Endogenous Technological Change: The Romer Model

The Solow model identified technological progress or improvements in total factor productivity (TFP) as the key determinant of growth in the long run, but did not provide any explanation of what determines it. In the technical language used by macroeconomists, long-run growth in the Solow framework is determined by something that is *exogenous* to the model.

In these notes, we consider a particular model that makes technological progress *endogeous*, meaning determined by the actions of the economic agents described in the model. The model, due to Paul Romer ("Endogenous Technological Change," *Journal of Political Economy*, 1990) starts by accepting the Solow model's result that technological progress is what determines long-run growth in output per worker. But, unlike the Solow model, Romer attempts to explain what determines technological progress.

TFP Growth as Invention of New Inputs

So what is this technology term A anyway? The Romer model takes a specific concrete view on this issue. Romer describes the aggregate production function as

$$Y = L_Y^{1-\alpha}\left(x_1^{\alpha} + x_2^{\alpha} + ... + x_A^{\alpha}\right) = L_Y^{1-\alpha}\sum_{i=1}^{A} x_i^{\alpha} \tag{1}$$

where L_Y is the number of workers producing output and the x_i's are different types of capital goods. The crucial feature of this production function is that diminishing marginal returns applies, not to capital as a whole, but separately to each of the individual capital goods (because $0 < \alpha < 1$).

If A was fixed, the pattern of diminishing returns to each of the separate capital goods would mean that growth would eventually taper off to zero. However, in the Romer model, A is not fixed. Instead, there are L_A workers engaged in R&D and this leads to the invention of new capital goods. This is described using a "production function" for the change in the number of capital goods:

$$\dot{A} = \gamma L_A^{\lambda} A^{\phi} \tag{2}$$

The change in the number of capital goods depends positively on the number of researchers (λ is an index of how slowly diminishing marginal productivity sets in for researchers) and also on the prevailing value of A itself. This latter effect stems from the "giants shoulders" effect.[1] For instance, the invention of a new piece of software will have relied on the previous invention of the relevant computer hardware, which itself relied on the previous invention of semiconductor chips, and so on.

Romer's model contains a full description of the factors that determines the fraction of workers that work in the research section. The research sector gets rewarded with patents that allow it to maintain a monopoly in the product it invents; wages are equated across sectors, so the research sector hire workers up to point where their value to it is as high as it is to producers of final output. In keeping with the spirit of the Solow model, I'm going to just treat the share of workers in the research sector as an exogenous parameter (but will discuss later some of the factors that should determine this share). So, we have

$$L = L_A + L_Y \tag{3}$$

$$L_A = s_A L \tag{4}$$

And again we assume that the total number of workers grows at an exogenous rate n:

$$\frac{\dot{L}}{L} = n \tag{5}$$

[1] Stemming from Isaac Newton's observation "If I have seen farther than others, it is because I was standing on the shoulders of giants."

Simplifying the Aggregate Production Function

We can define the aggregate capital stock as

$$K = \sum_{i=1}^{A} x_i \tag{6}$$

Again, we'll treat the savings rate as exogenous and assume

$$\dot{K} = s_K Y - \delta K \tag{7}$$

One observation that simplifies the analysis of the model is the fact that all of the capital goods play an identical role in the production process. For this reason, we can assume that the demand from producers for each of these capital goods is the same, implying that

$$x_i = \bar{x} \qquad i = 1, 2, ...A \tag{8}$$

This means that the production function can be written as

$$Y = A L_Y^{1-\alpha} \bar{x}^{\alpha} \tag{9}$$

Note now that

$$K = A\bar{x} \Rightarrow \bar{x} = \frac{K}{A} \tag{10}$$

so output can be re-expressed as

$$Y = A L_Y^{1-\alpha} \left(\frac{K}{A}\right)^{\alpha} = (A L_Y)^{1-\alpha} K^{\alpha} \tag{11}$$

This looks just like the Solow model's production function. The TFP term is written as $A^{1-\alpha}$ as opposed to just A as it was in our first handout, but this makes no difference to the substance of the model.

