Cross-Country Technology Diffusion

So far, we've been discussing how the invention of new technologies promotes economic growth by pushing out the "technological frontier" and allowing capital to be allocated across new and old technologies with diminishing returns setting in. This is clearly an important aspect of economic growth. However, we should remember that only a very few countries in the world are "on the technological frontier"—most places are not relying on Apple to invent a new gadget to promote efficiency. One way to illustrate this point is to estimate the level of total factor productivity for different countries in the world.

An important paper that did these calculations and used them to shed light on cross-country income differences is the paper on the reading list by Hall and Jones (1999). The basis of the study is a "levels accounting" exercise that starts from the following production function

$$Y_i = K_i^{\alpha} \left(h_i A_i L_i\right)^{1-\alpha} \tag{1}$$

Like the BLS multifactor productivity calculations that we discussed a few lectures ago, Hall and Jones account for the effect of education on the productivity of the labour force. Specifically, they construct measures of *human capital* based on estimates of the return to education—this is the h_i in the above equation.

Hall and Jones show that their production function can be re-formulated as

$$\frac{Y_i}{L_i} = \left(\frac{K_i}{Y_i}\right)^{\frac{\alpha}{1-\alpha}} h_i A_i \tag{2}$$

Hall and Jones then constructed a measure h_i using evidence on levels of educational attainment and they also set $\alpha = 1/3$. This allowed them to use (2) to express all cross-country differences in output per worker in terms of three multiplicative terms: capital intensity, human capital per worker, and technology or total factor productivity. They found that output per worker in the richest five countries was 31.7 times that in the poorest five countries. This was explained as follows:

- Differences in capital intensity contributed a factor of 1.8.
- Differences in human capital contributed a factor of 2.2
- The remaining difference—a factor of 8.3—was due to differences in TFP.

The results from this paper show that differences in total factor productivity, rather than differences in factor accumulation, are the key explanation of cross-country variations in income levels. A more detailed table of Hall and Jones's calculations is reproduced on the next page. These calculations show that most countries are very far from the technological frontier, so their growth is not likely to be reliant on the invention of new technologies.

Table from Hall-Jones Paper

TABLE I
PRODUCTIVITY CALCULATIONS: RATIOS TO U. S. VALUES

Country	Y/L	Contribution from $(K/Y)^{\alpha/(1-\alpha)}$	H/L	A
United States	1.000	1.000	1.000	1.000
Canada	0.941	1.002	0.908	1.034
Italy	0.834	1.063	0.650	1.207
West Germany	0.818	1.118	0.802	0.912
France	0.818	1.091	0.666	1.126
United Kingdom	0.727	0.891	0.808	1.011
Hong Kong	0.608	0.741	0.735	1.115
Singapore	0.606	1.031	0.545	1.078
Japan	0.587	1.119	0.797	0.658
Mexico	0.433	0.868	0.538	0.926
Argentina	0.418	0.953	0.676	0.648
U.S.S.R.	0.417	1.231	0.724	0.468
India	0.086	0.709	0.454	0.267
China	0.060	0.891	0.632	0.106
Kenya	0.056	0.747	0.457	0.165
Zaire	0.033	0.499	0.408	0.160
Average, 127 countries:	0.296	0.853	0.565	0.516
Standard deviation:	0.268	0.234	0.168	0.325
Correlation with Y/L (logs)	1.000	0.624	0.798	0.889
Correlation with A (logs)	0.889	0.248	0.522	1.000

The elements of this table are the empirical counterparts to the components of equation (3), all measured as ratios to the U. S. values. That is, the first column of data is the product of the other three columns.

Leaders and Followers

The Romer model probably should not be thought of as a model of growth in any one particular country. No country uses only technologies that were invented in that country; rather, products invented in one country end up being used all around the world. Thus, the model is best thought of as a model of the leading countries in the world economy. How then should long-run growth rates be determined for individual countries? By itself, the Romer model has no clear answer, but it suggests a model in which ability to learn about the usage of new technologies should plays a key role in determining output per worker.

We will now describe such a model. The mathematics of the model are also formally equivalent to a well-known model of Nelson and Phelps (*AER*, 1966), though the application there is different, their subject being the diffusion of technological knowledge over time within an individual country.

