Determinants of Total Factor Productivity

The Solow model identified technological progress or improvements in total factor productivity (TFP) as the key determinant of growth in the long run, but did not provide any explanation of what determines it. In these notes, we discuss one interpretation of what is meant by technological progress – the invention and applications of new technologies. We discuss the past and future of technological inventions and then briefly present a model of what determines differences in the level of technological efficiency across countries.

TFP Growth as Invention of New Inputs

So what is this technology term A anyway? A well-known paper by Paul Romer ("Endogenous Technological Change," *Journal of Political Economy*, 1990) provided a specific concrete view on this issue. Romer describes the aggregate production function as

$$Y = L_Y^{1-\alpha} \left(x_1^\alpha + x_2^\alpha + ... + x_A^\alpha \right) = L_Y^{1-\alpha} \sum_{i=1}^{A} x_i^\alpha \tag{1}$$

where L_Y is the number of workers producing output and the x_i's are different types of capital goods. The crucial feature of this production function is that diminishing marginal returns applies, not to capital as a whole, but separately to each of the individual capital goods (because $0 < \alpha < 1$). (Note that I'm dropping the time subscripts t here because this model already has equations with a lot of terms.)

If A was fixed, the pattern of diminishing returns to each of the separate capital goods would mean that growth would eventually taper off to zero. However, in the Romer model, A is not fixed. Instead, there are L_A workers engaged in Research and Development and this leads to the invention of new capital goods, so there is an increase in A each period. In other words, the total labour force is

$$L = L_A + L_Y \tag{2}$$

so workers either produce goods or work in the research sector trying to invent new goods.

Simplifying the Aggregate Production Function

We can define the aggregate capital stock as the sum of all the different capital inputs

$$K = \sum_{i=1}^{A} x_i \tag{3}$$

Because all the capital goods play an identical role in the production process, we can assume that the demand from producers for each of these capital goods is the same. Call this amount that is demanded of each capital good $\bar{x}$ so

$$x_i = \bar{x} \qquad i = 1, 2, ...A \tag{4}$$

This means that the production function can be written as

$$Y = AL_Y^{1-\alpha}\bar{x}^{\alpha} \tag{5}$$

Note now that

$$K = A\bar{x} \Rightarrow \bar{x} = \frac{K}{A} \tag{6}$$

so output can be re-expressed as

$$Y = AL_Y^{1-\alpha}\left(\frac{K}{A}\right)^{\alpha} = (AL_Y)^{1-\alpha} K^{\alpha} \tag{7}$$

This looks just like the Solow model's production function. The TFP term is written as $A^{1-\alpha}$ as opposed to just A as it was in our growth accounting notes, but this makes no difference to the substance of the model. In this model, total factor productivity growth comes from the invention of new technologies. Without the invention of new technologies, additional capital accumulation would have to be spread among the same set of existing capital inputs and diminishing marginal returns would set in and growth would grind to a halt. The invention of these technologies means that additional capital accumulated can be spread across a wider amount of inputs. This prevents diminishing marginal returns setting in for any of the individual capital inputs and means there is continuous ongoing growth.

A Vision of Economic Growth and Some Tradeoffs

Romer's vision of economic growth is one in which growth is driven by the invention of new technologies that work as inputs into the production process. His paper provides a full model that spells out a number of extra elements that we don't have time to cover here but I can give you jist. He describes why it is that new technologies are invented: He outlines a process in which researchers work at trying to invent new technologies so that, when successful, they can make monopoly profits on their new inventions. He assumes that the process of invention of new technologies works according to a sort of "production function for ideas" of the form

$$\frac{dA}{dt} = \gamma L_A^{\lambda} A^{\phi} \tag{8}$$

The change in the number of capital goods depends positively on the number of researchers (λ is an index of how slowly diminishing marginal productivity sets in for researchers) and also on the prevailing value of A itself. This latter effect stems from the "giants shoulders" effect.[1] For instance, the invention of a new piece of software will have relied on the previous invention of the relevant computer hardware, which itself relied on the previous invention of

[1] Stemming from Isaac Newton's observation "If I have seen farther than others, it is because I was standing on the shoulders of giants."

semiconductor chips, and so on.

Romer's model assumes a labour market in which workers can move freely between the output and research sectors. With a common wage across both sectors, this means the marginal return to a worker in the research sector ends up being the same as the marginal return to a worker in the output sector. Because he assumes diminishing marginal returns to labour in the production of both output and research, this implies a downward-sloping demand curve for labour. The condition that marginal returns to labour are equated across sectors determines how may workers are employed in each sector, i.e. if something raises the productivity of workers in the research sector, this sector will hire more workers until diminishing marginal productivity of these workers brings their marginal product back in line with the marginal product in the output sector.

Romer's work describes two trade-offs that policy-makers have to think about when promoting growth:

- Present versus Future: At any point in time, governments could incentivize people to go into education and research with the hope of inventing new technologies that will raise productivity over time. However, these people will then not be producing the goods and services that affect people's current standard of living.

- Competition versus Growth: In general, Romer's model points to outcomes in which there is too little R&D activity. People who invent a great new product add to the "giants shoulders" effect and that influences future inventions but usually not in a way that allows the full stream of profits from these future inventions to flow back to those who came up with the earlier inventions. Laws to strengthen patent protection may raise the incentives to conduct R&D. This points to a potential conflict between policies aimed at raising macroeconomic growth and microeconomic policies aimed at reducing the inefficiencies due to monopoly power: Some amount of monopoly power for patent-holders may be necessary if we want to induce a high level of R&D and thus a high level of output.

Robert Gordon on The Past and Future of New Technologies

Many of the facts about economic history back up Romer's vision of economic growth. Robert Gordon's paper "Is US economic growth over? Faltering innovation confronts the six head-winds" provides an excellent description of the various phases of technological invention and also provides an interesting perspective on the potential for future technological progress. Gordon highlights how economic history can be broken into different periods based on how the invention of technologies have impacted the economy.

The First Industrial Revolution: "centered in 1750-1830 from the inventions of the steam engine and cotton gin through the early railroads and steamships, but much of the impact of railroads on the American economy came later between 1850 and 1900. At a minimum it took 150 years for IR1 to have its full range of effects."