Steady-State Growth in The Romer Model

You can use the same arguments as before to show that this economy converges to a steady-state growth path in which capital and output grow at the same rate. So, we can derive the steady-state growth rate as follows. Re-write the production function as

$$Y = (A s_Y L)^{1-\alpha} K^{\alpha} \tag{12}$$

where

$$s_Y = 1 - s_A \tag{13}$$

Our usual procedure for taking growth rates give us

$$\frac{\dot{Y}}{Y} = (1-\alpha)\left(\frac{\dot{A}}{A} + \frac{\dot{s_Y}}{s_Y} + \frac{\dot{L}}{L}\right) + \alpha \frac{\dot{K}}{K} \tag{14}$$

Now use the fact that the steady-state growth rates of capital and output are the same to derive that this steady-state growth rate is given by

$$\left(\frac{\dot{Y}}{Y}\right)^* = (1-\alpha)\left(\frac{\dot{A}}{A} + \frac{\dot{s_Y}}{s_Y} + \frac{\dot{L}}{L}\right) + \alpha \left(\frac{\dot{Y}}{Y}\right)^* \tag{15}$$

Finally, because the share of labour allocated to the non-research sector cannot be changing along the steady-state path (otherwise the fraction of researchers would eventually go to zero or become greater than one, which would not be feasible) we have

$$\left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}\right)^* = \frac{\dot{A}}{A} \tag{16}$$

The steady-state growth rate of output per worker equals the steady-state growth rate of A. The only difference from the Solow model is that writing the TFP term as $A^{1-\alpha}$ makes this growth rate $\frac{\dot{A}}{A}$ as opposed to $\frac{1}{1-\alpha}\frac{\dot{A}}{A}$.

Deriving the Steady-State Growth Rate

The big difference relative to the Solow model is that the A term is determined within the model as opposed to evolving at some fixed rate unrelated to the actions of the agents in the model economy. To derive the steady-state growth rate in this model, note that the growth rate of the number of capital goods is

$$\frac{\dot{A}}{A} = \gamma \left(s_A L\right)^{\lambda} A^{\phi-1} \tag{17}$$

The steady-state of this economy features A growing at a constant rate. This can only be the case if the growth rate of the right-hand-side of (17) is zero. Using our usual procedure for calculating growth rates of Cobb-Douglas-style items, we get

$$\lambda \left(\frac{\dot{s}_A}{s_A} + \frac{\dot{L}}{L} \right) - (1-\phi)\frac{\dot{A}}{A} = 0 \tag{18}$$

Again, in steady-state, the growth rate of the fraction of researchers ($\frac{\dot{s}_A}{s_A}$) must be zero. So, along the model's steady-state growth path, the growth rate of the number of capital goods (and hence output per worker) is

$$\left(\frac{\dot{A}}{A}\right)^{*} = \frac{\lambda n}{1-\phi} \tag{19}$$

The long-run growth rate of output per worker in this model depends on positively on three factors:

- The parameter λ, which describes the extent to which diminishing marginal productivity sets in as we add researchers.
- The strength of the "standing on shoulders" effect, ϕ. The more past inventions help to boost the rate of current inventions, the faster the growth rate will be.

- The growth rate of the number of workers n. The higher this, the faster the economy adds researchers. This may seem like a somewhat unusual prediction, but it holds well if one takes a very long view of world economic history. Prior to the industrial revolution, growth rates of population and GDP per capita were very low. The past 200 years have seen both population growth and economic growth rates increases. See the figures on the next two pages (the first comes from Greg Clark's book *A Farewell to Alms* which provides a very interesting discussion of pre-Industrial-Revolution economies.)