The Model

We will assume that there is a "lead" country in the world economy that has technology level, A_t at time t which grows at rate g every period, so that

$$\frac{\dot{A}_t}{A_t} = g \tag{3}$$

All other countries in the world, indexed by j, have technology levels given by $A_{jt} < A_t$. The growth rate of technology in country j is determined by

$$\frac{\dot{A}_{jt}}{A_{jt}} = \lambda_j + \sigma_j \frac{(A_t - A_{jt})}{A_{jt}} \tag{4}$$

where $\lambda_j < g$ and $\sigma_j > 0$. This tells us that technology growth in all countries apart from the lead country is determined by two factors

- Learning: The second term says that their technology level will grow faster the bigger is the percentage gap between its level of technology, A_{jt} and the level of the leader, A_t. The larger is the parameter σ_j, the better the country is at learning about the technologies being applied in the lead country.

- The first term, λ_j indicates the country's capacity for increasing its level of technology without learning from the leader. We impose the condition $\lambda_j < g$. This means that country j can't grow faster than the lead country without the learning that comes from having lower technology than the frontier.

Exponential Growth

You've probably heard about exponential functions before but, even if you have, it's worth a quick reminder. The number $e \approx 2.71828$ is a very special number such that the function

$$\frac{de^x}{dx} = e^x \tag{5}$$

One way to see why the number is 2.718 is to use something called the Taylor series approximation for a function, which states that you can approximate a function $f(x)$ as

$$f(x) = f(a) + f'(x)(x-a) + \frac{1}{2}f''(x))(x-a)^2 + \frac{1}{3!}f'''(x))(x-a)^3 + ...\frac{1}{n!}f^n(x))(x-a)^n + ... \tag{6}$$

where $n! = (1)(2)(3)...(n-1)(n)$. If there is a number, e that has the property that $e^x = f(x) = f'(x)$, then that means that all derivatives also equal e^x. In this case, we have

$$e^x = e^a + e^a(x-a) + \frac{1}{2}e^a(x-a)^2 + \frac{1}{3!}e^a(x-a)^3 + ... \tag{7}$$

Setting $x = 1, a = 0$, this becomes

$$e = 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + ... \tag{8}$$

This converges to 2.71828. Ok, that's not on the test but worth knowing. Now note that

$$\frac{de^{gt}}{dt} = \frac{de^{gt}}{d(gt)}\frac{d(gt)}{dt} = ge^{gt} \tag{9}$$

Now let's relate this back to our model. The fact that the lead country has growth such that

$$\frac{dA_t}{dt} = \dot{A}_t = gA_t \tag{10}$$

means that this country is characterised by what is known as exponential growth, i.e.

$$A_t = A_0 e^{gt} \tag{11}$$

We write the first term as A_0 because $e^{(g)(0)} = 1$ so whatever term multiplies e^{gt} that is the value that A_t takes in the first period.

Dynamics of Technology

Now we are going to try to figure out how the technology variable behaves in the follower country. First, lets take equation (4) and multiply across by A_{jt} to get

$$\dot{A}_{jt} = \lambda_j A_{jt} + \sigma_j \left(A_t - A_{jt}\right) \tag{12}$$

This is what is known as a first-order linear differential equation (differential equation because it involves a derivative; first-order because it only involves a first derivative; linear because it doesn't involve any terms taken to powers than are not one.) These equations can be solved to illustrate how A_j changes over time. To do this, we first draw some terms together to re-write it as

$$\dot{A}_{jt} + \left(\sigma_j - \lambda_j\right) A_{jt} = \sigma_j A_t \tag{13}$$

Recalling equation (11) for the technology level of the leader country, this differential equation can be re-written as

$$\dot{A}_{jt} + \left(\sigma_j - \lambda_j\right) A_{jt} = \sigma_j A_0 e^{gt} \tag{14}$$

Now we'll move on to illustrating how people figure out how an A_{jt} that satisfies this equation needs to behave.

One Possible Solution

Let's think about what we learned about exponential functions to help us see what form a potential solution might take. The derivative of A_{jt} with respect to time plus $(\sigma_j - \lambda_j)$ times A_{jt} can be written as a multiple of the exponential function.

