

## Sticky Prices and the Phillips Curve

One of the themes of the first part of this course was that the behaviour of prices was crucial in determining how the macro-economy responded to shocks. In the IS-LM model, we needed to assume that prices were “sticky” in the short-run to obtain real effects for fiscal and monetary policy but we assumed that prices were flexible in the long-run so that the economy returned to its full employment level over time. In the IS-MP-PC theory, we formalised this idea a bit more: This model featured prices that adjusted over time in response to the real economy according to a Phillips curve.

In these notes, we will return to the topic of price setting and the relationship over time between inflation and the business cycle. We will emphasise the role of price flexibility and expectations.

### Evidence on Price Stickiness

When we discussed IS-LM, we assumed that the price level did not keep moving to constantly equate GDP with the level of output consistent with a natural rate of unemployment. Instead, we assumed that prices only changed gradually over time in response to the real economy. The idea that prices may be “sticky” has a long history in Keynesian macroeconomics but, until recent decades, there was comparatively little evidence on the extent to which prices changed over time.

This has changed since the statistical agencies have made available the micro-data that underlie Consumer Price Indices. To construct CPIs, these agencies collect large numbers of quotes of prices on individual items (e.g. they can tell you the price in April of a bottle of Heinz ketchup at a particular store). These individual price quote data can be used to assess

how often individual prices are changed.

Studies of this type now exist for a large number of countries. For example, Bils and Klenow 2004 paper provided evidence for consumer prices in the United States.<sup>1</sup> An important finding from this research is that the data show a very wide range of the frequency with which different prices change. Figure 1 shows a histogram from Bils and Klenow's paper showing the distribution of the percentage probability that any price changes in a month. These vary from prices that only have a one percent probability of changing each month ("Coin-operated apparel laundry and dry cleaning") to those that have an 80 percent probability of changing each month (gasoline).

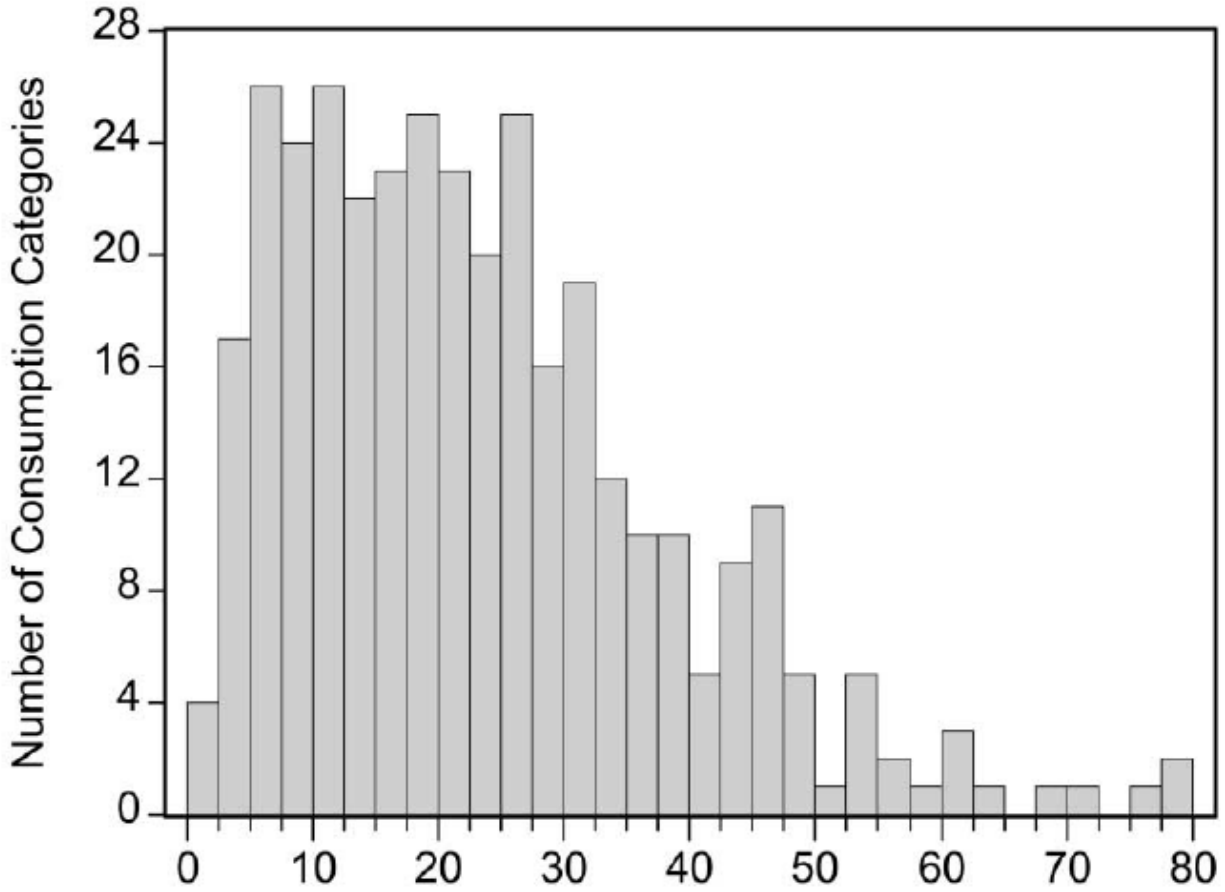
The table on the following page shows the median price duration is about four months. In other words, half of the prices quoted in the CPI index change more than every four months, while the other half change less than every four months. Research for the euro area has shown that price durations are even longer in Europe. For example, Alvarez et al (2006) report a median price duration for the euro area of 10.6 months.<sup>2</sup>

