Before Growth: The Malthusian Model

We have devoted the last few weeks to studying economies that grow steadily over time. For many countries around the world, that has been a reasonable description of their behaviour since the start of the Industrial Revolution. However, prior to around the year 1800, there is very little evidence of steady growth in income levels. The chart on the next page is taken from a book called *A Farewell to Alms* by economic historian Greg Clark. It summarises world economic history as a long period in which living standards fluctuated over time showing no growth trend before the Industrial Revolution lead to steady growth over time (though Clark notes that this take-off did not occur in all countries and some remain exceptionally poor).

Measurement of living standards is an imprecise business even in modern economies with well-resourced statistical agencies. So it's hardly surprising that there is a lot of controversy over Clark's particular interpretation of the evidence as implying no trend growth at all in living standards prior to 1800. Other studies show slow but gradual increases in living standards prior to the Industrial Revolution but all agree that the average rate of economic growth was very low before 1800. In addition, Figure 2 shows that global population growth was extremely slow until 1800 and then increased to much higher rates.

What explains these patterns? Our previous models would suggest the pace of technological progress must have been slower before the Industrial Revolution and this is true. But cumulatively, there was a lot of technological progress in the years prior to 1800 with many advances made in science and in the organisation of economic life. One might have expected this to translate into growth in average living standards over time but the evidence suggests such progress was limited. In these notes, we will present the Malthusian model, which explains how the world works very differently when rates of technological progress are slow.

Figure 1: World Economic History (from Greg Clark's book)

