MA Macroeconomics
8. Rational Expectations, Consumption and Asset Prices

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A Model of Optimising Consumers

- We will now move on to another example involving the techniques developed in the last topic.
- Here, we will look at the question of how a consumer with rational expectations will plan their spending over a lifetime.
- Along the way, we will
  1. Show how consumption depends on net wealth and expectations of future income.
  2. Illustrate some pitfalls in using econometrics to assess the effects of policy.
  3. Discuss the impact of tax cuts on consumption spending.
  4. Discuss the link between consumption spending and the return on various financial assets.

The Household Budget Constraint

- Let A_t be household assets, Y_t be labour income, and C_t stand for consumption spending. Stock of assets changes by

$$A_{t+1} = (1 + r_{t+1})(A_t + Y_t - C_t)$$

  where r_{t+1} is the return on household assets at time $t+1$.

- Note that Y_t is *labour* income (income earned from working) not total income because total income also includes the capital income earned on assets (i.e. total income is $Y_t + r_{t+1}A_t$.)

- This can be written as a first-order difference equation in our standard form

$$A_t = C_t - Y_t + \frac{A_{t+1}}{1 + r_{t+1}}$$

- Assume that agents have rational expectations and that return on assets equals a constant, r:

$$A_t = C_t - Y_t + \frac{1}{1+r} E_t A_{t+1}$$

The Intertemporal Budget Constraint

- We have another first-order stochastic difference equation

$$A_t = C_t - Y_t + \frac{1}{1+r} E_t A_{t+1}$$

- Using the same repeated substitution method as before, we get

$$A_t = \sum_{k=0}^{\infty} \frac{E_t \left(C_{t+k} - Y_{t+k} \right)}{(1+r)^k}$$

- We are assuming $\frac{E_t A_{t+k}}{(1+r)^k}$ goes to zero as k gets large.
- One way to understand this equation is to re-writing it as

$$\sum_{k=0}^{\infty} \frac{E_t C_{t+k}}{(1+r)^k} = A_t + \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{(1+r)^k}$$

- This is called the *intertemporal budget constraint*. The present value sum of current and future household consumption must equal the current stock of financial assets plus the present value sum of current and future labour income.

Optimising Consumers

- We will assume that consumers wish to maximize a welfare function of the form
$$W = \sum_{k=0}^{\infty} \left(\frac{1}{1+\beta} \right)^k U\left(C_{t+k}\right)$$
where $U\left(C_t\right)$ is the instantaneous utility obtained at time t, and β is a positive number that describes the fact that households prefer a unit of consumption today to a unit tomorrow.
- If the future path of labour income is known, consumers choose a path for consumption to maximise the following Lagrangian:
$$L = \sum_{k=0}^{\infty} \left(\frac{1}{1+\beta} \right)^k U\left(C_{t+k}\right) + \lambda \left[A_t + \sum_{k=0}^{\infty} \frac{Y_{t+k}}{(1+r)^k} - \sum_{k=0}^{\infty} \frac{C_{t+k}}{(1+r)^k} \right]$$
- For every current and future value of consumption, C_{t+k}, this yields a first-order condition of the form
$$\left(\frac{1}{1+\beta} \right)^k U'\left(C_{t+k}\right) - \frac{\lambda}{(1+r)^k} = 0$$

Consumption Euler Equation

- For $k = 0$, this implies

$$U'(C_t) = \lambda$$

- For $k = 1$, it implies

$$U'(C_{t+1}) = \left(\frac{1+\beta}{1+r}\right)\lambda$$

- Putting these two equations together, we get

$$U'(C_t) = \left(\frac{1+r}{1+\beta}\right)U'(C_{t+1})$$

- When there is uncertainty about future labour income, this optimality condition can just be re-written as

$$U'(C_t) = \left(\frac{1+r}{1+\beta}\right)E_t\left[U'(C_{t+1})\right]$$

This implication of the first-order conditions for consumption is sometimes known as an *Euler equation*.

