Sample Questions for the Final Exam
Final Edition

This handout provides sample questions for the midterm, to be updated throughout the term. Here are some tips for answering these questions.

- **The Bottom Line**: Try to understand the key economic points behind the models. This is crucial. While ideally I would like everyone in the class to understand all the technicalities of the models presented, I know this isn't possible. Even if you can't answer a question in full using the equations or graphs presented, an answer that shows you have studied the material and have a good idea what the key points are can still score reasonably well.

- **Maths and Graphs**: Some of the questions can be answered using both equations and graphs and a perfect answer might have both. If, however, you have problems studying equations, an answer with a graph only or a less-technical description of what is going on can still score reasonably well.

- **Multi-Part Questions**: Read all parts of the question and attempt to answer each part. If a question contains two elements (e.g. "Discuss ... " and "Why is ...?") then I am expecting you to address both elements and answers that ignore one of them will score poorly.

- **Whelan's Golden Rule of Exams**: Please answer the required amount of questions. Even if you don't know much about a question, write something. I do not want to fail people but the rules are that if you write nothing, you get zero. Even a short and very poor answer could still get 30%. The difference between this and zero could be the difference between passing and failing.

1. **The Rational Expectations Permanent Income Hypothesis**

(a) Starting from the household budget constraint

$$A_{t+1} = (1+r)(A_t + Y_t - C_t)$$

where A_t is the value of household assets, Y_t is after-tax labour income, and C_t is consumption expenditures, derive the intertemporal budget constraint

$$\sum_{k=0}^{\infty} \frac{E_t C_{t+k}}{(1+r)^k} = A_t + \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{(1+r)^k}$$

(b) State the assumptions required to derive the random walk hypothesis that expected changes in consumption should be unpredictable, i.e. that $C_t = E_t C_{t+1}$ and sketch out how this result is derived.

(c) Show that the random walk hypothesis implies that current consumption depends in a specific fashion on current assets and on current and expected future labour income.

(d) Now suppose that consumers expect after-tax labour income to grow at rate g forever (where $g < r$). Maintaining the random walk assumption, what does this imply for the relationship between consumption, labour income and assets?

(e) Could econometric estimates of the relationship derived in (d) be used to assess the effects of a temporary tax cut? Explain your answer.

2. **Consumption and Asset Pricing**

Consider a consumer maximising a welfare function of the form

$$W = \sum_{k=0}^{\infty} \left(\frac{1}{1+\beta} \right)^k U\left(C_{t+k}\right)$$

where $U\left(C_t\right)$ is the instantaneous utility obtained at time t, and β is a positive number that describes the fact that households prefer a unit of consumption today to a unit tomorrow. When consumers can invest in multiple assets, the following optimality condition must hold for consumption spending and *any* asset with return $R_{i,t}$ that they choose to invest in.

$$U'\left(C_t\right) = E_t \left[\left(\frac{R_{i,t+1}}{1+\beta} \right) U'\left(C_{t+1}\right) \right]$$

(a) Explain the logic behind this optimality condition.

(b) Assume consumers have a Constant Relative Risk Aversion (CRRA) utility function

$$U(C_t) = \frac{1}{1-\theta} C_t^{1-\theta}$$

Use the optimality condition above to derive the risk-free interest rate.

(c) Derive and explain the Consumption Capital Asset Pricing Model (C-CAPM).

(d) How well does the C-CAPM do in explaining the high average rate of return on equities and the low average risk-free interest rate?

3. **The Calvo Model**

Consider an economy in which only a random fraction $(1-\theta)$ of firms are able to reset their price each period. Firms that get to set their price do so by choosing a log-price, z_t, that minimizes the "loss function"

$$L(z_t) = \sum_{k=0}^{\infty} (\theta\beta)^k E_t \left(z_t - p^*_{t+k}\right)^2$$

where β is between zero and one, and p^*_{t+k} is the log of the optimal price that the firm would set in period $t+k$ if there were no price rigidity.

(a) Explain the logic behind why the firm wishes to minimise this particular loss function.

(b) Derive the optimal price z_t that is set by the firm solving this problem.

(c) Show how to use the optimal price to derive the New-Keynesian Phillips curve.

(d) Discuss the implications of the New-Keynesian Phillips curve for a central bank that is seeking to reduce inflation.

4. **The Solow Model**

(a) Use a graph to illustrate the effect of an increase in the savings rate on output in the Solow model.

(b) Consider the Cobb-Douglas production function

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Derive an expression for the growth rate of output in this economy as a function of the growth rates of A_t, K_t, and L_t.

(c) Derive an expression for the growth rate of the capital stock in the Solow model.

(d) Using the equations for output growth and capital growth, show that the capital-output ratio converges towards a specific long-run value. What determines this long-run capital-output ratio?

(e) What do the dynamic properties of the capital-output ratio imply for the behaviour of output-per-worker in this economy?

5. **The Romer Model**

The production function in the Romer model is

$$Y = L_Y^{1-\alpha} \sum_{i=1}^{A} x_i^{\alpha}$$

where L_Y is the number of workers producing output and the x_i's are different types of capital goods.

(a) In equilibrium in this model, the demand for each of the capital goods is the same. Show that this allows the production function to be re-written as

$$Y = (AL_Y)^{1-\alpha} K^{\alpha}$$

(b) Technology in the Romer model evolves according to

$$\dot{A} = \gamma L_A^{\lambda} A^{\phi}$$

where L_A is the number of workers in the research sector. What is the intuition for this equation?

(c) Show that, along this economy's steady-state growth path, the growth rate of output per worker equals the growth rate of technology.

(d) What is the steady-state growth rate of technology in this model? Does the growth rate of technology tend to converge towards its steady-state rate?

6. **Technology Diffusion**

This questions relates to the following model. There is a "lead" country in the world economy that has technology level, A_t at time t which grows at rate g every period, so that

$$\frac{\dot{A}_t}{A_t} = g$$

All other countries in the world, indexed by j, have technology levels given by $A_{jt} < A_t$. The growth rate of technology in country j is determined by

$$\frac{\dot{A}_{jt}}{A_{jt}} = \lambda_j + \sigma_j \frac{(A_t - A_{jt})}{A_{jt}}$$

where $\lambda_j < g$ and $\sigma_j > 0$.

(a) Describe the intuition behind the formula for the growth rate of technology in the follower country, including the interpretation of the λ_j and σ_j.

(b) Show how to solve for the dynamics of technology in country j.

(c) What happens over time to the ratio of the level of technology in country j to the level of technology in the leading country?

(d) What happens to a country when its technology is growing at a steady rate and then there is an increase in its value of σ_j?