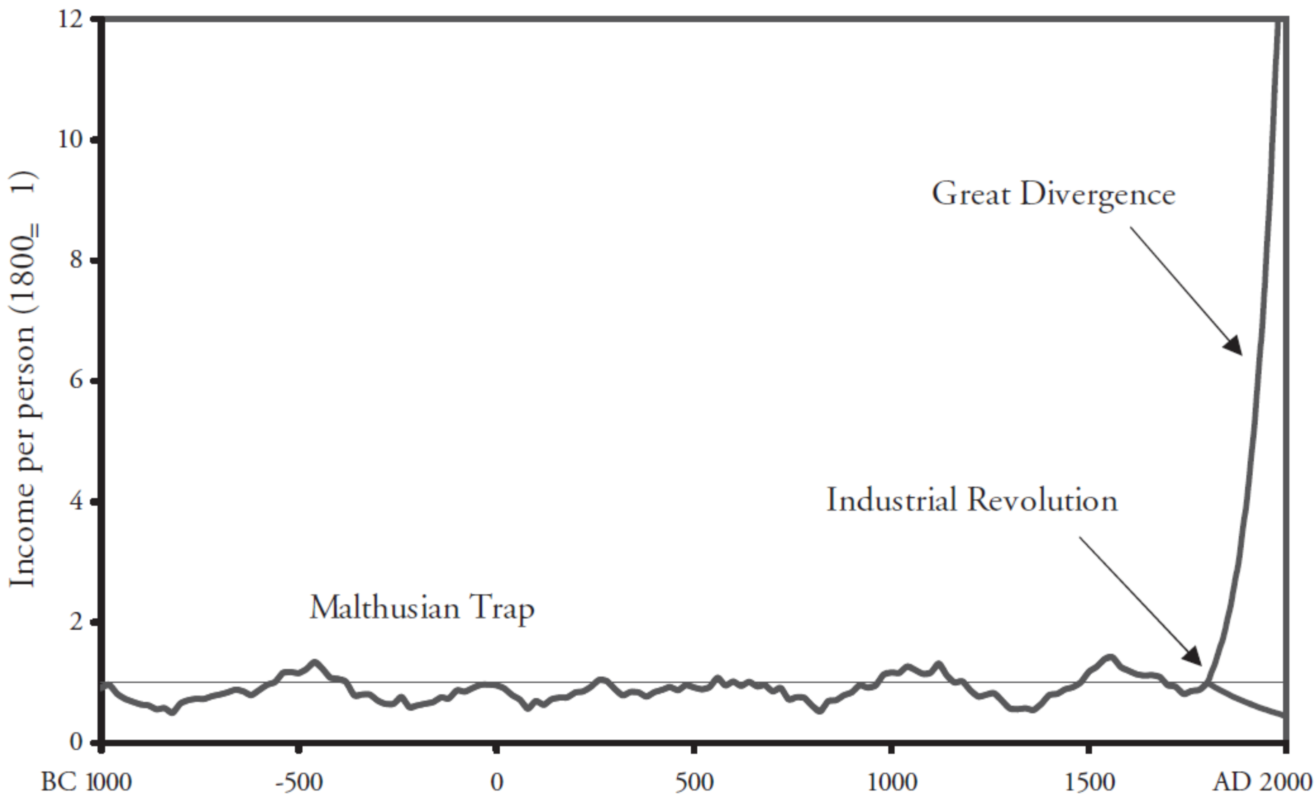
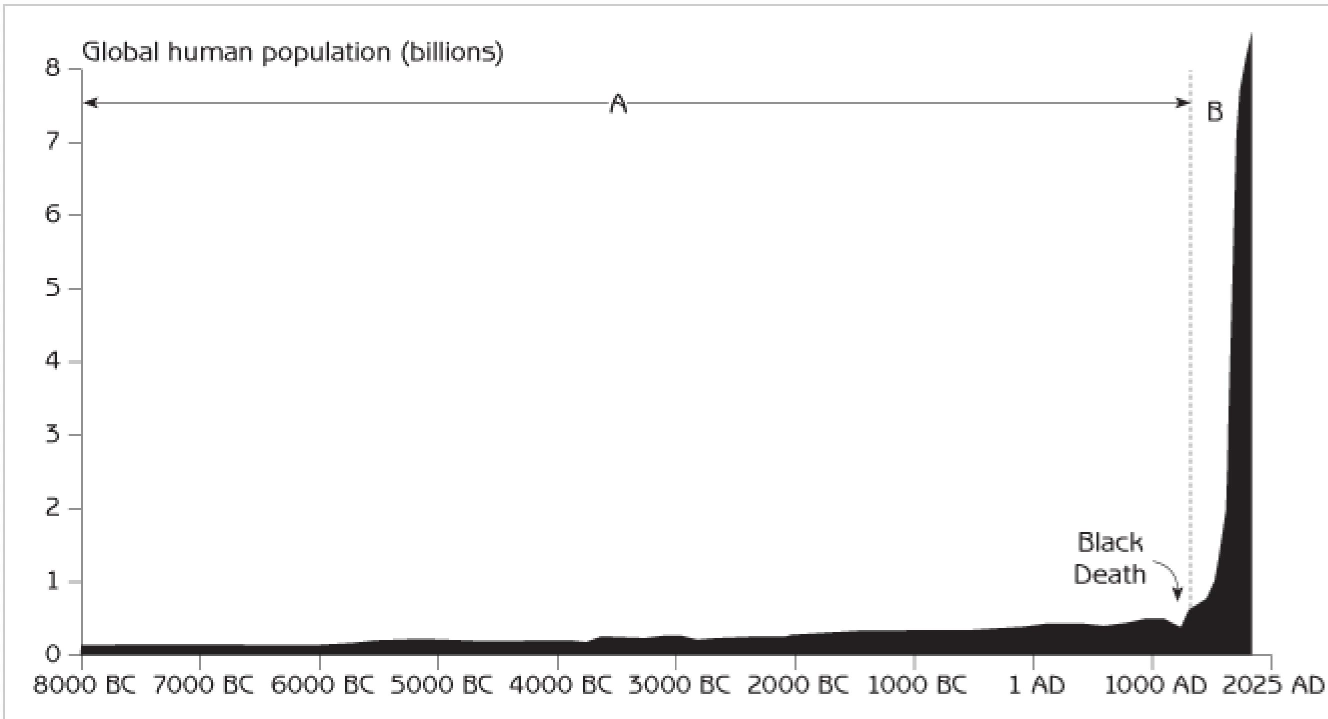


Figure 1.1 World economic history in one picture. Incomes rose sharply in many countries after 1800 but declined in others.

Figure 2: Global Population



Life Expectancy and Income Levels

The Malthusian model has two key elements: A negative relationship between income levels and the size of population and a positive relationship between income levels and population *growth*. Let's start with the second relationship

By definition, population growth increases with birth rates and falls with death rates. Death rates, in turn, are the key determinant of life expectancy. Throughout history, there has been a strong relationship between a country's average level of income per capita and its average life expectancy. This relationship still holds strongly today. Figure 3 shows a chart taken from a wonderful website called Gapminder which allows you to make animated charts showing developments over time and around the world in income levels, health outcomes and lots of other areas.