The Second Industrial Revolution: "within the years 1870-1900 created within just a few years the inventions that made the biggest difference to date in the standard of living. Electric light and a workable internal combustion engine were invented in a three-month period in late 1879. The number of municipal waterworks providing fresh running water to urban homes multiplied tenfold between 1870 and 1900. The telephone, phonograph, and motion pictures were all invented in the 1880s. The benefits of IR2 included subsidiary and complementary inventions, from elevators, electric machinery and consumer appliances; to the motorcar, truck, and airplane; to highways, suburbs, and supermarkets; to sewers to carry the wastewater away. All this had been accomplished by 1929, at least in urban America, although it took longer to bring the modern household conveniences to small towns and farms. Additional follow-up inventions continued and had their main effects by 1970, including television, air conditioning, and the interstate highway system. The inventions of IR2 were so important and far-reaching that they took a full 100 years to have their main effect."

The Third Industrial Revolution: "is often associated with the invention of the web and internet around 1995. But in fact electronic mainframe computers began to replace routine and repetitive clerical work as early as 1960."

Gordon's paper is very worth reading for understanding how the innovations associated with the "second industrial revolution" completely altered people's lives. He describes life in 1870 as follows

> most aspects of life in 1870 (except for the rich) were dark, dangerous, and involved backbreaking work. There was no electricity in 1870. The insides of dwelling units were not only dark but also smoky, due to residue and air pollution from candles and oil lamps. The enclosed iron stove had only recently been invented and much cooking was still done on the open hearth. Only the proximity of the hearth or stove was warm; bedrooms were unheated and family members carried warm bricks with them to bed.
>
> But the biggest inconvenience was the lack of running water. Every drop of water for laundry, cooking, and indoor chamber pots had to be hauled in by the housewife, and wastewater hauled out. The average North Carolina housewife in 1885 had to walk 148 miles per year while carrying 35 tonnes of water.

Gordon believes that the technological innovations associated with computer technologies are far less important than those associated with the "second industrial revolution" and that growth may sputter out over time. Figure 1 repeats a chart from Gordon's paper showing the growth rate of per capita GDP for the world's leading economies (first the UK, then the US). It shows growth accelerating until 1950 and declining thereafter. Figure 2 shows a hypothetical chart in which Gordon projects a continuing fall-off in growth.

To illustrate why he believes modern inventions don't match up with past improvements, Gordon offers the following thought experiment.

> You are required to make a choice between option A and option B. With option A you are allowed to keep 2002 electronic technology, including your Windows 98 laptop accessing Amazon, and you can keep running water and indoor toilets; but you can't use anything invented since 2002.
>
> Option B is that you get everything invented in the past decade right up to Facebook, Twitter, and the iPad, but you have to give up running water and indoor toilets. You have to haul the water into your dwelling and carry out the waste. Even at 3am on a rainy night, your only toilet option is a wet and perhaps muddy walk to the outhouse. Which option do you choose?

You probably won't be surprised to find out that most people pick option B.

Gordon also discusses other factors likely to hold back growth in leading countries such as the leveling off of a long-run pattern of educational achievement, an aging population and energy-related constraints. It's worth noting, though, that while Gordon's paper is very well researched and well argued, economists are not very good at forecasting the invention of new technologies or their impact on the economy. For all we know, the next "industrial revolution" could be around the corner to spark a new era of rapid growth. Joel Mokyr's article "Is technological progress a thing of the past?" (linked to on the website) is a good counterpart to Gordon's scepticism.