The Random Walk Theory of Consumption

- In an important 1978 paper, Robert Hall discussed a specific case of the consumption Euler equation. He assumed

$$\begin{aligned} U(C_t) &= aC_t + bC_t^2 \\ r &= \beta \end{aligned}$$

- In this case, the Euler equation becomes

$$a + 2bC_t = E_t [a + 2bC_{t+1}]$$

- Thus which simplifies to

$$C_t = E_t C_{t+1}$$

- Because, the Euler equation holds for all time periods, we have

$$C_t = E_t (C_{t+k}) \qquad k = 1, 2, 3, ...$$

- All future expected values of consumption equal the current value. Because it implies that changes in consumption are unpredictable, this is sometimes called the *random walk* theory of consumption.

The Rational Expectations Permanent Income Hypothesis

- Consumption changes are unpredictable but what determines the level of consumption each period? Insert $E_t C_{t+k} = C_t$ into the intertemporal budget constraint to get

$$\sum_{k=0}^{\infty} \frac{C_t}{(1+r)^k} = A_t + \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{(1+r)^k}$$

- Now we can use the geometric sum formula to turn this into a more intuitive formulation:

$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k} = \frac{1}{1 - \frac{1}{1+r}} = \frac{1+r}{r}$$

- So, Hall's assumptions imply the following equation, which we will term the *Rational Expectations Permanent Income Hypothesis*:

$$C_t = \frac{r}{1+r} A_t + \frac{r}{1+r} \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{(1+r)^k}$$

Implications of RE-PIH

- The Rational Expectations Permanent Income Hypothesis

$$C_t = \frac{r}{1+r} A_t + \frac{r}{1+r} \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{(1+r)^k}$$

states that the current value of consumption is driven by three factors:

1. The expected present discounted sum of current and future labour income.
2. The current value of household assets. This "wealth effect" is likely to be an important channel through which financial markets affect the macroeconomy.
3. The expected return on assets: This determines the coefficient, $\frac{r}{1+r}$, that multiplies both assets and the expected present value of labour income. In this model, an increase in this expected return raises this coefficient, and thus boosts the propensity to consume from wealth.

An Example: Constant Expected Growth in Income

- Suppose households expect labour income to grow at a constant rate g:

$$E_t Y_{t+k} = (1+g)^k \, Y_t$$

- This implies

$$C_t = \frac{r}{1+r} A_t + \frac{r Y_t}{1+r} \sum_{k=0}^{\infty} \left(\frac{1+g}{1+r} \right)^k$$

- As long as $g < r$ (and we will assume it is) then we can use the geometric sum formula to simplify this expression

$$\sum_{k=0}^{\infty} \left(\frac{1+g}{1+r} \right)^k = \frac{1}{1 - \frac{1+g}{1+r}} = \frac{1+r}{r-g}$$

- This implies a consumption function of the form

$$C_t = \frac{r}{1+r} A_t + \frac{r}{r-g} Y_t$$

- Note that the higher is expected future growth in labour income g, the larger is the coefficient on today's labour income and thus the higher is consumption.

A Warning About Econometrics and Policy Evaluation

- Consider an economy where households have always expected their after-tax labour income to grow at rate g.
- Now suppose the government decide to introduce a one-period income tax cut that boosts after-tax labour income by one unit.
- They ask an econometrician to figure out how much this will raise consumption. The econometrican goes to the data which previously has been characterised by

$$C_t = \frac{r}{1+r} A_t + \frac{r}{r-g} Y_t$$

and says the answer is $\frac{r}{r-g}$.
- In reality, that relationship only works when people expect labour income growth of g and that won't hold anymore when there is a once-off temporary tax cut. The true model is still

$$C_t = \frac{r}{1+r} A_t + \frac{r}{1+r} \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{(1+r)^k}$$

so consumption will only go up by $\frac{r}{1+r}$.

The Lucas Critique

- How badly does the econometrician get it wrong?
- Suppose $r = 0.06$ and $g = 0.02$. In this case, the economic advisor concludes that the effect of a dollar of tax cuts is an extra 1.5 $\left(= \frac{.06}{.06 - .02}\right)$ dollars of consumption. In reality, the tax cut will produce only an extra 0.057 $\left(= \frac{.06}{1.06}\right)$ dollars of extra consumption. This is a big difference.
- This may seem like a cooked-up example. But the idea that coefficients in statistical relationships depend upon expectations and that these expectations may change when policy change is not so strange.
- In a famous 1976 paper, Robert Lucas argued that this kind of problem could often lead to econometric analysis providing the wrong answer to various questions about how policy changes would affect the economy.
- This idea that econometric models may be limited in usefulness when analysing policy change (and that it may be better to use theoretically-founded models that incorporate how people formulate expectations) is now known as the **Lucas critique** of econometric policy evaluation.