Looked at this way, we might guess that one possible solution for an A_{jt} process that will satisfy this equation is something of the form $B_j e^{gt}$ where B_j is some unknown coefficient. Indeed, it turns out that this is the case. Let's figure out what B_j must be. It must satisfy

$$gB_j e^{gt} + (\sigma_j - \lambda_j) B_j e^{gt} = \sigma_j A_0 e^{gt} \tag{15}$$

Canceling the e^{gt} terms, we see that

$$B_j = \frac{\sigma_j A_0}{\sigma_j + g - \lambda_j} \tag{16}$$

So, this solution takes the form

$$A_{jt}^p = B_j e^{gt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j} \right) A_0 e^{gt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j} \right) A_t \tag{17}$$

A General Solution

Is that it or could we add on an additional term and still get a solution? Suppose we look for a solution of the form

$$A_{jt} = B_j e^{gt} + D_{jt} \tag{18}$$

Then the solution would have to obey

$$gBe^{gt} + \dot{D}_{jt} + (\sigma_j - \lambda_j)\left(Be^{gt} + D_{jt}\right) = \sigma_j A_0 e^{gt} \quad (19)$$

All the terms in e^{gt} cancel out because, by construction of B_j, they satisfy equation (15). This means the additional term D_{jt} must satisfy

$$\dot{D}_{jt} + (\sigma_j - \lambda_j) D_{jt} = 0 \quad (20)$$

Again using the properties of the exponential function, this equation is satisfied by anything of the form

$$D_{jt} = D_{j0} e^{-(\sigma_j - \lambda_j)t} \quad (21)$$

where D_{j0} is a parameter that can take on any value. So, given the differential equation (12), all possible solutions for technology in country j must take the form

$$A_{jt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) A_t + D_{j0} e^{-(\sigma_j - \lambda_j)t} \quad (22)$$

where D_{j0} is an arbitrary parameter than can take any value.

Properties of the Solution

Now we like to examine the properties of this solution. Does technology in the follower country catch up and, if not, where does it end up and why? To answer these questions, it is useful to express A_{jt} as a ratio of the frontier level of technology. This can be written as

$$\frac{A_{jt}}{A_t} = \frac{\sigma_j}{\sigma_j + g - \lambda_j} + \frac{D_{j0}}{A_t} e^{-(\sigma_j - \lambda_j)t} \quad (23)$$

Now using the fact that $A_t = A_0 e^{gt}$, this becomes

$$\frac{A_{jt}}{A_t} = \frac{\sigma_j}{\sigma_j + g - \lambda_j} + \frac{D_{j0}}{A_0} e^{-(\sigma_j + g - \lambda_j)t} \quad (24)$$

To understand the properties of this solution, recall that we assumed $\lambda_j < g$, which means that on its own (without catch-up growth) the follower country's level of technology grows slower than the leader country and also that $\sigma_j > 0$ (some learning takes place). Putting these two assumptions together, we can say

$$\sigma_j + g - \lambda_j > 0 \tag{25}$$

That means that

$$e^{-(\sigma_j + g - \lambda_j)t} \to 0 \quad \text{as} \quad t \to \infty \tag{26}$$

This means that the second term in (24) tends towards zero. So, over time, as this term disappears, the country converges towards a level of technology that is a constant ratio, $\frac{\sigma_j}{\sigma_j + g - \lambda_j}$ of the frontier level, and its growth rate tends towards g.

Note that $g - \lambda_j > 0$ also means that

$$0 < \frac{\sigma_j}{\sigma_j + g - \lambda_j} < 1 \tag{27}$$

so each country never actually catches up to the leader but instead converges to some fraction of the lead country's technology level. This makes sense if you think about it. Because of their inferiority at developing their own technologies ($\lambda_j < g$) the follower countries will always be falling further behind the leader unless there is a gap between their level of technology and the leader. So, to have a steady-state in which everyone's technology is growing at the same rate, the followers must all have technology levels below that of the leader.

In addition, $g - \lambda_j > 0$ means that

$$\frac{d}{d\sigma_j}\left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) > 0 \tag{28}$$

The equilibrium ratio of the country's technology to the leader's depends positively on the "learning parameter" σ_j. The higher this parameter—the more fo the gap to the leader that it closes each period—then the close the ratio gets to one and the higher up the "pecking order" the country gets. It's also true that

$$\frac{d}{d\lambda_j}\left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) > 0 \tag{29}$$

In other words, the more growth the country can generate each period independent of learning from the leader, the higher will be its equilibrium ratio of technology relative to the leader.