Figure 1: World Economic History

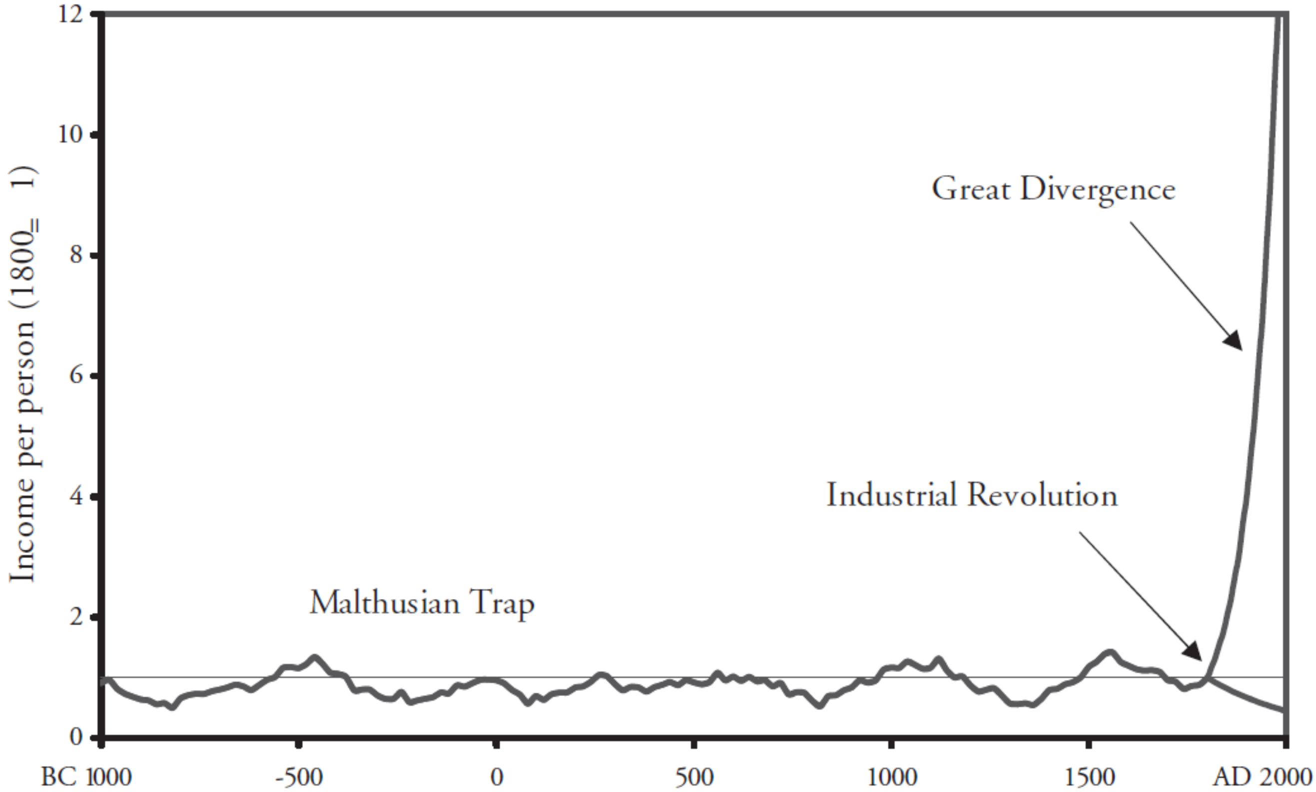
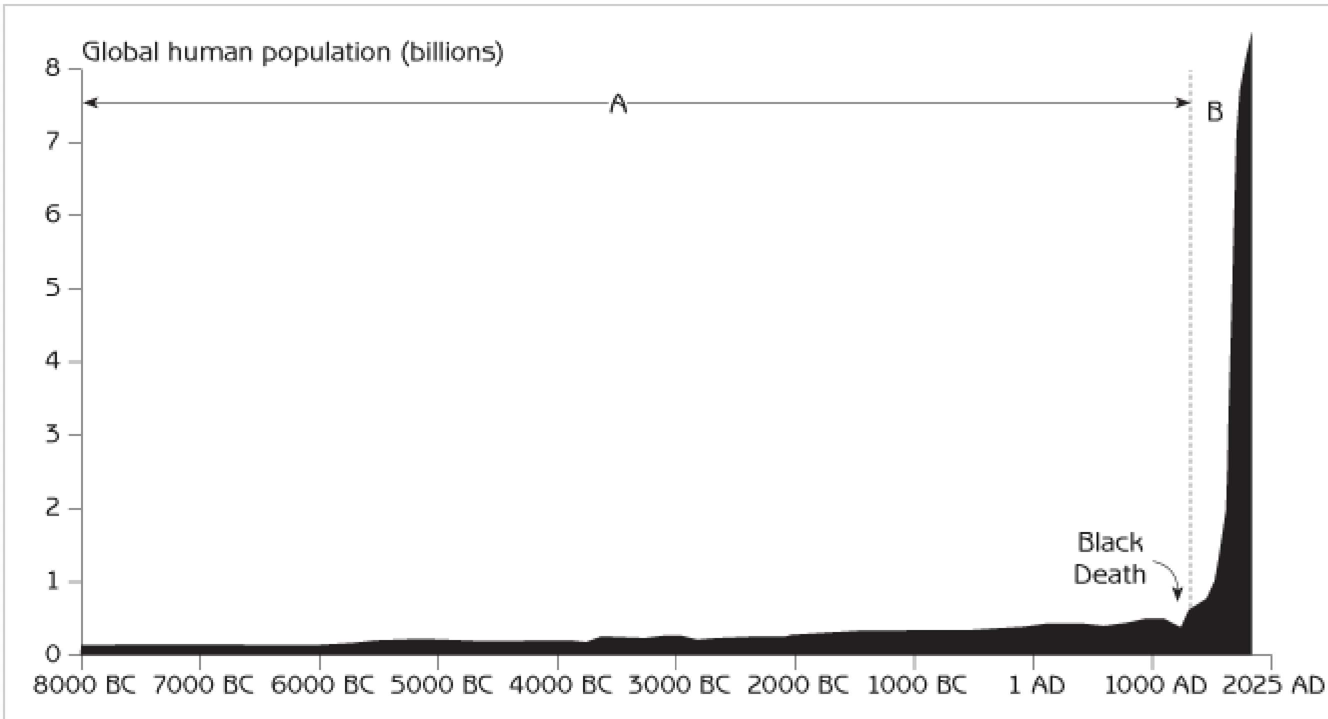