Explicitly Introducing Fiscal Policy

- Let's change the household budget constraint to explicitly incorporate taxes.
- The household budget constraint is now

$$A_{t+1} = (1 + r)(A_t + Y_t - T_t - C_t)$$

  where T_t is taxes paid by the household at time t.
- The household's intertemporal budget constraint becomes

$$\sum_{k=0}^{\infty} \frac{E_t C_{t+k}}{(1 + r)^k} = A_t + \sum_{k=0}^{\infty} \frac{E_t (Y_{t+k} - T_{t+k})}{(1 + r)^k}$$

- This equation makes it more explicit that households have to factor in all future levels of taxes when making their current spending decisions.

The Government's Budget Constraint

- Like households, governments also have budget constraints.
- The stock of public debt, D_t evolves over time according to

$$D_{t+1} = (1 + r)(D_t + G_t - T_t)$$

  where G_t is government spending and T_t is tax revenue.

- Applying the repeated-substitution method we can obtain an intertemporal version of the government's budget constraint.

$$\sum_{k=0}^{\infty} \frac{E_t T_{t+k}}{(1+r)^k} = D_t + \sum_{k=0}^{\infty} \frac{E_t G_{t+k}}{(1+r)^k}$$

- This states that the present discounted value of tax revenue must equal the current level of debt plus the present discounted value of government spending.
- In other words, in the long-run, the government must raise enough tax revenue to pay off its current debts as well as its current and future spending.

Ricardian Equivalence

- Remembering the household intertemporal budget constraint

$$\sum_{k=0}^{\infty} \frac{E_t C_{t+k}}{(1+r)^k} = A_t + \sum_{k=0}^{\infty} \frac{E_t \left(Y_{t+k} - T_{t+k} \right)}{(1+r)^k}$$

- And the governments intertemporal budget constraint

$$\sum_{k=0}^{\infty} \frac{E_t T_{t+k}}{(1+r)^k} = D_t + \sum_{k=0}^{\infty} \frac{E_t G_{t+k}}{(1+r)^k}$$

- The household intertemporal budget constraint becomes

$$\sum_{k=0}^{\infty} \frac{E_t C_{t+k}}{(1+r)^k} = A_t - D_t + \sum_{k=0}^{\infty} \frac{E_t \left(Y_{t+k} - G_{t+k} \right)}{(1+r)^k}$$

- Before, we had discussed how a temporary cut in taxes should have a small effect. This is a more extreme result — unless governments plan to change the profile of government spending, a cut to taxes today has no impact on consumption spending. Households anticipate that lower taxes today will just trigger higher taxes tomorrow.

Evidence on the RE-PIH

- There have been lots of macroeconomic studies on how well the RE-PIH fits the data.
- There are various reasons why the RE-PIH may not hold.
  1. Consumption smoothing may not be possible e.g. banks may not be willing to lend to people on the basis of their expected future income (i.e. there may be "liquidity constraints."
  2. People may not have rational expectations and may not plan their spending decisions in the calculating optimising fashion assumed by the theory.
- The 1980s saw a large amount of research on whether the RE-PIH fitted the data. The most common conclusion was that consumption was "excessively sensitive" to current disposable income.
- Campbell and Mankiw (1990) present a model in which a fraction of the households behave according to the RE-PIH while the rest simply consume all of their current income. They estimate the fraction of non-PIH consumers to be about a half. A common interpretation of this result is that liquidity constraints have an important impact on aggregate consumption.