Illustrating the Model

Going back to the equation for the ratio of technology in country j to the leader, equation (24), we noted already that the second term tends to disappear to zero over time. That doesn't mean it's unimportant. How a country behaves along its "transition path" depends on the value of the initial parameter D_{j0}.

- If $D_{j0} < 0$, then the term that is disappearing over time is a negative term that is a drag on the level of technology. This means that the country starts out below its equilbrium technology ratio, grows faster than the leader for some period of time with growth eventually tailing off to the growth rate of the leader.

- If $D_{j0} > 0$, then the term that is disappearing over time is a positive term that is boosting the level of technology. This means that the country starts out above its equilbrium technology ratio, grows slower than the leader for some period of time with growth eventually moves up towards the growth rate of the leader.

We have illustrated how these dynamics would work with the first two charts at the back of the notes. These charts show model simulations for a leader economy with $g = 0.02$ and a follower economy with $\lambda_j = 0.01$ and $\sigma_j = 0.04$. These values mean

$$\frac{\sigma_j}{\sigma_j + g - \lambda_j} = \frac{0.04}{0.04 + 0.02 - 0.01} = 0.8 \tag{30}$$

so the follower economy converges to a level of technology that is 20 percent below that of the leader. The first collection of charts show what happens when this economy has a value of $D_{j0} = -0.5$, so that it starts out with a technology level only 30 percent that of the leader. They grow faster than the leader country for a number of years before they approach the 0.8 equilibrium ratio and then their growth rate settles down to the same rate as that of the leader.

The second collection of charts show what happens when this economy has a value of $D_{j0} = 0.5$, so that it starts out with a technology level 30 percent above that of the leader, even though the equilibrium value is 20 percent below. Technology levels in this follower country never actually decline but they do go through a long-period of slow growth rates before eventually heading towards the same growth rate as the leader as they approach the 0.8 equilibrium ratio.

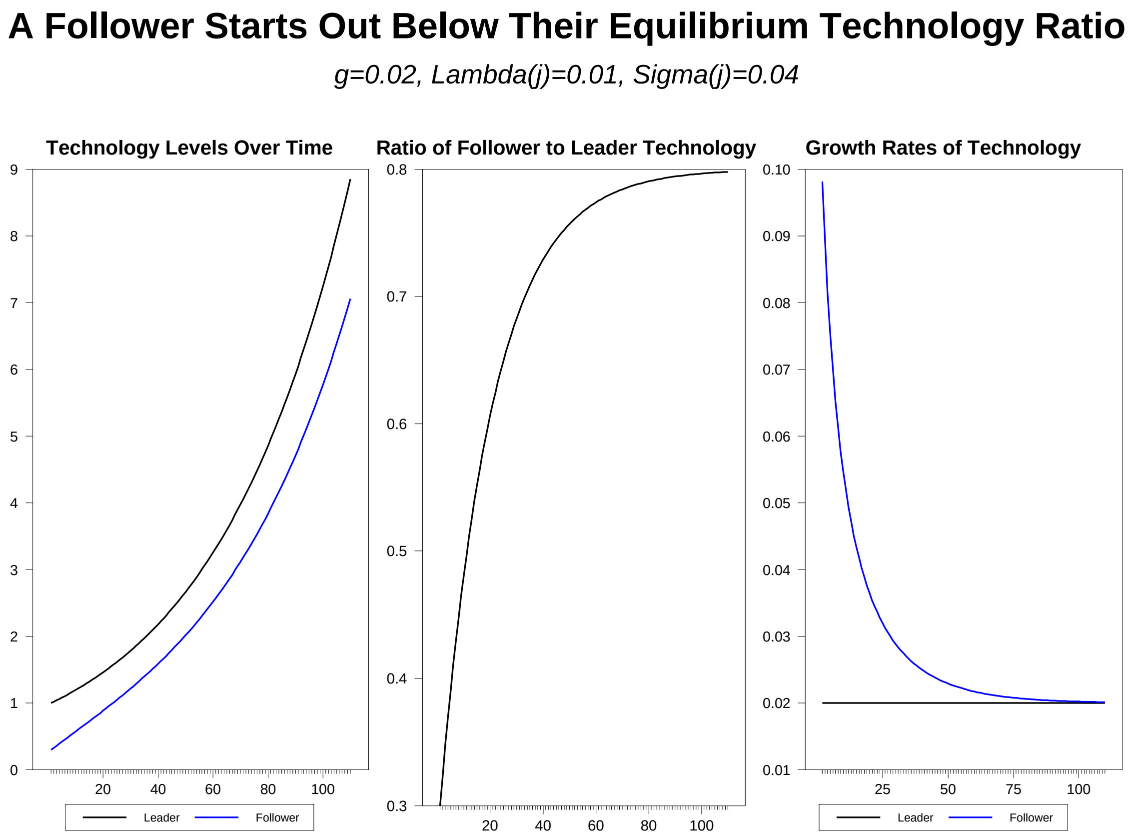
Finally, we show how the model may also be able to account for the sort of "growth miracles" that are occasionally observed when countries suddenly start experiencing rapid growth: If a country can increase its value of σ_j via education or science-related policies, its position in the steady-state distribution of income may move upwards substantially, with the economy then going through a phase of rapid growth. The third collection of charts show what happens when, in period 21, an economy changes from having $\sigma_j = 0.005$ to $\sigma_j = 0.04$. The equilbrium technology ratio changes from one-third to 0.8 and the economy experiences a long transitional period of rapid growth.