Figure 1.1 World economic history in one picture. Incomes rose sharply in many countries after 1800 but declined in others.

Figure 2: Global Population



The Steady-State Level of Output Per Worker

Just as with our discussion of the Solow model, we can decompose output per worker into a capital-output ratio component and a TFP component. In other words, one can re-arrange equation (11) to get

$$\frac{Y}{L_Y} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A \tag{20}$$

and use the fact that $L_Y = (1 - s_A)L$ to get

$$\frac{Y}{L} = (1 - s_A)\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A \tag{21}$$

Note that the s_A term reflects the reduction in the production of goods and services due to a fraction of the labour force being employed as researchers. One can also use the same arguments to show that, along the steady-state growth path the capital-output ratio is

$$\left(\frac{K}{Y}\right)^* = \frac{s_K}{n + \frac{\lambda n}{1-\phi} + \delta} \tag{22}$$

(The $\frac{\lambda n}{1-\phi}$ here takes the place of the $\frac{g}{1-\alpha}$ in the first handout's expression for the steady-state capital-output ratio because this is the new formula for the growth rate of output per worker).

Finally, we can also figure out the level of A along the steady-state growth path as follows. Along the steady-state path, we have

$$\frac{\dot{A}}{A} = \gamma \left(s_A L\right)^{\lambda} A^{\phi-1} = \frac{\lambda n}{1-\phi} \tag{23}$$

This latter equality can be re-arranged as

$$A^* = \left(\frac{\gamma\left(1-\phi\right)}{\lambda n}\right)^{\frac{1}{1-\phi}} \left(s_A L\right)^{\frac{\lambda}{1-\phi}} \tag{24}$$

So, along the steady-state growth path, output per worker is

$$\left(\frac{Y}{L}\right)^* = (1 - s_A)\left(\frac{s_K}{n + \frac{\lambda n}{1-\phi} + \delta}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\gamma\left(1-\phi\right)}{\lambda n}\right)^{\frac{1}{1-\phi}} \left(s_A L\right)^{\frac{\lambda}{1-\phi}} \tag{25}$$