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<sup>1</sup>Mark Bils and Peter Klenow (2004). "Some Evidence on the Importance of Sticky Prices" *Journal of Political Economy*, Volume 112, Number 5.

<sup>2</sup>Luis Alvarez, Emmanuel Dhyne, Marco Hoeberichts, Claudia Kwapil, Herve Le Bihan. Patrick Lunne-  
mann, Fernando Martins, Roberto Sabbatini, Harald Stahl, Philip Vermeulen and Jouko Vilmunen (2006).  
"Sticky Prices in the Euro Area: A Summary of New Micro-Evidence" *Journal of the European Economic  
Association*, Volume 4(2-3), pages 575-584.

Figure 1: The Distribution of Monthly Percent Probability of Price Changes



**Bils and Klenow Evidence on Price Durations**

TABLE 1  
MONTHLY FREQUENCY OF PRICE CHANGES BY YEAR,  
1995–2002

Year	Median Frequency (%)	Median Duration (Months)
1995	21.3	4.2
1996	20.8	4.3
1997	19.9	4.5
1998	21.2	4.2
1999	21.4	4.1
2000	21.7	4.1
2001–2	22.0	4.0

## **New Classical and New Keynesian Macroeconomics**

After Milton Friedman's critique of the Phillips curve, macroeconomists began to pay more attention to the question of how expectations were formed. In particular, a number of papers by Robert Lucas and Thomas Sargent introduced rational expectations into macroeconomic modelling. These early papers tended to assume that prices were perfectly flexible, which limited the ability of fiscal and monetary policy to influence output. This school of thought became labelled *New Classical* economics.

In a number of famous New Classical papers, Robert Lucas argued that monetary policy could still have short-run effects even if prices were flexible and people had rational expectations. Lucas's model relied on the idea that firms had a difficulty in the short-run distinguishing between movements in their prices and movements in the overall price levels. For this reason, an increase in the money supply that provoked an increase in prices could, in the short-run, provoke higher output because firms may believe this is increasing their relative price and making production more profitable. Lucas emphasised, however, that once people had rational expectations, the impact of policy on output could only be short-lived. In particular, he stressed that only *unpredictable* fiscal and monetary policies would have an impact because people with rational expectations would anticipate the impact of predictable policy on the price level.

Once we allow prices to be sticky, however, these points no longer hold. Because some prices will not change even after the government changes fiscal or monetary policy, these policies will have the traditional short-run impacts described in the IS-LM model even if people have rational expectations. There are lots of different ways of formulating the idea that prices may be sticky. Some of the best known formulations were those introduced in papers in

the late seventies by John Taylor and Stanley Fischer.<sup>3</sup> These papers assumed that only a certain fraction of firms set prices each period but those who did change their prices would set them in an optimal manner using rational expectations. This work, which combined rational expectations with sticky prices, invented what is now known as *New Keynesian economics*.

### Pricing à la Calvo

The New Keynesian literature contains a number of different formulations of sticky prices. For the rest of these notes, we will use a formulation of sticky prices known as *Calvo pricing*, after the economist who first introduced it.<sup>4</sup> Though not the most realistic formulation of sticky prices, it turns out to provide analytically convenient expressions, and has implications that are very similar to those of more realistic (but more complicated) formulations.