Figure 1: Gordon on the Growth Rate of Leading Economies

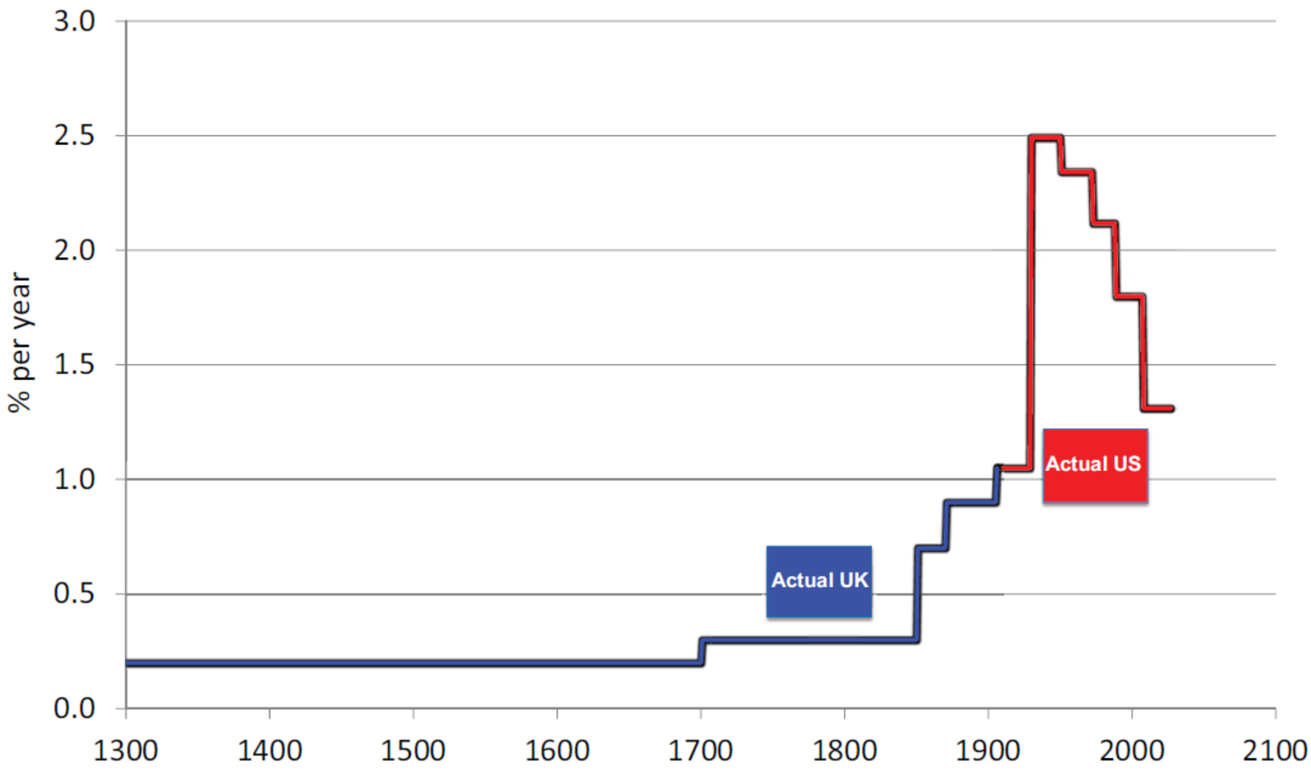
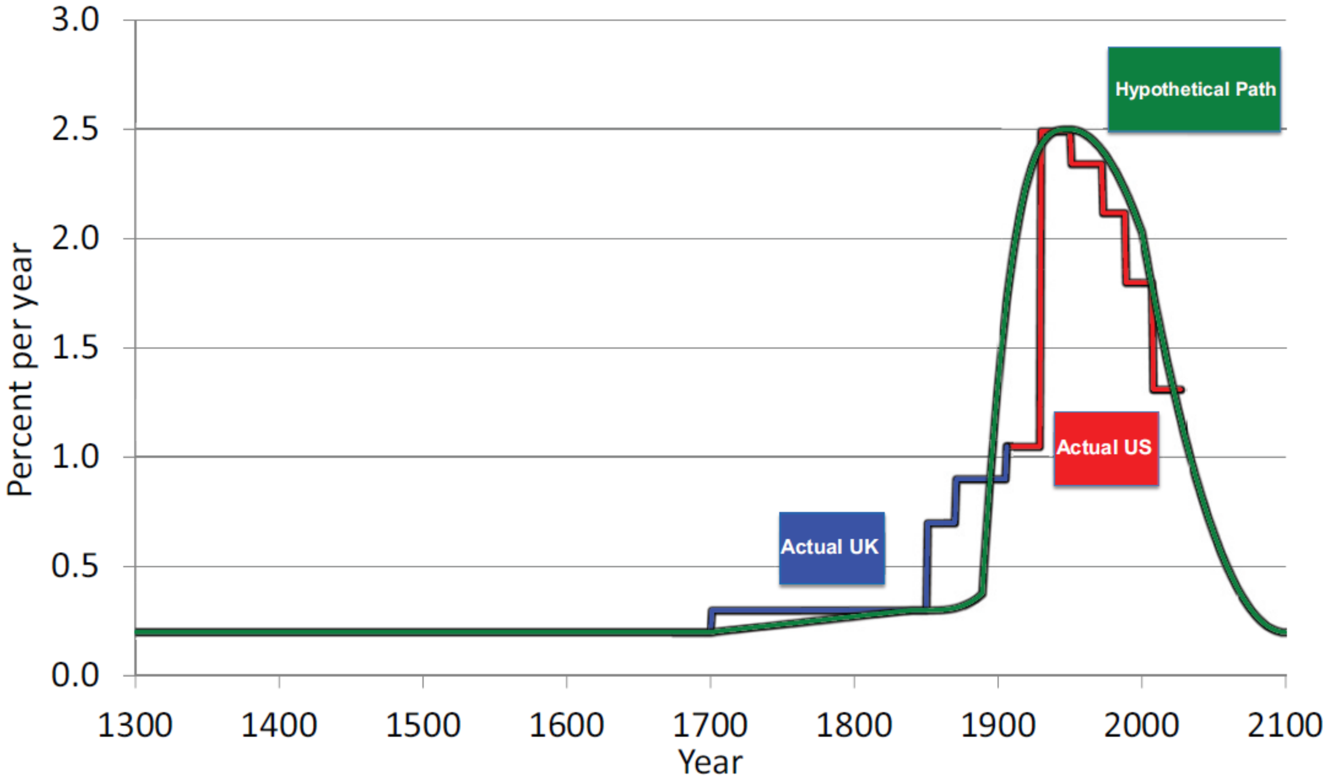


Figure 2: Gordon's Hypothetical Path for Growth



Cross-Country Differences in Output Per Worker

So far, we've been discussing how the invention of new technologies promotes economic growth by pushing out the "technological frontier" and allowing capital to be allocated across new and old technologies with diminishing returns setting in. This is clearly an important aspect of economic growth. However, we should remember that only a very few countries in the world are "on the technological frontier"—most places are not relying on Apple to invent a new gadget to promote efficiency. One way to illustrate this point is to estimate the level of total factor productivity for different countries in the world.

An important paper that did these calculations and used them to shed light on cross-country income differences is the paper on the reading list by Hall and Jones (1999). The basis of the study is a "levels accounting" exercise that starts from the following production function

$$Y_i = K_i^{\alpha} \left(h_i A_i L_i\right)^{1-\alpha} \tag{9}$$

Like the BLS multifactor productivity calculations that we discussed a few lectures ago, Hall and Jones account for the effect of education on the productivity of the labour force. Specifically, they construct measures of *human capital* based on estimates of the return to education—this is the h_i in the above equation.

Hall and Jones show that their production function can be re-formulated as

$$\frac{Y_i}{L_i} = \left(\frac{K_i}{Y_i}\right)^{\frac{\alpha}{1-\alpha}} h_i A_i \tag{10}$$

Hall and Jones then constructed a measure h_i using evidence on levels of educational attainment and they also set $\alpha = 1/3$. This allowed them to use (10) to express all cross-country differences in output per worker in terms of three multiplicative terms: capital intensity, human capital per worker, and technology or total factor productivity. They found that output per worker in the richest five countries was 31.7 times that in the poorest five countries. This was explained as follows:

- Differences in capital intensity contributed a factor of 1.8.
- Differences in human capital contributed a factor of 2.2
- The remaining difference—a factor of 8.3—was due to differences in TFP.

The results from this paper show that differences in total factor productivity, rather than differences in factor accumulation, are the key explanation of cross-country variations in income levels. A more detailed table of Hall and Jones's calculations is reproduced on the next page. These calculations show that most countries are very far from the technological frontier, so their growth is not likely to be reliant on the invention of new technologies.

Table from Hall-Jones Paper

TABLE I
PRODUCTIVITY CALCULATIONS: RATIOS TO U. S. VALUES

Country	Y/L	Contribution from $(K/Y)^{\alpha/(1-\alpha)}$	Contribution from H/L	Contribution from A
United States	1.000	1.000	1.000	1.000
Canada	0.941	1.002	0.908	1.034
Italy	0.834	1.063	0.650	1.207
West Germany	0.818	1.118	0.802	0.912
France	0.818	1.091	0.666	1.126
United Kingdom	0.727	0.891	0.808	1.011
Hong Kong	0.608	0.741	0.735	1.115
Singapore	0.606	1.031	0.545	1.078
Japan	0.587	1.119	0.797	0.658
Mexico	0.433	0.868	0.538	0.926
Argentina	0.418	0.953	0.676	0.648
U.S.S.R.	0.417	1.231	0.724	0.468
India	0.086	0.709	0.454	0.267
China	0.060	0.891	0.632	0.106
Kenya	0.056	0.747	0.457	0.165
Zaire	0.033	0.499	0.408	0.160
Average, 127 countries:	0.296	0.853	0.565	0.516
Standard deviation:	0.268	0.234	0.168	0.325
Correlation with Y/L (logs)	1.000	0.624	0.798	0.889
Correlation with A (logs)	0.889	0.248	0.522	1.000

The elements of this table are the empirical counterparts to the components of equation (3), all measured as ratios to the U. S. values. That is, the first column of data is the product of the other three columns.