Convergence Dynamics for A

We noted already that the arguments showing that the capital-output ratio tends to converge towards its steady-state are the same here as in the Solow model. What about the A term? How do we know, for instance, that A always reverts back eventually to the path given by equation (24)? To see that this is the case, let

$$g_A = \frac{\dot{A}}{A} = \gamma \left(s_A L\right)^{\lambda} A^{\phi-1} \tag{26}$$

The growth rate of theh right-hand-side of this equation is

$$\frac{\dot{g}_A}{g_A} = \lambda \left(\frac{\dot{s}_A}{s_A} + n\right) - \left(1 - \phi\right) g_A \tag{27}$$

One can use this equation to show that g_A will be falling whenever

$$g_A > \frac{\lambda n}{1 - \phi} + \frac{\lambda}{1 - \phi} \frac{\dot{s}_A}{s_A} \tag{28}$$

So, apart from periods when the share of researchers is changing, the growth rate of A will be declining whenever it is greater than its steady-state value of $\frac{\lambda n}{1-\phi}$. The same argument works in reverse when g_A is below its steady-state value. Thus, the growth rate of A displays convergent dynamics, always tending back towards its steady-state value. And equation (24) tells us exactly what the *level* of A has to be if the growth rate of A is at its steady-state value.

Optimal R&D?

We haven't discussed the various factors that may determine the share of the labour force allocated to the research sectors, s_A. However, in equation (25) we have diagnosed two separate offsetting effects that s_A has on output: A negative one caused by the fact the researchers don't actually produce output, and a positive one due to the positive effect of the share of researchers on the level of technology.

Equation (25) looks very complicated but it looks simpler if we just take all the terms that don't involve s_A and bundle them together calling them X and also write $Z = \frac{\lambda}{1-\phi}$. In this case, the equation becomes

$$\left(\frac{Y}{L}\right)^* = X\left(1-s_A\right)\left(s_A\right)^Z \tag{29}$$

Written like this, it is a relatively simple calculus problem to figure out the level of s_A that maximises the level of output per worker along the steady-state growth path. In other words, one can can differentiate equation (25) with respect to s_A, set equal to zero, and solve to obtain that this optimizing share of researchers is

$$s_A^{**} = \frac{Z}{1+Z} = \frac{\frac{\lambda}{1-\phi}}{1+\frac{\lambda}{1-\phi}} = \frac{\lambda}{1-\phi+\lambda} \tag{30}$$

When one fills in the model to determine s_A endogenously, does the economy generally arrive at this optimal level? No. The reason for this is that research activity generates *externalities* that affect the level of output per worker, but which are not taken into account by private individuals or firms when they make the choice of whether or not to conduct research. Looking at the "ideas" production function, equation (2), one can see both positive and negative externalities:

- A positive externality due to the "giants shoulders" effect. Researchers don't take into account the effect their inventions have in boosting the future productivity of other researchers. The higher is θ, the more likely it is that the R&D share will be too low.

- A negative externality due to the fact that $\lambda < 1$, so diminishing marginal productivity applies to the number of researchers.

Whether there is too little or too much research in the economy relative to the optimal level depends on the strength of these various externalities. However, using empirical estimates of the parameters of equation (2), Charles Jones and John Williams have calculated that it is far more likely that the private sector will do too little research relative to the social optimum.[2]

To give some insight into this result, note that the steady-state growth rate in this model is $\frac{\lambda n}{1-\phi}$, so $\frac{\lambda}{1-\phi}$ is the ratio of the growth rate of output per worker to the growth rate of population. Suppose this equals one, so growth in output per worker equals growth in population—perhaps a reasonable ballpark assumption. In this case $\frac{\lambda}{1-\phi} = 1$ and the optimal share of researchers is one-half. Indeed, for any reasonable steady-state growth rate, the optimal share of researchers is very high, so it is hardly surprising that the economy does not automatically generate this share.