The form of price rigidity faced by the Calvo firm is as follows. Each period, only a random fraction  $(1-\theta)$  of firms are able to reset their price; all other firms keep their prices unchanged. When firms do get to reset their price, they must take into account that the price may be fixed for many periods. We assume they do this by choosing a log-price,  $z_t$ , that minimizes the “loss function”

$$L(z_t) = \sum_{k=0}^{\infty} (\theta\beta)^k E_t \left( z_t - p_{t+k}^* \right)^2 \quad (1)$$

where  $\beta$  is between zero and one, and  $p_{t+k}^*$  is the log of the optimal price that the firm would set in period  $t+k$  if there were no price rigidity.

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<sup>3</sup>Stanley Fischer (1977), “Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule,” *Journal of Political Economy*, 85, 191-205, and John Taylor (1979), “Staggered Wage Setting in a Macro Model,” *American Economic Review*, Papers and Proceedings, Vol. 69, 108-113.

<sup>4</sup>Guillermo Calvo, “Staggered Contracts in a Utility-Maximizing Framework” *Journal of Monetary Economics*, September 1983.

This expression probably looks a bit intimidating, so it's worth discussing it a bit to explain what it means. The loss function has a number of different elements:

- The term  $E_t (z_t - p_{t+k}^*)^2$  describes the expected loss in profits for the firm at time  $t + k$  due to the fact that it will not be able to set a frictionless optimal price that period. This quadratic function is intended just as an approximation to some more general profit function. What is important here is to note that because the firm may be stuck with the price  $z_t$  for some time, it will lose profits relative to what it would have been able to obtain if there were no price rigidities.
- The summation  $\sum_{k=0}^{\infty}$  shows that the firm considers the implications of the price set today for all possible future periods.
- However, the fact that  $\beta < 1$  implies that the firm places less weight on future losses than on today's losses. A dollar today is worth more than a dollar tomorrow because it can be re-invested. By the same argument, a dollar lost today is more important than a dollar lost tomorrow.
- Future losses are actually discounted at rate  $(\theta\beta)^k$ , not just  $\beta^k$ . This is because the firm only considers the *expected* future losses from the price being fixed at  $z_t$ . The chance that the price will be fixed until  $t + k$  is  $\theta^k$ . So the period  $t + k$  loss is weighted by this probability. There is no point in the firm worrying too much about losses that might occur from having the wrong price far off in the future, when it is unlikely that the price will remain fixed for that long.

### The Optimal Reset Price

After all that, the actual solution for the optimal value of  $z_t$ , (i.e. the price chosen by the firms who get to reset) is quite simple. Each of the terms featuring the choice variable  $z_t$ —that is, each of the  $(z_t - p_{t+k}^*)^2$  terms—need to be differentiated with respect to  $z_t$  and then the sum of these derivatives is set equal to zero. This means

$$L'(z_t) = 2 \sum_{k=0}^{\infty} (\theta\beta)^k E_t (z_t - p_{t+k}^*) = 0 \quad (2)$$

Separating out the  $z_t$  terms from the  $p_{t+k}^*$  terms, this implies

$$\left[ \sum_{k=0}^{\infty} (\theta\beta)^k \right] z_t = \sum_{k=0}^{\infty} (\theta\beta)^k E_t p_{t+k}^* \quad (3)$$

Now, we can use our old pal the geometric sum formula to simplify the left side of this equation. In other words, we use the fact that

$$\sum_{k=0}^{\infty} (\theta\beta)^k = \frac{1}{1 - \theta\beta} \quad (4)$$

to re-write the equation as

$$\frac{z_t}{1 - \theta\beta} = \sum_{k=0}^{\infty} (\theta\beta)^k E_t p_{t+k}^* \quad (5)$$

implying a solution of the form

$$z_t = (1 - \theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k E_t p_{t+k}^* \quad (6)$$

Stated in English, all this equation says is that the optimal solution is for the firm to set its price equal to a weighted average of the prices that it would have expected to set in the future if there weren't any price rigidities. Unable to change price each period, the firm chooses to try to keep close “on average” to the right price.