Leaders and Followers

The Romer model probably should not be thought of as a model of growth in any one particular country. No country uses only technologies that were invented in that country; rather, products invented in one country end up being used all around the world. Thus, the model is best thought of as a model of the leading countries in the world economy. How then should long-run growth rates be determined for individual countries? By itself, the Romer model has no clear answer, but it suggests a model in which ability to learn about the usage of new technologies should plays a key role in determining output per worker.

We will now sketch out such a model and describes its solution. We will assume that there is a "lead" country in the world economy that has technology level, A_t at time t which grows at rate g every period, so that

$$\frac{dA_t}{dt} = gA_t \tag{11}$$

All other countries in the world, indexed by j, have technology levels given by $A_{jt} < A_t$. The growth rate of technology in country j is determined by

$$\frac{dA_{jt}}{dt} = \lambda_j A_{jt} + \sigma_j \left(A_t - A_{jt}\right) \tag{12}$$

where $\lambda_j < g$ and $\sigma_j > 0$. This tells us that technology growth in all countries apart from the lead country is determined by two factors

- Learning: The second term says that their technology level will grow faster the bigger is the percentage gap between its level of technology, A_{jt} and the level of the leader, A_t. The larger is the parameter σ_j, the better the country is at learning about the technologies being applied in the lead country.

- The first term, λ_j indicates the country's capacity for increasing its level of technology without learning from the leader. We impose the condition $\lambda_j < g$. This means that country j can't grow faster than the lead country without the learning that comes from having lower technology than the frontier.

The Model's Solution

Characterising how A_{jt} behaves over time requires solving what are known as differential equations and I don't have time to teach those methods in this class. (I have provided a technical appendix the gives the derivation but this is not examinable material.) However, what I can say is the solution to this model shows that A_{jt} gradually converges over time to

$$A_{jt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j} \right) A_t \tag{13}$$

so each country never actually catches up to the leader but instead converges to some fraction of the lead country's technology level. This makes sense if you think about it. Because of their inferiority at developing their own technologies ($\lambda_j < g$) the follower countries can't overtake the leader country. However, they can grow faster than their own rate of technological invention as long as there is a gap between them and the leader. So their outcome is one in which they grow at the same rate g as the leader but maintain GDP that is a constant percentage lower than that of the leader.

In addition, $g - \lambda_j > 0$ means that

$$\frac{d}{d\sigma_j} \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j} \right) > 0 \tag{14}$$

The equilibrium ratio of the country's technology to the leader's depends positively on the "learning parameter" σ_j. The higher this parameter—the more fo the gap to the leader that it closes each period—then the closer the ratio gets to one and the higher up the "pecking order" the country gets. It's also true that

$$\frac{d}{d\lambda_j}\left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) > 0 \tag{15}$$

In other words, the more growth the country can generate each period independent of learning from the leader, the higher will be its equilibrium ratio of technology relative to the leader.

Illustrating the Model

To be a bit more precise, we can show that the model's solution takes the form

$$A_{jt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) A_t + D_{j0} e^{-(\sigma_j - \lambda_j)t} \tag{16}$$

where D_{j0} is an arbitrary parameter than can take any value and which determines the starting position of the economy. The second term tends to disappear to zero over time. That doesn't mean it's unimportant. How a country behaves along its "transition path" depends on the value of the initial parameter D_{j0}.

- If $D_{j0} < 0$, then the term that is disappearing over time is a negative term that is a drag on the level of technology. This means that the country starts out below its equilbrium technology ratio, grows faster than the leader for some period of time with growth eventually tailing off to the growth rate of the leader.

- If $D_{j0} > 0$, then the term that is disappearing over time is a positive term that is boosting the level of technology. This means that the country starts out above its equilbrium technology ratio, grows slower than the leader for some period of time with growth eventually moves up towards the growth rate of the leader.

We have illustrated how these dynamics would work in Figures 3 to 5. These charts show model simulations for a leader economy with $g = 0.02$ and a follower economy with $\lambda_j = 0.01$ and $\sigma_j = 0.04$. These values mean

$$\frac{\sigma_j}{\sigma_j + g - \lambda_j} = \frac{0.04}{0.04 + 0.02 - 0.01} = 0.8 \tag{17}$$

so the follower economy converges to a level of technology that is 20 percent below that of the leader. The first collection of charts show what happens when this economy has a value of $D_{j0} = -0.5$, so that it starts out with a technology level only 30 percent that of the leader. They grow faster than the leader country for a number of years before they approach the 0.8 equilibrium ratio and then their growth rate settles down to the same rate as that of the leader.

The second collection of charts show what happens when this economy has a value of $D_{j0} = 0.5$, so that it starts out with a technology level 30 percent above that of the leader, even though the equilibrium value is 20 percent below. Technology levels in this follower country never actually decline but they do go through a long-period of slow growth rates before eventually heading towards the same growth rate as the leader as they approach the 0.8 equilibrium ratio.

Finally, we show how the model may also be able to account for the sort of "growth miracles" that are occasionally observed when countries suddenly start experiencing rapid growth: If a country can increase its value of σ_j via education or science-related policies, its position in the steady-state distribution of income may move upwards substantially, with the economy then going through a phase of rapid growth. The third collection of charts show what happens when, in period 21, an economy changes from having $\sigma_j = 0.005$ to $\sigma_j = 0.04$. The equilbrium technology ratio changes from one-third to 0.8 and the economy experiences a long transitional period of rapid growth.