This result points to the potential for policy interventions to boost the rate of economic growth by raising the number of researchers. For instance, laws to strengthen patent protection may raise the incentives to conduct R&D. This points to a potential conflict between macroeconomic policies aimed at raising growth and microeconomic policies aimed at reducing the inefficiencies due to monopoly power: Some amount of monopoly power for patent-holders may be necessary if we want to induce a high level of R&D and thus a high level of output.

[2] Charles I. Jones and John C. Williams, "Too Much of a Good Thing? The Economics of Investment in R&D", *Journal of Economic Growth*, March 2000, Vol. 5, No. 1, pp. 65-85.

Robert Gordon on The Past and Future of New Technologies

Many of the facts about economic history back up Romer's vision of economic growth. Robert Gordon's paper "Is US economic growth over? Faltering innovation confronts the six headwinds" provides an excellent description of the various phases of technological invention and also provides an interesting perspective on the potential for future technological progress. Gordon highlights how economic history can be broken into different periods based on how the invention of technologies have impacted the economy.

The First Industrial Revolution: "centered in 1750-1830 from the inventions of the steam engine and cotton gin through the early railroads and steamships, but much of the impact of railroads on the American economy came later between 1850 and 1900. At a minimum it took 150 years for IR1 to have its full range of effects."

The Second Industrial Revolution: "within the years 1870-1900 created within just a few years the inventions that made the biggest difference to date in the standard of living. Electric light and a workable internal combustion engine were invented in a three-month period in late 1879. The number of municipal waterworks providing fresh running water to urban homes multiplied tenfold between 1870 and 1900. The telephone, phonograph, and motion pictures were all invented in the 1880s. The benefits of IR2 included subsidiary and complementary inventions, from elevators, electric machinery and consumer appliances; to the motorcar, truck, and airplane; to highways, suburbs, and supermarkets; to sewers to carry the wastewater away. All this had been accomplished by 1929, at least in urban America, although it took longer to bring the modern household conveniences to small towns and farms. Additional follow-up inventions continued and had their main effects by 1970, including television, air conditioning, and the interstate highway system. The inventions of IR2 were so important and far-reaching that they took a full 100 years to have their main effect."

The Third Industrial Revolution: "is often associated with the invention of the web and internet around 1995. But in fact electronic mainframe computers began to replace routine and repetitive clerical work as early as 1960."

Gordon's paper is very worth reading for understanding how the innovations associated with the "second industrial revolution" completely altered people's lives. He describes life in 1870 as follows

> most aspects of life in 1870 (except for the rich) were dark, dangerous, and involved backbreaking work. There was no electricity in 1870. The insides of dwelling units were not only dark but also smoky, due to residue and air pollution from candles and oil lamps. The enclosed iron stove had only recently been invented and much cooking was still done on the open hearth. Only the proximity of the hearth or stove was warm; bedrooms were unheated and family members carried warm bricks with them to bed.
>
> But the biggest inconvenience was the lack of running water. Every drop of water for laundry, cooking, and indoor chamber pots had to be hauled in by the housewife, and wastewater hauled out. The average North Carolina housewife in 1885 had to walk 148 miles per year while carrying 35 tonnes of water.

Gordon believes that the technological innovations associated with computer technologies are far less important than those associated with the "second industrial revolution" and that growth may sputter out over time. Figure 1 repeats a chart from Gordon's paper showing the growth rate of per capita GDP for the world's leading economies (first the UK, then the US). It shows growth accelerating until 1950 and declining thereafter. Figure 2 shows a hypothetical chart in which Gordon projects a continuing fall-off in growth.

To illustrate why he believes modern inventions don't match up with past improvements, Gordon offers the following thought experiment.

> You are required to make a choice between option A and option B. With option A you are allowed to keep 2002 electronic technology, including your Windows 98 laptop accessing Amazon, and you can keep running water and indoor toilets; but you can't use anything invented since 2002.
>
> Option B is that you get everything invented in the past decade right up to Facebook, Twitter, and the iPad, but you have to give up running water and indoor toilets. You have to haul the water into your dwelling and carry out the waste. Even at 3am on a rainy night, your only toilet option is a wet and perhaps muddy walk to the outhouse. Which option do you choose?