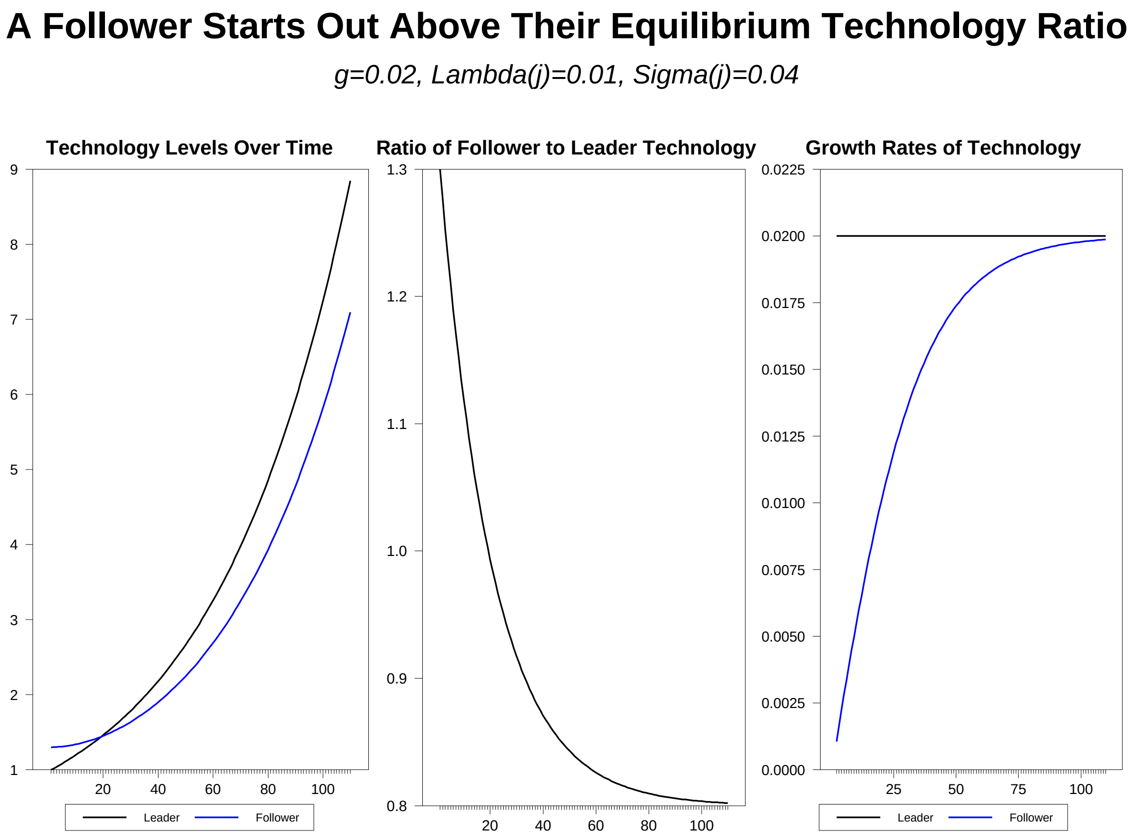
An important message from this model is that for most countries, it is not their ability to invent new capital goods that is key to high living standards, but rather their ability to learn from those countries that are more technologically advanced.