Figure 3: Converging from Below

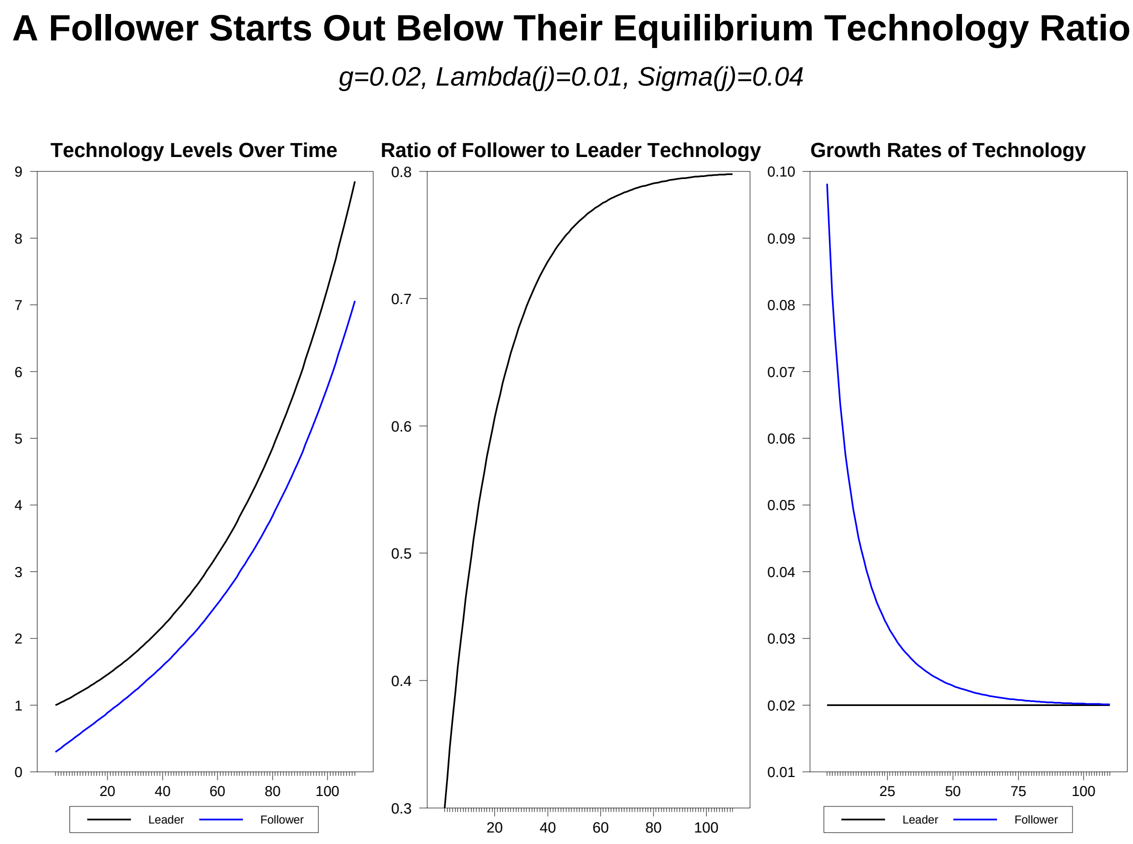


Figure 4: Converging from Above

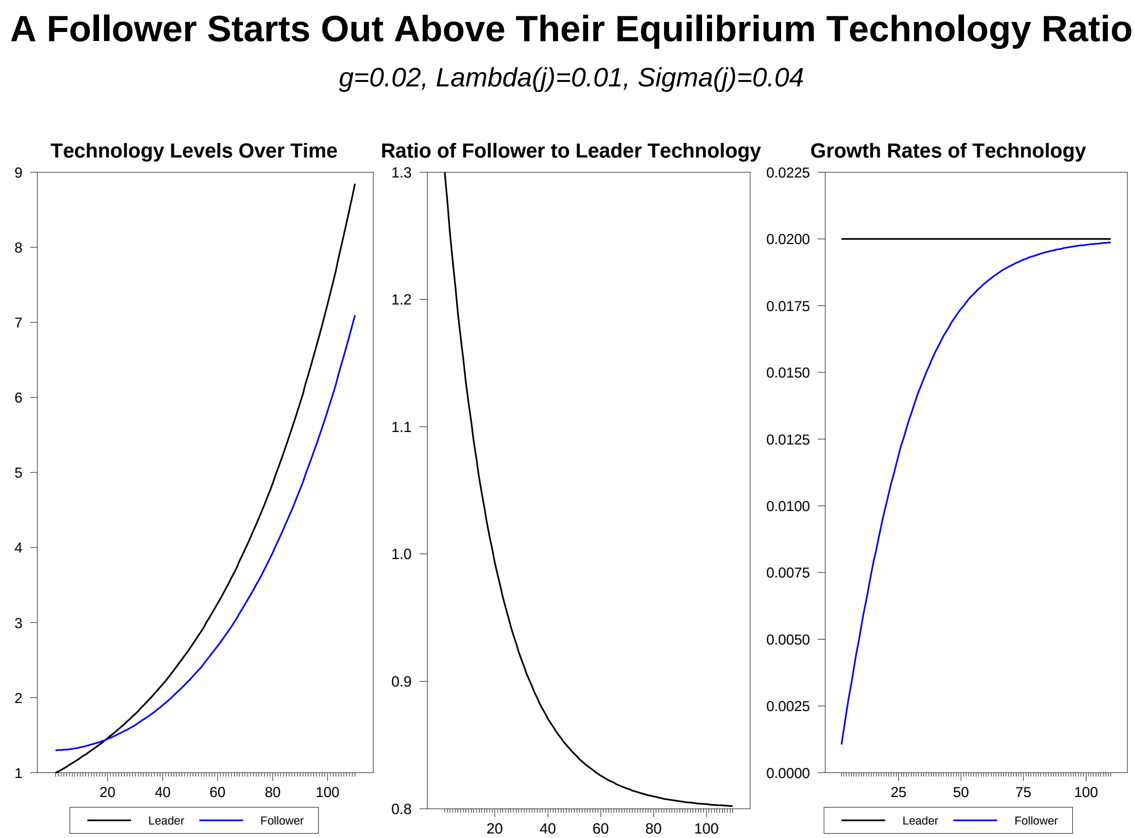
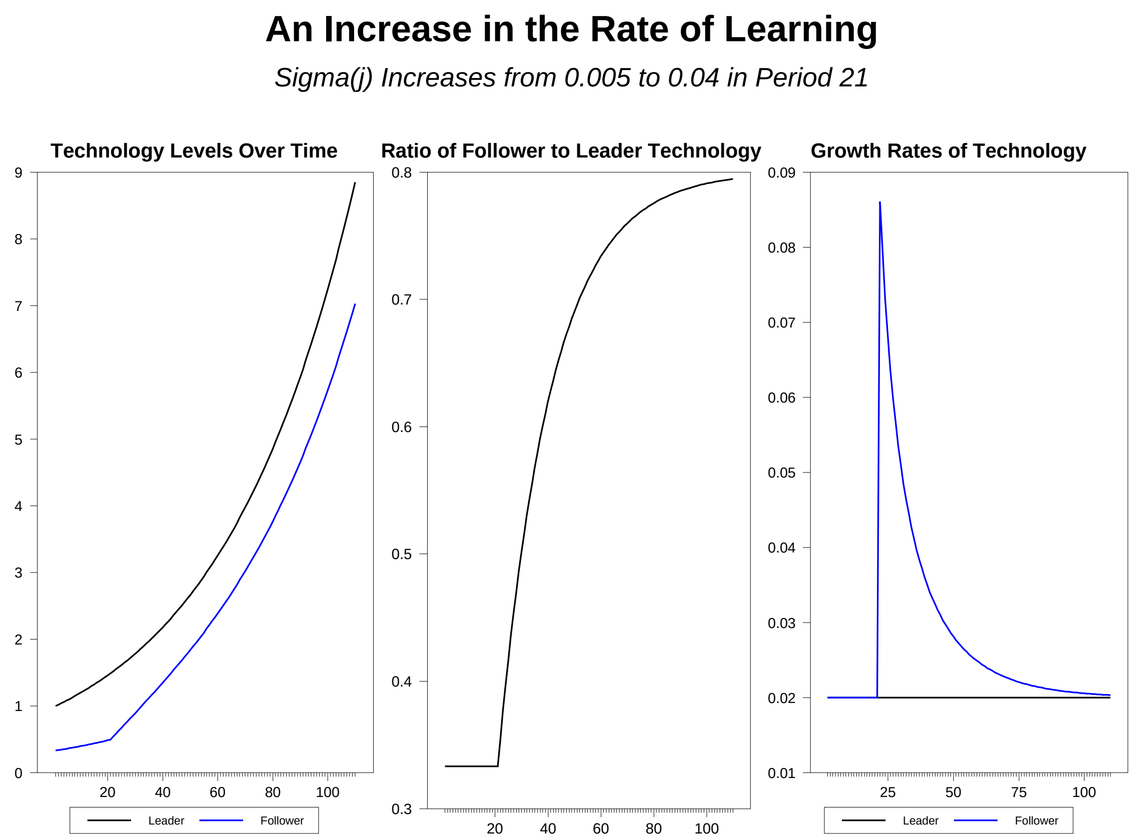


