

$$G_t^Y = g + \alpha G_t^K + (1-\alpha) n \tag{28}$$

This means all variations in the growth rate of output are due to variations in the growth rate for capital. If output is growing at a constant rate, then capital must also be growing at a constant rate. And we know that the capital-output ratio tends to move towards a specific equilibrium value. So along a steady growth path, the growth rate of output equals the growth rate of capital. Thus, the previous equation can be re-written

$$G_t^Y = g + \alpha G_t^Y + (1-\alpha)\, n \tag{29}$$

which can be simplified to

$$G_t^Y = \frac{g}{1-\alpha} + n \tag{30}$$

The growth rate of output per worker is

$$G_t^Y - n = \frac{g}{1-\alpha} \tag{31}$$

So the economy tends to converge towards a steady growth path and the growth rate of output per worker along this path is $\frac{g}{1-\alpha}$. Without growth in technological efficiency, there can be no steady growth in output per worker.

A Useful Formula for Output Per Worker

In this case of the Cobb-Douglas production function, output per worker can be written as

$$\frac{Y_t}{L_t} = A_t \left(\frac{K_t}{L_t}\right)^{\alpha} \tag{32}$$

In other words, output per worker is a function of technology and of capital per worker. A drawback of this representation is that we know that increases in A_t also increase capital per worker, so this has the misleading implications about the role of capital accumulation discussed above. It is useful, then, to derive an alternative characterisation of output per worker, one that we will use again. First, we'll define the capital-output ratio as

$$x_t = \frac{K_t}{Y_t} \tag{33}$$

So, the production function can be expressed as

$$Y_t = A_t \left(x_t Y_t\right)^{\alpha} L_t^{1-\alpha} \tag{34}$$

Here, we are using the fact that

$$K_t = x_t Y_t \tag{35}$$

Dividing both sides of this expression by Y_t^{α}, we get

$$Y_t^{1-\alpha} = A_t x_t^{\alpha} L_t^{1-\alpha} \tag{36}$$

Taking both sides of the equation to the power of $\frac{1}{1-\alpha}$ we arrive at

$$Y_t = A_t^{\frac{1}{1-\alpha}} x_t^{\frac{\alpha}{1-\alpha}} L_t \tag{37}$$

So, output per worker is

$$\frac{Y_t}{L_t} = A_t^{\frac{1}{1-\alpha}} x_t^{\frac{\alpha}{1-\alpha}} \tag{38}$$

This equation states that all fluctuations in output per worker are due to either changes in technological progress or changes in the capital-output ratio. When considering the relative role of technological progress or policies to encourage accumulation, we will see that this decomposition is more useful than equation (32) because the level of technology does not affect x_t in the long run while it does affect $\frac{K_t}{L_t}$. So, this decomposition offers a cleaner picture of the part of growth due to technology and the part that is not.

A Formal Model of Convergence Dynamics

Because A_t is assumed to grow at a constant rate each period, this means that all of the interesting dynamics for output per worker in this model stem from the behaviour of the capital-output ratio. We will now describe in more detail how this ratio behaves. Before doing so, I want to introduce a new piece of terminology that we will use in the next few lectures.

A useful mathematical shorthand that saves us from having to write down derivatives with respect to time everywhere is to write

$$\dot{Y}_t = \frac{dY_t}{dt} \tag{39}$$

What we are really interested in, though, is *growth rates* of series, so we need to scale this by the level of output itself. Thus, $\frac{\dot{Y}_t}{Y_t}$, and this is our mathematical expression for the growth rate of a series. For our Cobb-Douglas production function, we can use the result we derived earlier to express the growth rate of output as

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} + \alpha\frac{\dot{K}_t}{K_t} + (1-\alpha)\frac{\dot{L}_t}{L_t} \tag{40}$$

The Solow model assumes

$$\frac{\dot{A}_t}{A_t} = g \tag{41}$$

$$\frac{\dot{N}_t}{N_t} = n \tag{42}$$

So this can be re-written as

$$\frac{\dot{Y}_t}{Y_t} = g + \alpha\frac{\dot{K}_t}{K_t} + (1-\alpha)n \tag{43}$$

Similarly, because

$$x_t = K_t Y_t^{-1} \tag{44}$$

its growth rate can be written as

$$\frac{\dot{x}_t}{x_t} = \frac{\dot{K}_t}{K_t} - \frac{\dot{Y}_t}{Y_t} \tag{45}$$