Evidence on Ricardian Equivalence: Macro

- There is also a large literature devoted to testing the Ricardian equivalence hypothesis. In addition to the reasons the RE-PIH itself may fail, there are other reasons why Ricardian equivalence may not hold.
  1. People don't live forever and so may not worry about future tax increases that could occur in the far future.
  2. Taxes take a more complicated form than the simple lump-sum payments presented above.
  3. The interest rate in the government's budget constraint may not be the same as the interest rate in the household's constraint.
  4. People may often be unable to tell whether tax changes are temporary or permanent.
- Most macro studies find effects of fiscal policy are quite different from the Ricardian equivalence predictions.
- The evidence generally suggests that tax cuts and increases in government spending tend to boost the economy.

Evidence on Ricardian Equivalence: Micro

- Perhaps more interesting are micro-studies of explicitly temporary tax cuts or rebates. These generally find people spend more of the increase in income than the PIH predicts.

  1. Parker et al (2013) studied effects of rebate cheques mailed to households and estimate that people spent between 50 and 90 percent of the rebate in the three-month period after they receive the payment.
  2. Other studies show people increasing spending after spend more in response to transitory changes in their social security taxes or once-off tax rebates.
  3. Often the people doing the extra spending are well-off households that are probably not subject to liquidity constraints.

- Still, people don't go on a splurge every time they get a large payment. Hsieh (2003) examines how people in Alaska responded to large anticipated annual payments that they received from a state fund that depends largely on oil revenues. He finds that Alaskan households respond to these payments in line with the predictions of the PIH, smoothing out their consumption over the year.

Certainty Equivalence

- Let's keep assumption that $r = \beta$, so the Euler equation is
$$U'(C_t) = E_t[U'(C_{t+1})]$$
You might think that this expression is consistent with constant expected consumption but it is not.
- This is because for functions F generally $E(F(X)) \neq F(E(X))$. For concave functions—those with negative second derivatives—a famous result known as Jensen's inequality states that $E(F(X)) < F(E(X))$.
- In this example, we are looking at the properties of $E_t[U'(C_{t+1})]$. Whether or not marginal utility is concave or convex depends on its second derivative, so it depends upon the third derivative of the utility function U'''.
- Most standard utility functions have positive third derivatives implying convex marginal utility and thus $E_t[U'(C_{t+1})] > U'(E_t C_{t+1})$.
- Quadratic utility function was a special case because it has $U''' = 0$, its marginal utility is neither concave or convex and the Jensen relationship is an equality. In this very particular case, the utility function displays *certainty equivalence*: The uncertain outcome is treated the same way as if people were certain of achieving its average value.

An Example Without Certainty Equivalence

- Suppose consumers have a utility function of the form
$$U(C_t) = -\frac{1}{\alpha}\exp\left(-\alpha C_t\right)$$
where exp is the exponential function.
- This implies marginal utility of the form
$$U'\left(C_t\right) = \exp\left(-\alpha C_t\right)$$
- In this case, the Euler equation becomes
$$\exp\left(-\alpha C_t\right) = E_t\left(\exp\left(-\alpha C_{t+1}\right)\right)$$
- Now suppose C_{t+1} is perceived to have a normal distribution with mean $E_t(C_{t+1})$ and variance σ^2. A useful result from statistics is that if a variable X is normally distributed has mean μ and variance σ^2:
$$X \sim N\left(\mu, \sigma^2\right)$$
then one can show that
$$E\left(\exp(X)\right) = \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

Uncertainty-Induced Tilt

- This result implies that

$$E_t\left(\exp\left(-\alpha C_{t+1}\right)\right) = \exp\left(E_t\left(-\alpha C_{t+1}\right) + \frac{Var\left(-\alpha C_{t+1}\right)}{2}\right)$$
$$= \exp\left(-\alpha E_t\left(C_{t+1}\right) + \frac{\alpha^2\sigma^2}{2}\right)$$

- So, the Euler equation can be written as

$$\exp\left(-\alpha C_t\right) = \exp\left(-\alpha E_t\left(C_{t+1}\right) + \frac{\alpha^2\sigma^2}{2}\right)$$

- Taking logs of both sides this becomes

$$-\alpha C_t = -\alpha E_t\left(C_{t+1}\right) + \frac{\alpha^2\sigma^2}{2}$$

which simplifies to

$$E_t\left(C_{t+1}\right) = C_t + \frac{\alpha\sigma^2}{2}$$

- Even though expected marginal utility is flat, consumption tomorrow is expected to be higher than consumption today.