Figure 5: Faster Learning



Institutions and Efficiency

We have documented huge differences in total factor productivity across countries. What determines these differences? One answer is provided by the combination of the Romer model and the leader-follower model. According to these models, large differences in TFP reflect variations in the extent to which countries have adopted the latest technologies.

However, this is perhaps too mechanistic a view of what generates cross-country differences in efficiency. TFP doesn't just reflect the technologies a country's people use. It is a measure of the efficiency with which an economy makes use of its resources and there are a whole range of other factors that can affect this. For example:

- *Bureaucratic Inefficiency and Corruption*: Satisfaction of bureaucratic requirements and bribing of officials can be important diversions of resources in poor economies.
- *Crime*: Time spent on crime does not produce output. Neither do resources devoted to protecting inviduals and firms from crime.
- *Restrictions on Market Mechanisms*: Protectionism, price controls, and central planning can all lead to resources being allocated in an inefficient manner.

In addition, while technology adoption certainly has an impact on differences in TFP, this still leaves open the question of what drives the pace of technology adoption in poorer countries. Ultimately, the models so far don't answer the question of the *deeper determinants* of economic success. We will now discuss on the idea that the ultimate explanation for patterns of economic efficiency relates to differences in institutions.

Douglass North and Institutions

There is now a large literature that focuses on the idea that differences in *institutions* provides the key to understanding TFP differences across countries. Economic activity does not take place in a vacuum. Firms need to take account of the legal and regulatory environment, the tax system, and the services provided by government as well as the political setting that determines these institutions.

The work of economic historian Douglass North, winner of the 1993 Nobel prize for economics, was particularly influential in stressing the key importance of good institutions for economic growth. There is a link to one of North's papers on the class website. The introduction gives a flavour of his arguments:

> A theory of institutional change is essential for further progress in the social sciences in general and economics in particular. Essential because neo-classical theory (and other theories in the social scientist's toolbag) at present cannot satisfactorily account for the very diverse performance of societies and economies both at a moment of time and over time. The explanations derived from neo-classical theory are not satisfactory because, while the models may account for most of the differences in performance between economies on the basis of differential investment in education, savings rates, etc., they do not account for why economies would fail to undertake the appropriate activities if they had a high payoff. Institutions determine the payoffs. While the fundamental neo-classical assumption of scarcity and hence competition has been robust (and is basic to this analysis), the assumption of a frictionless exchange process has led economic theory astray. Institutions are the structure that humans impose on human interaction and therefore define the

> incentives that (together with the other constraints (budget, technology, etc.) determine the choices that individuals make that shape the performance of societies and economies over time.

He goes to discuss the link between institutions and the profit-maximising decisions that people will take:

> Institutions consist of formal rules, informal constraints (norms of behavior, conventions, and self imposed codes of conduct) and the enforcement characteristics of both ... If institutions are the rules of the game, organizations are the players. They are groups of individuals engaged in purposive activity. The constraints imposed by the institutional framework (together with the other constraints) define the opportunity set and therefore the kind of organizations that will come into existence ... If the highest rates of return in a society are to be made from piracy, then organizations will invest in knowledge and skills that will make them better pirates; if organizations realize the highest payoffs by increasing productivity then they will invest in skills and knowledge to achieve that objective.

This paper contains a discussion of some aspects of the US's institutional history that have been positive for economic growth. Much of North's other work focuses on the development of institutions that made some countries such as the UK successful early developers through the industrial revolution while others lagged.

An Example of the Importance of Institutions

Korea provides an extreme example of the importance of institutions in determining the success of an economy. After World War II, Korea was split into a northern zone that became the Democratic People's Republic of Korea, a Soviet-style socialist republic, while South Korea became a capitalist economy.

North Korea received external support from the USSR for many years but no longer receives external aid. It remains a centrally planned economy with only one political party. The economy has failed to prosper and there are reliable reports of large amounts of death from famine in the 1990s. In contrast, South Korea has been a huge economic success and is home to many globally successful corporations such as Samsung and Hyundai.