To get an expression for the growth rate of the capital stock, w re-write the capital accumulation equation as

$$\dot{K}_t = sY_t - \delta K_t \tag{46}$$

and divide across by K_t on both sides

$$\frac{\dot{K}_t}{K_t} = s\frac{Y_t}{K_t} - \delta \tag{47}$$

This means we write, the growth rate of the capital stock as

$$\frac{\dot{K}_t}{K_t} = \frac{s}{x_t} - \delta \tag{48}$$

Now using equation (43) for output growth and equation (48) for capital growth, we can derive a useful equation for the dynamics of the capital-output ratio:

$$\frac{\dot{x}_t}{x_t} = (1-\alpha)\frac{\dot{K}_t}{K_t} - g - (1-\alpha)n \tag{49}$$

$$= (1-\alpha)\left(\frac{s}{x_t} - \frac{g}{1-\alpha} - n - \delta\right) \tag{50}$$

This dynamic equation has a very important property: The growth rate of x_t depends negatively on the value of x_t. In particular, when x_t is over a certain value, it will tend to decline, and when it is under that value it will tend to increase. This provides a specific illustration of the convergent dynamics of the capital-output ratio.

What is the long-run steady-state value of x_t, which we will label x^*? It is the value consistent with $\frac{\dot{x}}{x} = 0$. This implies that

$$\frac{s}{x^*} - \frac{g}{1-\alpha} - n - \delta = 0 \tag{51}$$

This solves to give

$$x^* = \frac{s}{\frac{g}{1-\alpha} + n + \delta} \tag{52}$$

Given this equation, we can derive a more intuitive-looking expression to describe the convergence properties of the capital-output ratio. The dynamics of x_t are given by

$$\frac{\dot{x}_t}{x_t} = (1-\alpha)(\frac{s}{x_t} - \frac{g}{1-\alpha} - n - \delta) \tag{53}$$

Multiplying and dividing the right-hand-side of this equation by $(\frac{g}{1-\alpha} + n + \delta)$:

$$\frac{\dot{x}_t}{x_t} = (1-\alpha)(\frac{g}{1-\alpha} + n + \delta)\left(\frac{s/x_t - \frac{g}{1-\alpha} - n - \delta}{\frac{g}{1-\alpha} + n + \delta}\right) \tag{54}$$

The last term inside the brackets can be simplified to give

$$\frac{\dot{x}_t}{x_t} = (1-\alpha)(\frac{g}{1-\alpha} + n + \delta)\left(\frac{1}{x_t}\frac{s}{\frac{g}{1-\alpha} + n + \delta} - 1\right) \tag{55}$$

$$= (1-\alpha)(\frac{g}{1-\alpha} + n + \delta)\left(\frac{x^*}{x_t} - 1\right) \tag{56}$$

$$= (1-\alpha)(\frac{g}{1-\alpha} + n + \delta)\left(\frac{x^* - x_t}{x_t}\right) \tag{57}$$

This equation states that each period the capital-output ratio closes a fraction equal to $\lambda = (1-\alpha)(\frac{g}{1-\alpha} + n + \delta)$ of the gap between the current value of the ratio and its steady-state value.

Illustrating Convergence Dynamics

Often, the best way to understand dynamic models is to load them onto the computer and see them run. This is easily done using spreadsheet software such as Excel or econometrics-oriented packages such as RATS. Figures 1 to 3 provide examples of the behaviour over time of two economies, one that starts with a capital-output ratio that is half the steady-state level, and other that starts with a capital output ratio that is 1.5 times the steady-state level.

The parameters chosen were $s = 0.2, \alpha = \frac{1}{3}, g = 0.02, n = 0.01, \delta = 0.06$. Together these parameters are consistent with a steady-state capital-output ratio of 2. To see, this plug these values into (52):

$$x^* = \left(\frac{K}{Y}\right)^* = \frac{s}{\frac{g}{1-\alpha} + n + \delta} = \frac{0.2}{1.5 * 0.02 + 0.01 + 0.06} = 2 \tag{58}$$

Figure 9 shows how the two capital-output ratios converge, somewhat slowly, over time to their steady-state level. This slow convergence is dictated by our choice of parameters: Our "convergence speed" is:

$$\lambda = (1 - \alpha)(\frac{g}{1 - \alpha} + n + \delta) = \frac{2}{3}(1.5 * 0.02 + 0.01 + 0.06) = 0.067 \tag{59}$$

So, the capital-output ratio converges to its steady-state level at a rate of about 7 percent per period. These are fairly standard parameter values for annual data, so this should be understood to mean 7 percent per year.