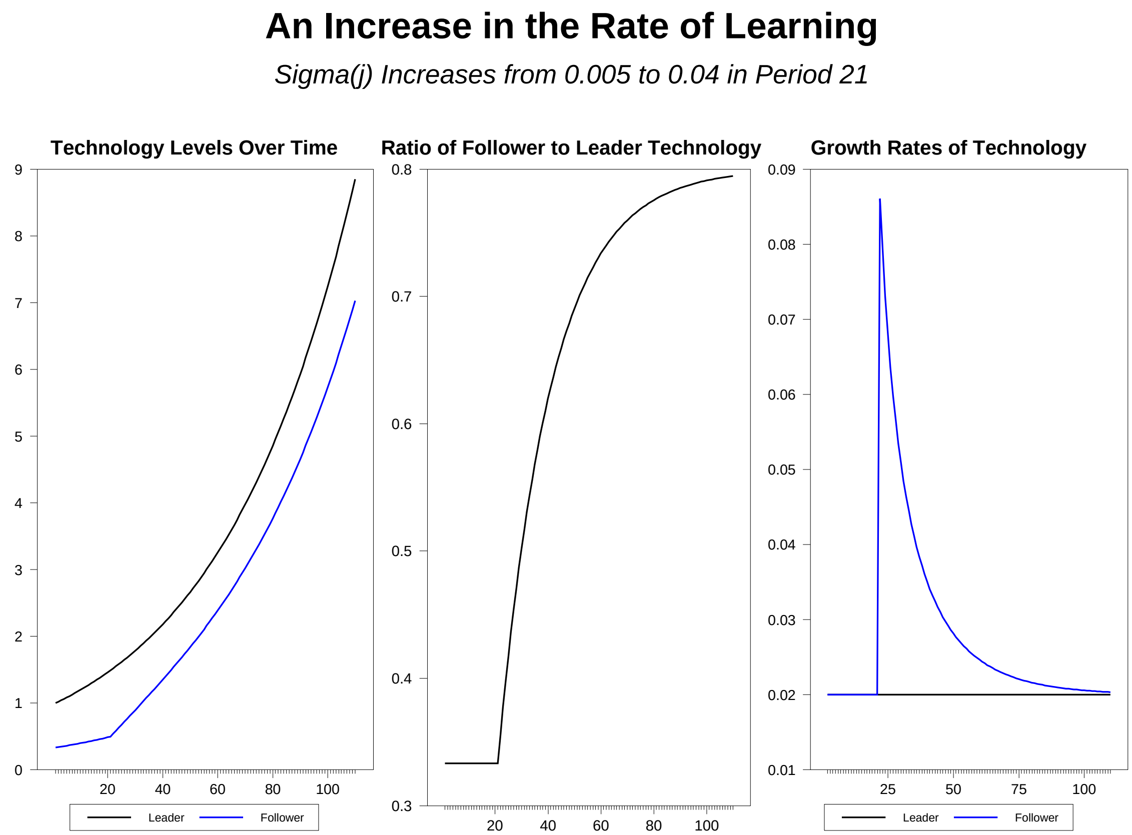
Things to Understand from these Notes

Here's a brief summary of the things that you need to understand from these notes.

1. Evidence on the sources of cross-country differences in output per worker.
2. The model's assumptions and the meaning of its parameters.
3. Exponential growth: The properties of the function e^{gt}.
4. The model's differential equation and its two-part solution method.
5. Properties of the solution: How dynamics depend on σ_j, λ_j and A_0^g.
6. How the model can explain long periods of rapid growth or protacted slumps.
7. "What if" scenarios: What happens if a parameter changes?


A Follower Starts Out Below Their Equilibrium Technology Ratio
g=0.02, Lambda(j)=0.01, Sigma(j)=0.04
Technology Levels Over Time
Ratio of Follower to Leader Technology
Growth Rates of Technology
Leader
Follower
Leader
Follower


A Follower Starts Out Above Their Equilibrium Technology Ratio
g=0.02, Lambda(j)=0.01, Sigma(j)=0.04
Technology Levels Over Time
Ratio of Follower to Leader Technology
Growth Rates of Technology
Leader
Follower


An Increase in the Rate of Learning
Sigma(j) Increases from 0.005 to 0.04 in Period 21
Technology Levels Over Time
Ratio of Follower to Leader Technology
Growth Rates of Technology
Leader
Follower
Leader
Follower