The figure on the next page illustrates the gap between North and South Korea. While the two areas began with few substantive differences, sharing a common culture and identity, their different economic institutions mean that they are now completely different. Viewed from the sky, you can see development all over South Korea while North Korea is almost fully dark because of a lack of electricity.

Figure 6: The Korean Peninsula at Night

An Econometric Approach

The historical approach adopted by North and isolated examples of extreme events (such as the Korean split) been very valuable in highlighting cases where good institutions have facilitated economic growth and where bad institutions have prevented it. More recently, there has been an attempt to assess the role of institutions in economic development using more formal econometric techniques. An early paper in this literature was the 1999 *Quarterly Journal of Economics* paper by Robert Hall and Charles I. Jones (Recall that we previously discussed this paper's calculations of the sources of differences in output per worker). They used the term *social infrastructure* to describe the institutions that affect incentives to produce and invest. Their approach was to collect data on a large number of countries and then estimate regressions of the form

$$\frac{Y_i}{L_i} = \alpha + \beta S_i + \epsilon_i \tag{18}$$

where $\frac{Y}{L}$ is output per worker in country i and S_i is a variable that aims to measure the extent to which institutions in country i facilitate economic activity. Hall and Jones constructed their S_i variable as an average of two different variables:

1. An "index of government antidiversion policies". This is an average of five different variables relating to (i) law and order (ii) bureaucratic quality (iii) corruption (iv) risk of expropriation, and (v) government repudiation of contracts.

2. An index that focuses on the openness of a country to trade with other countries

There are two potentially serious econometric problems when assessing the linkage between productivity and institutions. The first is *endogeneity.* Do countries get rich because they have good institutions or do countries have good institutions because they are rich? The latter linkage certainly exists. Citizens in richer countries have substantial incentives to keep good institutions that promote productive efficiency because they would have lot to lose if their markets ceased to work well; these incentives may be substantial weaker in the world's poorer countries. Hall and Jones thus describe their "social infrastructure" variable as being determined by

$$S_i = \gamma + \delta \frac{Y_i}{L_i} + \theta X_i + \eta_i \tag{19}$$

In this case, a simple OLS regression of $\frac{Y_i}{L_i}$ on S_i will produce a positive estimate of β—the effect of institutions on output per worker—even if the true value of β was zero.

The second econometric problem is *measurement error*. The variables used as measures of institutional quality can only ever be proxies, and possibly poor proxies, for the true measure of institutional quality that actually affects economic output. The use of proxies like this is the same as using variables that are affected by measurement error. One of the standard results from econometrics is that measurement error can result in downward bias in coefficients. In other words, the OLS coefficient might be less than the true coefficient.

So the presence of these econometric problems means OLS estimation will produce biased estimates, though whether the bias is upwards or downwards depends on the source of the bias. The usual solution to these econometric problems is estimation via instrumental variables. This means estimating β from

$$\frac{Y_i}{L_i} = \alpha + \beta \hat{S}_i + \epsilon_i \tag{20}$$

where $\hat{S}_i$ is the fitted value from a regression of S on a set of instruments (exogenous variables that that may be correlated with the institutions variable but that are not affected by the country's level of output per worker). By focusing on variations in institutions related to exogenous factors that are not determined by output per worker, the researcher can try to identify the true causal effect of institutions.

Hall and Jones's Findings

Finding good instruments for this problem can be tricky. Many of the papers in this literature have focused on either *geography* or *history* as their inspiration for truly exogenous sources of variations in institutions.

- A country's geography is certainly exogenous—it is not influenced by a country's level of prosperity. But certain types of geographical features may be correlated with whether a country has good institutions or not. Hall and Jones used the country's distance from the equator as an instrument. Other papers have also used coastal access, average temperature, rainfall and soil quality.

- In relation to history, many countries around the world were colonised by various European countries and their current institutions (e.g. whether a country uses a French or English legal systems) are often determined, in a somewhat random fashion, by which countries colonised them. Hall and Jones used instruments measuring the fraction of people speaking English as a native language and a variable measuring the fraction of people speaking other Western European languages.

Using their selected instrument set, Hall and Jones found a positive and significant effect of their "social infrastructure" variable when estimating using IV methods, with the coefficient being higher than the OLS estimate. They concluded from this that there is a large causal effect from institutions to productivity and that the measurement error is a more important source of bias in their OLS regressions than is endogeneity.

Some Other Papers

There is now a large empirical literature on this topic. Some examples:

- Acemoglu, Johnson, and Robinson (*AER*, 2001) assess the effect on GDP per capita of institutions, proxied by a measure of "protection against expropriation risk." They use a new instrument measuring settler mortality in different European colonies. They argue that countries where mortality for initial settlers was low were places where Europeans were more likely to settle and set up good institutions, with the reverse working when settler mortality was high. With this variable as an instrument, they find a very strong effect of certain measures of institutions on output per capita.

- Rodrik, Subramanian and Trebbi (*Journal of Economic Growth*, 2004). These authors assess the role of institutions (as proxied by a variable measuring the strength of the rule of law), openness to trade and geography (as measured by distance from the equator). To be able to assess whether geography has a direct effect on income per capita, they use other variables such as the AJR settler mortality variable and language-related variables as instruments. They conclude that institutions, in the form of their rule of law variable, are the key determinant of economic success and do not find a significant role for trade or geography.

- Gillanders and Whelan (2014) compare the effect of the Rule of Law variable preferred by Rodrik, Subramanian and Trebbi with a new variable that measures the "ease of doing business". Both are institutional variables but they measure different types of institutions. This paper also applies IV methods using geographical variables as instruments and concludes that it is the ease of doing business that is the key determinant of output per capita rather than Rule of Law variable.

Things to Understand from these Notes

Here's a brief summary of the things that you need to understand from these notes.

1. The Romer model's production function.
2. The Romer model's interpretation of total factor productivity.
3. Policy trade-offs suggested by the Romer model.
4. Robert Gordon on the history and future of technological innovation.
5. The assumptions of the leader-follower model.
6. The properties of the solution to the leader-follower model.
7. How non-technological factors influence total factor productivity.
8. Douglass North on institutions.
9. How Korea illustrates the importance of institutions.
10. Hall and Jones's approach to assessing the links between institutions and economic success.
11. The econometric problems that Hall and Jones confronted and their findings.
12. Findings of other papers in this literature.