Figure 10 shows how output per worker evolves over time in these two economies. Both economies exhibit growth, but the capital-poor economy grows faster during the convergence period than the capital-rich economy. These output per worker differentials may seem a little small on this chart, but the Figure 11 shows the behaviour of the growth rates, and this chart makes it clear that the convergence dynamics can produce substantially different growth rates depending on whether an economy is above or below its steady-state capital-output ratio. During the initial transition periods, the capital-poor economy grows at rates over 6 percent, while the capital-rich economy grows at under 2 percent. Over time, both economies converge towards the steady-state growth rate of 3 percent.

Figure 9: Convergence Dynamics for the Capital-Output Ratio

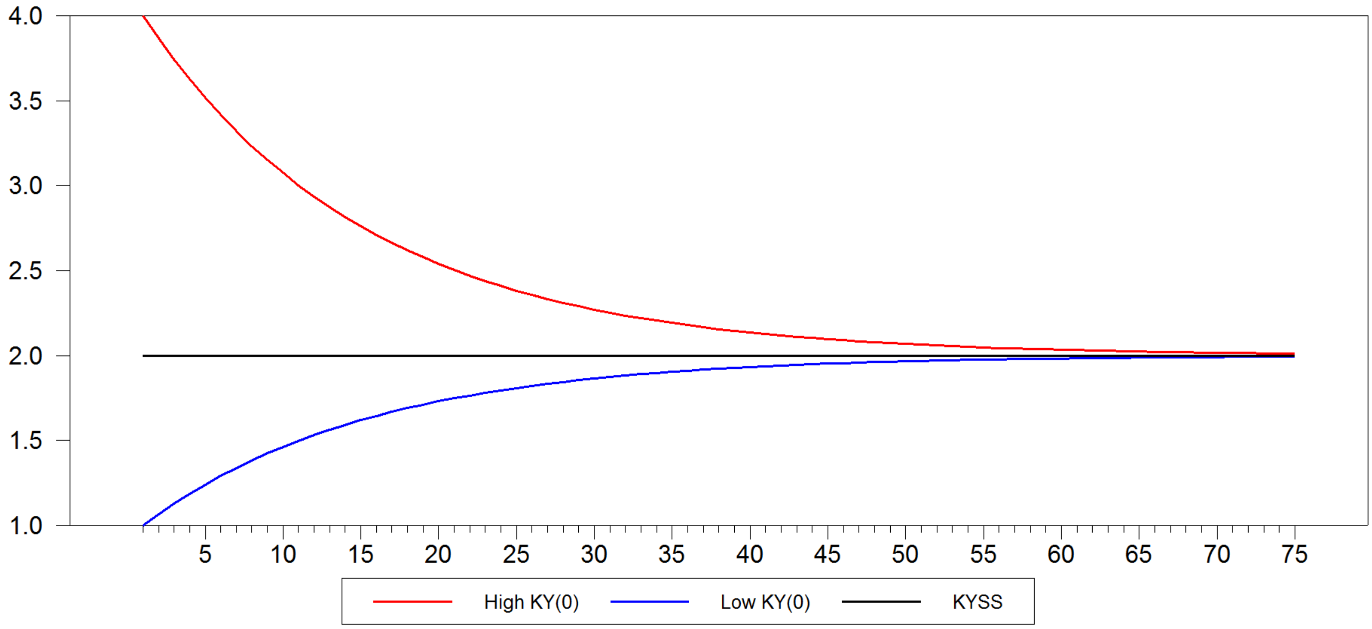


Figure 10: Convergence Dynamics for Output Per Worker

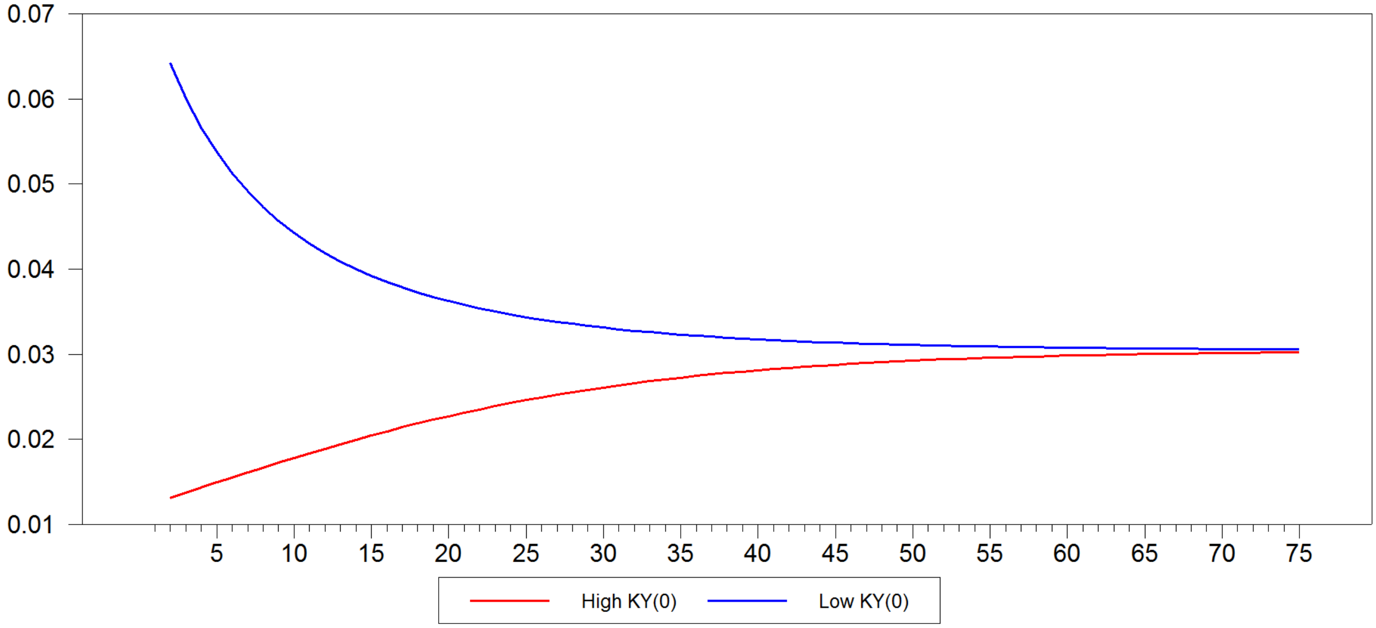
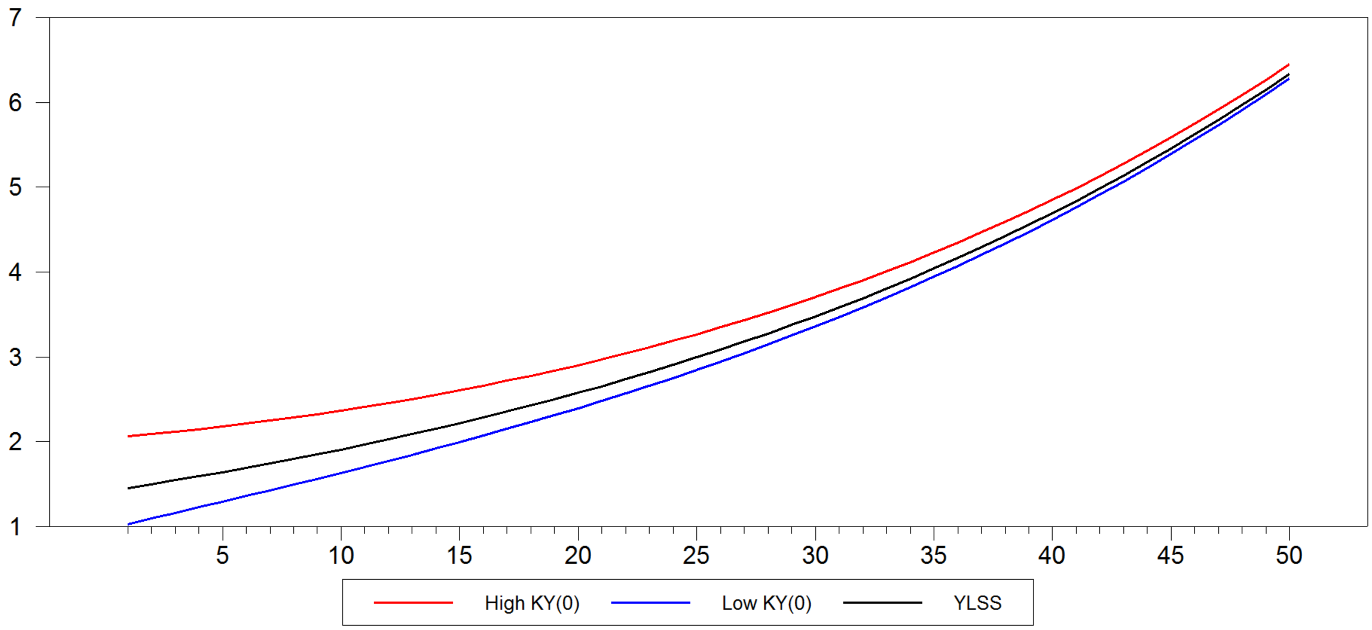