Precautionary Savings

- Uncertainty induces an "upward tilt" to the consumption profile. And this upward tilt has an affect on today's consumption: We cannot sustain higher consumption tomorrow without having lower consumption today.
- We can calculate exactly what the effect of uncertainty is on consumption today. The Euler equation implies that

$$E_t\left(C_{t+k}\right) = C_t + \frac{k\alpha\sigma^2}{2}$$

- Inserting this into the intertemporal budget constraint, we get

$$\sum_{k=0}^{\infty} \frac{C_t}{(1+r)^k} + \frac{\alpha\sigma^2}{2} \sum_{k=1}^{\infty} \frac{k}{(1+r)^k} = A_t + \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{(1+r)^k}$$

  we can show that

$$\sum_{k=1}^{\infty} \frac{k}{(1+r)^k} = \frac{1+r}{r^2}$$

Precautionary Savings

- So, the intertemporal budget constraint simplifies to

$$\sum_{k=0}^{\infty} \frac{C_t}{(1+r)^k} + \frac{1+r}{r^2}\frac{\alpha\sigma^2}{2} = A_t + \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{(1+r)^k}$$

- Taking the same steps as before, consumption today is

$$C_t = \frac{r}{1+r} A_t + \frac{r}{1+r} \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{(1+r)^k} - \frac{\alpha\sigma^2}{2r}$$

- This is exactly as before apart from an additional "precautionary savings" term $-\frac{\alpha\sigma^2}{2r}$. The more uncertainty there is, the more lower the current level of consumption will be.
- This particular result obviously relies on very specific assumptions about the form of the utility function and the distribution of uncertain outcomes. However, since almost all utility function feature positive third derivatives, the key property underlying the precautionary savings result—marginal utility averaged over the uncertain outcomes being higher than at the average level of consumption—will generally hold.

Euler Equation With Time-Varying Returns

- Up to now we assumed consumers expect a constant return on assets. Here, we allow expected asset returns to vary.
- Using the same methods as in the last topic when looking at asset prices, we can derive a new intertemporal budget constraint via the repeated substitution method.

$$\sum_{k=0}^{\infty} \frac{E_t C_{t+k}}{\left(\prod_{m=1}^{k+1} (1 + r_{t+m})\right)} = A_t + \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{\left(\prod_{m=1}^{k+1} (1 + r_{t+m})\right)}$$

where $\prod_{n=1}^{h} x_i$ means the product of x_1, x_2 .... x_h.

- The optimisation problem of the consumer does not change much. Instead of the simple Euler equation, we get

$$U'(C_t) = E_t \left[\left(\frac{R_{t+1}}{1+\beta} \right) U'(C_{t+1}) \right]$$

where $R_t = 1 + r_t$.

Consumption and Rates of Return on Assets

- We used the Euler equation to derive the behaviour of consumption, given asset returns. However, Euler equations are also used to consider the determination of asset returns, taking consumption as given.
- When you extends the model to allow the consumer to allocate their wealth across multiple asset types, it turns out that equation just derived must hold for *all* of these assets, i.e. for a set of different asset returns $R_{i,t}$ each obeys

$$U'(C_t) = E_t \left[\left(\frac{R_{i,t+1}}{1+\beta} \right) U'(C_{t+1}) \right]$$

- For example, consider a pure risk-free asset that pays a guaranteed rate of return next period, call it $R_{f,t}$. With no uncertainty, this rate of return can be taken outside the expectation term, and the

$$U'(C_t) = \frac{R_{f,t+1}}{1+\beta} E_t \left[U'(C_{t+1}) \right]$$

- So, the risk-free interest rate should be determined as

$$R_{f,t+1} = \frac{(1+\beta) U'(C_t)}{E_t \left[U'(C_{t+1}) \right]}$$

Predictions for Relationships Between Asset Returns

- To think about the relationship between risk-free rates and returns on other assets, it is useful to use a well-known result from statistical theory, namely

$$E(XY) = E(X)E(Y) + Cov(X, Y)$$

- This allows us to re-write our Euler equation as follows

$$U'(C_t) = \frac{1}{1+\beta}\left[E_t(R_{i,t+1})E_t(U'(C_{t+1})) + Cov(R_{i,t+1}, U'(C_{t+1}))\right]$$