And what is this “frictionless optimal” price,  $p_t^*$ ? We will assume that the firm's optimal pricing strategy without frictions would involve setting prices as a fixed markup over marginal



cost:

$$p_t^* = \mu + mc_t \quad (7)$$

Thus, the optimal reset price can be written as

$$z_t = (1 - \theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k E_t(\mu + mc_{t+k}) \quad (8)$$

### The New-Keynesian Phillips Curve

Now, we can show how to derive the behaviour of aggregate inflation in the Calvo economy. The aggregate price level in this economy is just a weighted average of last period's aggregate price level and the new reset price, where the weight is determined by  $\theta$ :

$$p_t = \theta p_{t-1} + (1 - \theta) z_t, \quad (9)$$

This can be re-arranged to express the reset price as a function of the current and past aggregate price levels

$$z_t = \frac{1}{1 - \theta} (p_t - \theta p_{t-1}) \quad (10)$$

Now, let's examine equation (8) for the optimal reset price again. We have shown that the first-order stochastic difference equation

$$y_t = ax_t + bE_t y_{t+1} \quad (11)$$

can be solved to give

$$y_t = a \sum_{k=0}^{\infty} b^k E_t x_{t+k} \quad (12)$$

Examining equation (8), we can see that  $z_t$  must obey a first-order stochastic difference equation with

$$y_t = z_t \quad (13)$$

$$x_t = \mu + mc_t \quad (14)$$

$$a = 1 - \theta\beta \quad (15)$$

$$b = \theta\beta \quad (16)$$

In other words, we can write the reset price as

$$z_t = \theta\beta E_t z_{t+1} + (1 - \theta\beta)(\mu + mc_t) \quad (17)$$

Substituting in the expression for  $z_t$  in equation (10) we get

$$\frac{1}{1 - \theta} (p_t - \theta p_{t-1}) = \frac{\theta\beta}{1 - \theta} (E_t p_{t+1} - \theta p_t) + (1 - \theta\beta)(\mu + mc_t) \quad (18)$$

After a bunch of re-arrangements, this equation can be shown to imply

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \theta\beta)}{\theta} (\mu + mc_t - p_t) \quad (19)$$

where  $\pi_t = p_t - p_{t-1}$  is the inflation rate.

This equation is known as the *New-Keynesian Phillips Curve*. It states that inflation is a function of two factors:

- Next period's expected inflation rate,  $E_t \pi_{t+1}$ .
- The gap between the frictionless optimal price level  $\mu + mc_t$  and the current price level  $p_t$ . Another way to state this is that inflation depends positively on *real marginal cost*,  $mc_t - p_t$ .

Why is real marginal cost a driving variable for inflation? Firms in the Calvo model would like to keep their price as a fixed markup over marginal cost. If the ratio of marginal cost to

price is getting high (i.e. if  $mc_t - p_t$  is high) then this will spark inflationary pressures because those firms that are re-setting prices will, on average, be raising them.

### Real Marginal Cost and Output

For simplicity, we will denote the deviation of real marginal cost from its frictionless level of  $-\mu$  as

$$\hat{m}c_t^r = \mu + mc_t - p_t \quad (20)$$

so we can write the NKPC as

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} \hat{m}c_t^r \quad (21)$$

One problem with attempting to implement this model empirically, is that we don't actually observe data on real marginal cost. National accounts data contain information on the factors that affect *average* costs such as wages, but do not tell us about the cost of producing an additional unit of output. That said, it seems very likely that marginal costs are procyclical, and more so than prices. When production levels are high relative to potential output, there is more competition for the available factors of production, and this leads to increases in real costs, i.e. increases in the costs of the factors over and above increases in prices. Some examples of the procyclicality of real marginal costs are fairly obvious. For example, the existence of overtime wage premia generally means a substantial jump in the marginal cost of labour once output levels are high enough to require more than the standard workweek.

For these reasons, many researchers implement the NKPC using a measure of the *output gap* (the deviation of output from its potential level) as a proxy for real marginal cost. In other words, they assume a relationship such as

$$\hat{m}c_t^r = \lambda \tilde{y}_t \quad (22)$$

where  $\tilde{y}_t$  is the output gap. This implies a New-Keynesian Phillips curve of the form

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \tilde{y}_t \quad (23)$$

where

$$\gamma = \frac{\lambda(1-\theta)(1-\theta\beta)}{\theta} \quad (24)$$

And this approach can be implemented empirically using various measures for estimating potential output.<sup>5</sup>

### The “Asset-Price-Like” Behaviour of NKPC Inflation

The New-Keynesian approach assumes that firms have rational expectations. Thus, we can apply the repeated substitution method to equation (23) to arrive at

$$\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t \tilde{y}_{t+k} \quad (25)$$

Inflation today depends on the whole sequence of expected future output gaps. Thus, the NKPC sees inflation as behaving according to the classic “asset-price” logic that we saw with the dividend-discount stock price model.