Figure 11: Convergence Dynamics for the Growth Rate of Output Per Worker



Illustrating Changes in Key Parameters

Figures 12 to 14 examine what happens when the economy is moving along the steady-state path consistent with the parameters just given, and then one of the parameters is changed. Specifically, they examine the effects of changes in s, δ and g.

Consider first an increase in the savings rate to $s = 0.25$. This has no effect on the steady-state growth rate. But it does change the steady-state capital-output ratio from 2 to 2.5. So the economy now finds itself with too little capital relative to its new steady-state capital-output ratio. The growth rate jumps immediately and only slowly returns to the long-run 3 percent value. The faster pace of investment during this period gradually brings the capital-output ratio into line with its new steady-state level.

The increase in the savings rate permanently raises the level of output per worker relative to the path that would have occurred without the change. However, for our parameter values, this effect is not that big. This is because the long-run effect of the savings rate on output per worker is determined by $s^{\frac{\alpha}{1-\alpha}}$, which in this case is $s^{0.5}$. So in our case, 25 percent increase in the savings rate produces an 11.8 percent increase in output per worker ($1.25^{0.5} = 1.118$). More generally, a doubling of the savings rate raises output per worker by 41 percent ($2^{0.5} = 1.41$).

The charts also show the effect of an increase in the depreciation rate to $\delta = 0.11$. This reduces the steady-state capital-output ratio to 4/3 and the effects of this change are basically the opposite of the effects of the increase in the savings rate.

Finally, there is the increase in the rate of technological progress. I've shown the effects of a change from $g = 0.02$ to $g = 0.03$. This increases the steady-state growth rate of output per worker to 0.045. However, as the charts show there is another effect: A faster steady-state growth rate for output reduces the steady-state capital-output ratio. Why? The increase in

g raises the long-run growth rate of output; this means that each period the economy needs to accumulate more capital than before just to keep the capital-output ratio constant. Again, without a change in the savings rate that causes this to happen, the capital-output ratio will decline. So, the increase in g means that—as in the depreciation rate example—the economy starts out in period 25 with too much capital relative to its new steady-state capital-output ratio. For this reason, the economy doesn't jump straight to its new 4.5 percent growth rate of output per worker. Instead, after an initial jump in the growth rate, there is a very gradual transition the rest of the way to the 4.5 percent growth rate.

Figure 12: Capital-Output Ratios: Effect of Increases In ...