- This can be re-arranged to give

$$\frac{(1+\beta)U'(C_t)}{E_t[U'(C_{t+1})]} = E_t(R_{i,t+1}) + \frac{Cov(R_{i,t+1}, U'(C_{t+1}))}{E_t[U'(C_{t+1})]}$$

- Note now that the left-hand-side of this equation equals the risk-free rate. So, we have

$$E_t(R_{i,t+1}) = R_{f,t+1} - \frac{Cov(R_{i,t+1}, U'(C_{t+1}))}{E_t[U'(C_{t+1})]}$$

Consumption Capital Asset Pricing Model (C-CAPM)

- This relationship

$$E_t\left(R_{i,t+1}\right) = R_{f,t+1} - \frac{Cov\left(R_{i,t+1}, U'\left(C_{t+1}\right)\right)}{E_t\left[U'\left(C_{t+1}\right)\right]}$$

  is known as the **Consumption Capital Asset Pricing Model (C-CAPM)**.

- It predicts that expected rate of return on risky assets equals the risk-free rate *minus* a term that depends on the covariance of the risky return with the marginal utility of consumption.
- Most asset returns depend on payments generated by the real economy and so they are procyclical—they are better in expansions than during recessions.
- Usually assume diminishing marginal utility implies, so U' depends negatively on consumption and covariance term is negative for assets whose returns are positively correlated with consumption and these assets will have a higher rate of return than the risk free rate.
- Intuition: Consumers like assets that hedge against consumption variations. For investors to be induced into holding assets that are more positively correlated with consumption, the rate of return on these assets needs to be higher.

The Equity Premium Puzzle

- C-CAPM can be used to model why some assets, such as stocks, have high average returns while others, such as government bonds, have such low returns. However, it doesn't do very well as an empirical model.
- Most studies use simple utility functions such as the Constant Relative Risk Aversion (CRRA) preferences

$$U(C_t) = \frac{1}{1-\theta} C_t^{1-\theta}$$

  so marginal utility is

$$U'(C_t) = C_t^{-\theta}$$

- In this case, the consumption-CAPM equation becomes

$$E_t\left(R_{i,t+1}\right) = R_{f,t+1} - \frac{\text{Cov}\left(R_{i,t+1}, C_{t+1}^{-\theta}\right)}{E_t\left[C_{t+1}^{-\theta}\right]}$$

- For values of θ considered consistent with standard estimates of risk aversion, this covariance is not nearly big enough to justify the observed equity premium. It requires values such as $\theta = 25$, which turns out to imply people are incredibly risk averse.

The Low Risk-Free Rate

- One route that doesn't seem to work is arguing that people really are that risk averse, i.e. that $\theta = 25$ somehow is a good value. It implies a much higher risk-free rate than we actually see.
- Plugging the CRRA utility function into the equation for the risk free rate

$$R_{f,t+1} = \frac{(1+\beta)\, C_t^{-\theta}}{E_t\left[C_{t+1}^{-\theta}\right]}$$

- Neglecting uncertainty about consumption growth, this formula implies that on average, the risk-free rate should be

$$R_f = (1+\beta)\,(1+g_C)^{\theta}$$

  where g_C is the growth rate of consumption.
- Plugging in the average growth rate of consumption, a value of $\theta = 25$ would imply a far higher average risk-free rate than we actually see on government bonds.

Things to Understand From This Topic

1. The household budget constraint.
2. How to derive the intertemporal budget constraint.
3. How to set up and derive first-order conditions for optimal consumption.
4. How to derive the Rational Expectations/Permanent Income Hypothesis.
5. The Lucas Critique applied to temporary tax cuts.
6. The Ricardian equivalence hypothesis.
7. Evidence on temporary tax cuts.
8. Precautionary savings.
9. The first-order condition with time-varying asset returns.
10. The Consumption-CAPM model.
11. The equity premium and risk-free rate puzzles