Appendix: Solution to Leader-Follower Model

Before deriving this solution, we need to start with some background.

Exponential Growth

You've probably heard about exponential functions before. The number $e \approx 2.71828$ is a very special number such that the function

$$\frac{de^x}{dx} = e^x \tag{21}$$

One way to see why the number is 2.718 is to use something called the Taylor series approximation for a function, which states that you can approximate a function $f(x)$ as

$$f(x) = f(a) + f'(x)(x-a) + \frac{1}{2}f''(x))(x-a)^2 + \frac{1}{3!}f'''(x))(x-a)^3 + ...\frac{1}{n!}f^n(x))(x-a)^n + ... \tag{22}$$

where $n! = (1)(2)(3)...(n-1)(n)$. If there is a number, e that has the property that $e^x = f(x) = f'(x)$, then that means that all derivatives also equal e^x. In this case, we have

$$e^x = e^a + e^a(x-a) + \frac{1}{2}e^a(x-a)^2 + \frac{1}{3!}e^a(x-a)^3 + ... \tag{23}$$

Setting $x = 1, a = 0$, this becomes

$$e = 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + ... \tag{24}$$

This converges to 2.71828. Ok, that's not on the test but worth knowing. Now note that

$$\frac{de^{gt}}{dt} = \frac{de^{gt}}{d(gt)}\frac{dg}{dt} = ge^{gt} \tag{25}$$

Now let's relate this back to our model. The fact that the lead country has growth such that

$$\frac{dA_t}{dt} = gA_t \tag{26}$$

means that this country is characterised by what is known as exponential growth, i.e.

$$A_t = A_0 e^{gt} \tag{27}$$

We write the first term as A_0 because $e^{(g)(0)} = 1$ so whatever term multiplies e^{gt} that is the value that A_t takes in the first period.

Dynamics of Technology

Now we are going to try to figure out how the technology variable behaves in the follower country. The equation

$$\frac{dA_{jt}}{dt} = \lambda_j A_{jt} + \sigma_j \left(A_t - A_{jt}\right) \tag{28}$$

is what is known as a first-order linear differential equation (differential equation because it involves a derivative; first-order because it only involves a first derivative; linear because it doesn't involve any terms taken to powers than are not one.) These equations can be solved to illustrate how A_j changes over time. To do this, we first draw some terms together to re-write it as

$$\frac{dA_{jt}}{dt} + \left(\sigma_j - \lambda_j\right) A_{jt} = \sigma_j A_t \tag{29}$$

Recalling equation (27) for the technology level of the leader country, this differential equation can be re-written as

$$\frac{dA_{jt}}{dt} + \left(\sigma_j - \lambda_j\right) A_{jt} = \sigma_j A_0 e^{gt} \tag{30}$$

Now we'll move on to illustrating how people figure out how an A_{jt} that satisfies this equation needs to behave.

One Possible Solution

Let's think about what we learned about exponential functions to help us see what form a potential solution might take. The derivative of A_{jt} with respect to time plus $(\sigma_j - \lambda_j)$ times A_{jt} can be written as a multiple of the exponential function.

Looked at this way, we might guess that one possible solution for an A_{jt} process that will satisfy this equation is something of the form $B_j e^{gt}$ where B_j is some unknown coefficient. Indeed, it turns out that this is the case. Let's figure out what B_j must be. It must satisfy

$$gB_j e^{gt} + (\sigma_j - \lambda_j) B_j e^{gt} = \sigma_j A_0 e^{gt} \tag{31}$$

Canceling the e^{gt} terms, we see that

$$B_j = \frac{\sigma_j A_0}{\sigma_j + g - \lambda_j} \tag{32}$$

So, this solution takes the form

$$A_{jt}^p = B_j e^{gt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j} \right) A_0 e^{gt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j} \right) A_t \tag{33}$$

A General Solution

Is that it or could we add on an additional term and still get a solution? Suppose we look for a solution of the form

$$A_{jt} = B_j e^{gt} + D_{jt} \tag{34}$$

Then the solution would have to obey

$$gBe^{gt} + \dot{D}_{jt} + (\sigma_j - \lambda_j) \left(Be^{gt} + D_{jt} \right) = \sigma_j A_0 e^{gt} \tag{35}$$

All the terms in e^{gt} cancel out because, by construction of B_j, they satisfy equation (31). This means the additional term D_{jt} must satisfy

$$\dot{D}_{jt} + (\sigma_j - \lambda_j) D_{jt} = 0 \tag{36}$$

Again using the properties of the exponential function, this equation is satisfied by anything of the form

$$D_{jt} = D_{j0} e^{-(\sigma_j - \lambda_j)t} \tag{37}$$

where D_{j0} is a parameter that can take on any value. So, given the differential equation (28), all possible solutions for technology in country j must take the form

$$A_{jt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j} \right) A_t + D_{j0} e^{-(\sigma_j - \lambda_j)t} \tag{38}$$

where D_{j0} is an arbitrary parameter than can take any value.

Properties of the Solution

Now we like to examine the properties of this solution. Does technology in the follower country catch up and, if not, where does it end up and why? To answer these questions, it is useful to express A_{jt} as a ratio of the frontier level of technology. This can be written as

$$\frac{A_{jt}}{A_t} = \frac{\sigma_j}{\sigma_j + g - \lambda_j} + \frac{D_{j0}}{A_t} e^{-(\sigma_j - \lambda_j)t} \tag{39}$$

Now using the fact that $A_t = A_0 e^{gt}$, this becomes

$$\frac{A_{jt}}{A_t} = \frac{\sigma_j}{\sigma_j + g - \lambda_j} + \frac{D_{j0}}{A_0} e^{-(\sigma_j + g - \lambda_j)t} \tag{40}$$

To understand the properties of this solution, recall that we assumed $\lambda_j < g$, which means that on its own (without catch-up growth) the follower country's level of technology grows slower than the leader country and also that $\sigma_j > 0$ (some learning takes place). Putting these two assumptions together, we can say

$$\sigma_j + g - \lambda_j > 0 \tag{41}$$

That means that

$$e^{-(\sigma_j + g - \lambda_j)t} \to 0 \quad \text{as} \quad t \to \infty \tag{42}$$

This means that the second term in (40) tends towards zero. So, over time, as this term disappears, the country converges towards a level of technology that is a constant ratio, $\frac{\sigma_j}{\sigma_j + g - \lambda_j}$ of the frontier level, and its growth rate tends towards g.