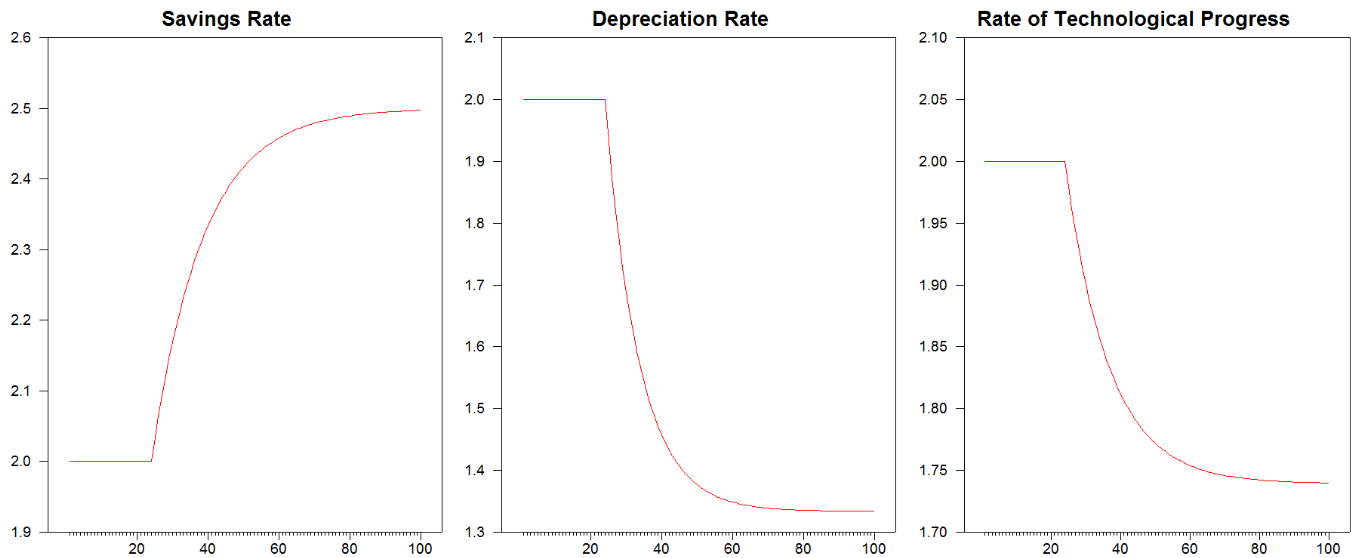


Figure 13: Growth Rates of Output Per Hour: Effect of Increases In ...

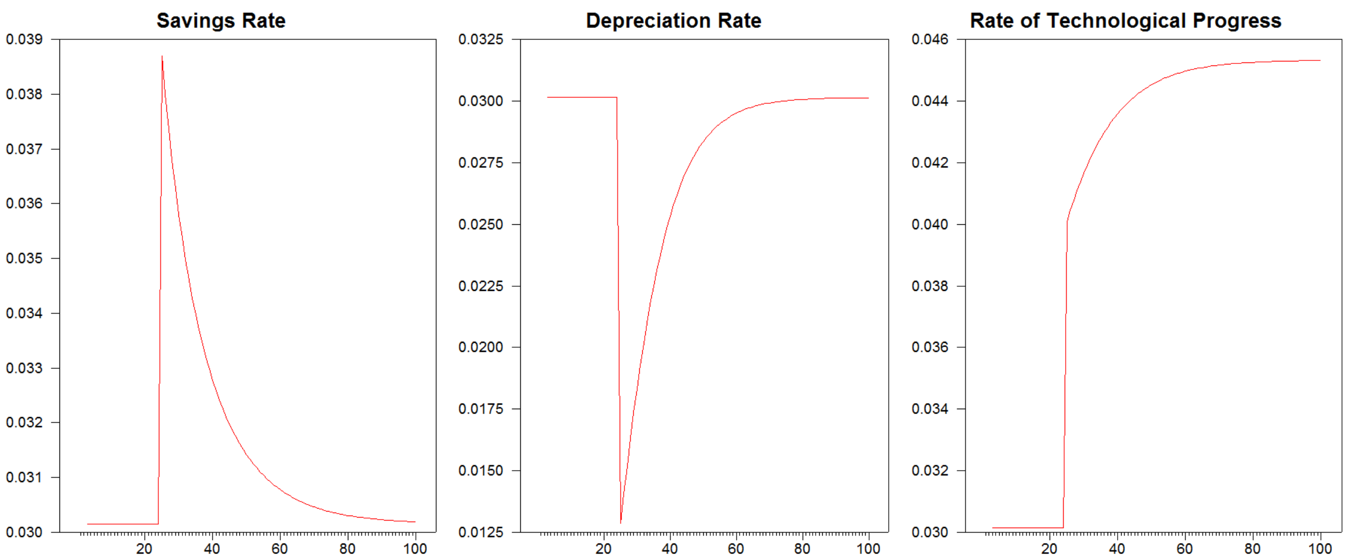
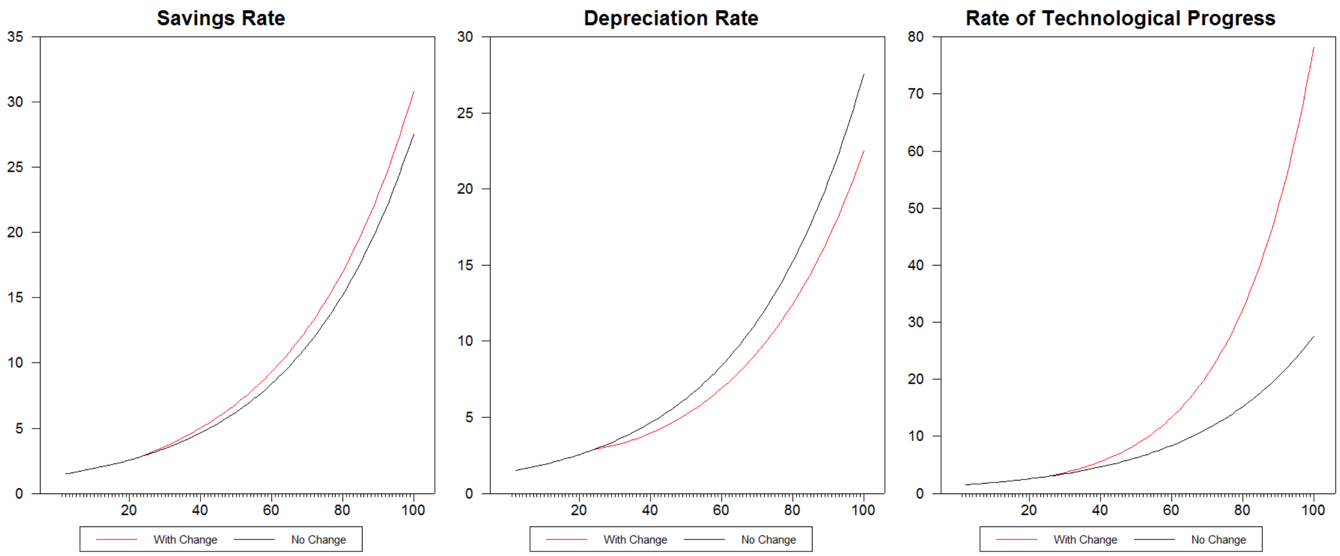