### The NKPC and the Lucas Critique

The vast majority of macroeconomists now accept Friedman’s critique of the original Phillips curve. Thus, it is widely accepted that inflation expectations will move upwards over time if output remains above its potential level, and that there is little or scope for policy-makers to choose a tradeoff between inflation and output. However, as we discussed in earlier lecture

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<sup>5</sup>Roberts (1995) shows that a number of other models of sticky prices also imply a formulation for inflation similar to the New Keynesian Phillips curve.

notes, there is empirical evidence for a relationship of the form

$$\pi_t = \pi_{t-1} + \alpha - \beta u_t \quad (26)$$

So there is a relationship between the *change* in inflation and the level of unemployment. In this formulation, the lagged inflation term reflects how last period's level of inflation changes people's expectations and so feeds into today's inflation. This so-called *accelerationist* Phillips curve fits the data quite well (or, more precisely, empirical approaches based on a weighted average of past inflation rates, not just last period's, fit the data well) and comes with its own well-known terminology. Specifically, economists often speak of the so-called NAIRU—the non-accelerating inflation rate of unemployment. This is the inflation rate consistent with constant inflation and it is defined implicitly by

$$\alpha - \beta u^* = 0 \Rightarrow u^* = \frac{\alpha}{\beta} \quad (27)$$

Empirical estimates of the NAIRU are often invoked in real-world policy discussions, with the policy recommendations made on the basis of whether unemployment is above or below this NAIRU level.<sup>6</sup>

The NKPC model provides a different view of this empirical relationship. While advocates of the NKPC will concede that the accelerationist model, equation (26), fits the data reasonably well, they view this as a so-called reduced-form relationship, not a structural relationship. In other words, if the true model is

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \tilde{y}_t \quad (28)$$

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<sup>6</sup>Note though the NAIRU terminology is actually a misnomer. If unemployment is below  $u^*$ , then inflation will be increasing, but not accelerating. The price level is what will be accelerating. Perhaps the NAIRU should be changed to the NAPLRU, but this isn't so catchy so the "slipped derivative" is probably here to stay.

then equation (26) might have a good statistical fit because  $\pi_{t-1}$  is likely to be correlated with  $E_t\pi_{t+1}$ . However, they would warn policy-makers not to rely on this relationship, because changes in policy may produce a break the correlation between  $E_t\pi_{t+1}$  and  $\pi_{t-1}$  and at this point the statistical accelerationist Phillips curve will break down.

### **The NKPC and Disinflation**

The NKPC also has important implications for how a government can approach reducing inflation. Consider again the accelerationist Phillips curve, equation (26). The fact that inflation depends on its own lagged values in this formulation means then it would be very difficult to reduce inflation quickly without a significant increase in unemployment. So, this Phillips curve suggests that gradualist policies are the best way to reduce inflation.

But the implications of the NKPC are completely different. There may be a statistical relationship between current and lagged inflation but the NKPC says that there is no structural relationship at all. Thus, there is no need for gradualist policies to reduce inflation. According to the NKPC, low inflation can be achieved immediately by the central bank announcing (and the public believing) that it is committing itself to eliminating positive output gaps in the future: This can be seen from equation (25).

Whether the empirical evidence fits with the NKPCs predictions is open for debate. For example, there has been plenty of evidence that reductions in inflation do tend to be costly in terms of lost output and high unemployment. Some, however, have put this down to the failure of governments and central banks to credibly convince the public of their commitment to lower inflation rates. There is also a modern econometric literature on estimating the NKPC which we will discuss in detail in the Advanced Macroeconomics course next term.

**Things to Understand from these Notes**

Here's a brief summary of the things that you need to understand from these notes.

1. The evidence on price stickiness.
2. New Classical macroeconomics.
3. New Keynesian macroeconomics.
4. The assumptions of the Calvo model.
5. The optimal reset price in the Calvo model.
6. How to derive the New Keynesian Phillips curve.
7. Real marginal cost and output gaps.
8. The NKPC and the Lucas Critique of the Phillips curve.
9. The NKPC and disinflation.