You probably won't be surprised to find out that most people pick option B.

Gordon also discusses other factors likely to hold back growth in leading countries such as the leveling off of a long-run pattern of educational achievement, an aging population and energy-related constraints. It's worth noting, though, that while Gordon's paper is very well researched and well argued, economists are not very good at forecasting the invention of new technologies or their impact on the economy. For all we know, the next "industrial revolution" could be around the corner to spark a new era of rapid growth. Joel Mokyr's article "Is technological progress a thing of the past?" (linked to on the website) is a good counterpart to Gordon's scepticism.

Figure 1: Gordon on the Growth Rate of Leading Economies

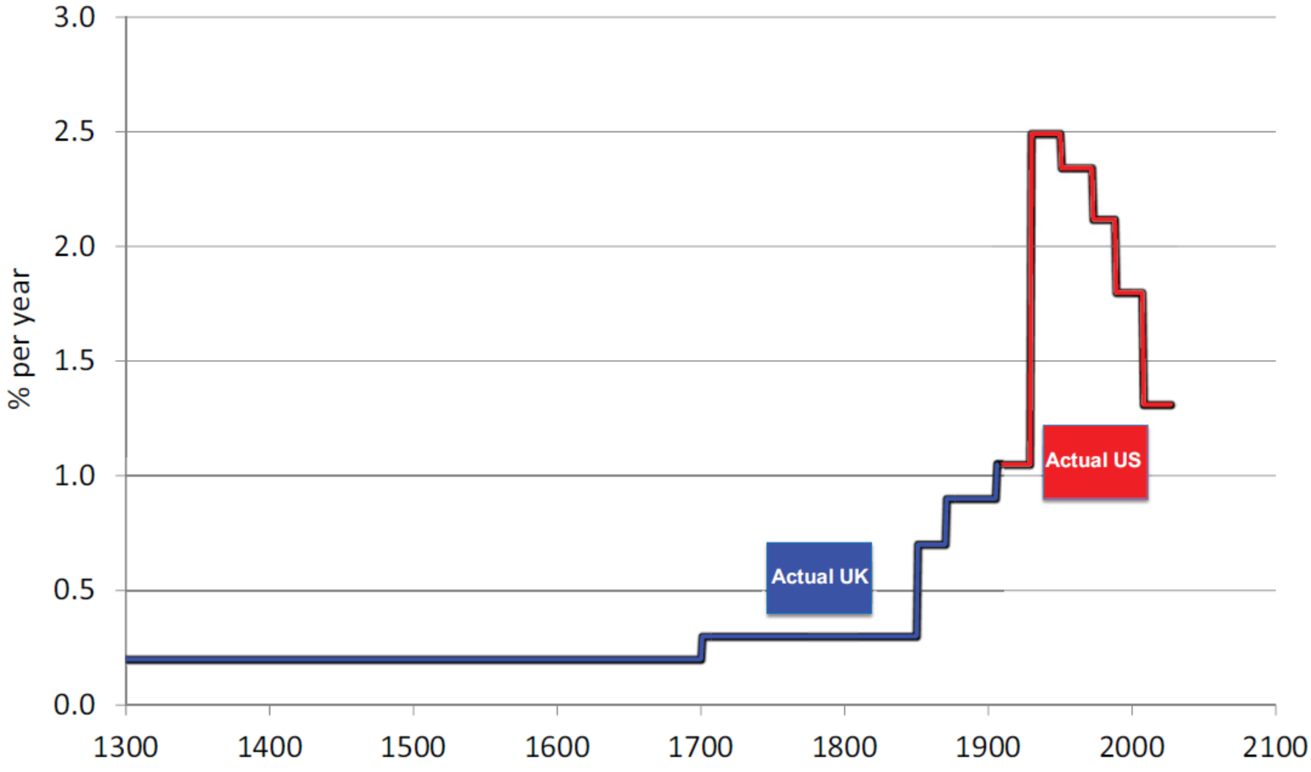
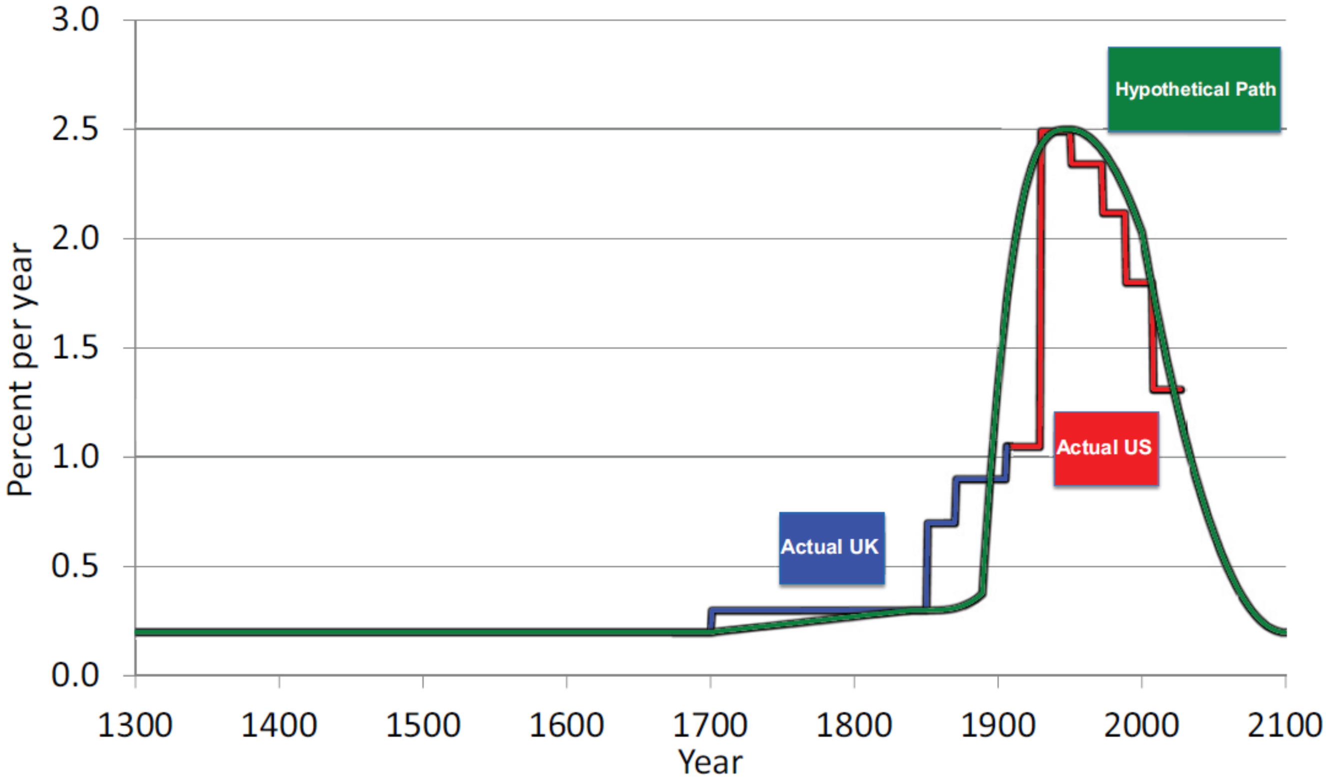


Figure 2: Gordon's Hypothetical Path for Growth



Things to Understand from these Notes

Here's a brief summary of the things that you need to understand from these notes.

1. The Romer model's production function.
2. The model's assumptions about how the number of capital goods changes.
3. How to simplify the aggregate production function.
4. How to derive the steady-state growth rate.
5. The steady-state level of output per worker.
6. Why A converges to its steady-state level.
7. The optimal level of R&D and why the observed level is probably below it.
8. Policy trade-offs suggested by the Romer model.
9. Robert Gordon on the history and future of technological innovation.