Figure 14: Output Per Hour: Effect of Increases In ...



Convergence Dynamics in Practice

The Solow model predicts that no matter what the original level of capital an economy starts out with, it will tend to revert to the equilibrium levels of output and capital indicated by the economy's underlying features. Does the evidence support this idea?

Unfortunately, history has provided a number of extreme examples of economies having far less capital than is consistent with their fundamental features. Wars have provided the "natural experiments" in which various countries have had huge amounts of their capital destroyed. The evidence has generally supported Solow's prediction that economies that experience negative shocks should tend to recover from these setbacks and return to their pre-shock levels of capital and output. For example, both Germany and Japan grew very strongly after the war, recovering prosperity despite the massive damage done to their stocks of capital by war bombing.

A more extreme example, perhaps, is study by Edward Miguel and Gerard Roland of the long-run impact of U.S. bombing of Vietnam in the 1960s and 1970s. Miguel and Roland found large variations in the extent of bombing across the various regions of Vietnam. Despite large differences in the extent of damage inflicted on different regions, Miguel and Roland found little evidence for lasting relative damage on the most-bombed regions by 2002. (Note this is not the same as saying there was no damage to the economy as a whole — the study is focusing on whether those areas that lost more capital than average ended up being poorer than average).

Things to Understand from these Notes

Here's a brief summary of the things that you need to understand from these notes.

1. The assumptions of the Solow model.
2. The rationale for diminishing marginal returns to capital accumulation.
3. Effects of changes in savings rate, depreciation rate and technology in the Solow model.
4. Why technological progress is the source of most growth.
5. Why growth accounting calculations can underestimate the role of technological progress.
6. Krugman on the Soviet Union.
7. The Solow model's predictions about convergent dynamics.
8. The formula for steady growth rate with a Cobb-Douglas production function.
9. The formula for the convergence rate with a Cobb-Douglas production function.
10. Historical examples of convergent dynamics.