Figure 3 shows the relationship between average life expectancy and real income per person. Each dot corresponds to a country, with the size of the dot corresponding to its population. The chart shows that in some of the poorest countries in the world in 2018, average life expectancy was as low as under 50 years of age while the richest countries tend to have average life expectancy of over 80 years. Figure 4 shows a relationship of this kind holding inside a large country: U.S. counties with higher income per capita have longer life expectancy.

Internationally, this pattern is partly related to the availability of medicines in advanced countries that allow people to live much longer. But it is more influenced by very high rates of child mortality. Figure 5 shows another Gapminder chart. This one shows that mortality among children under 5 is still very common in the world's poorest countries due to malnuitrition and poor public health systems.

This relationship between income levels and the rate of death among the population will be a key element of the version of the Malthusian model that we will cover.

Figure 3: Life Expectancy and Real GDP Per Capita Around the World in 2018

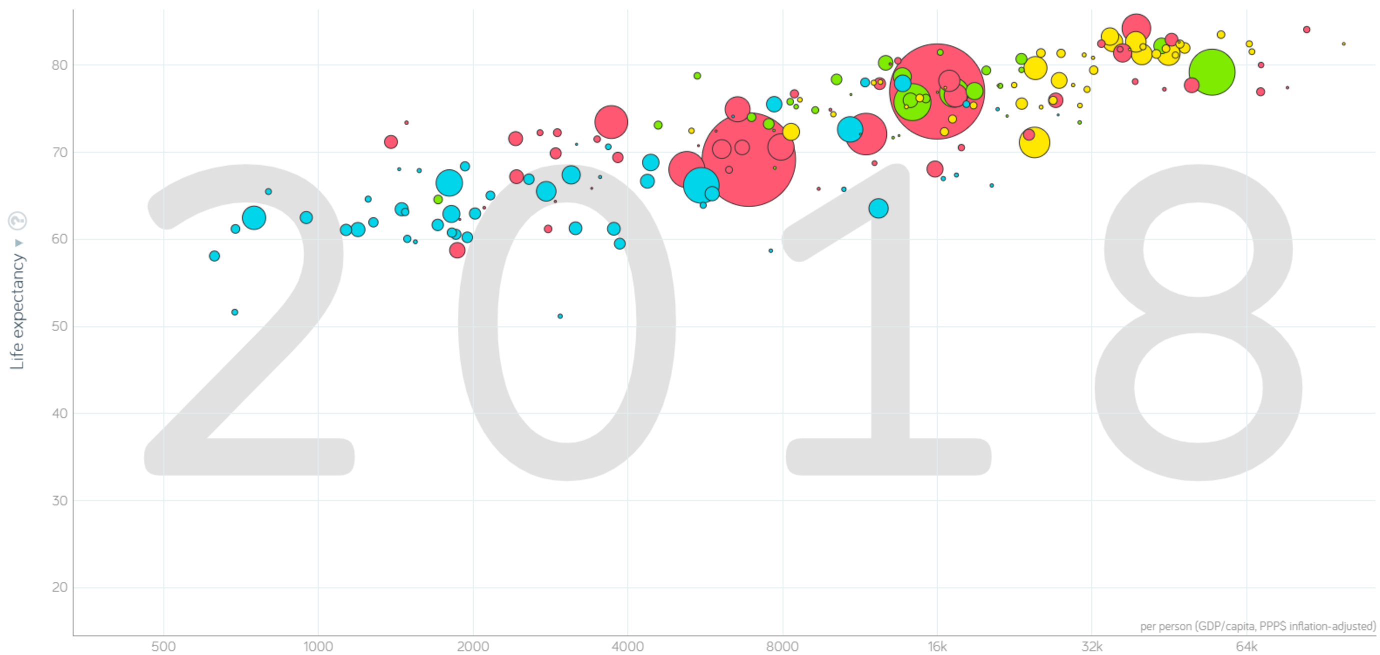


Figure 4: Life Expectancy and Income Levels: U.S. Counties

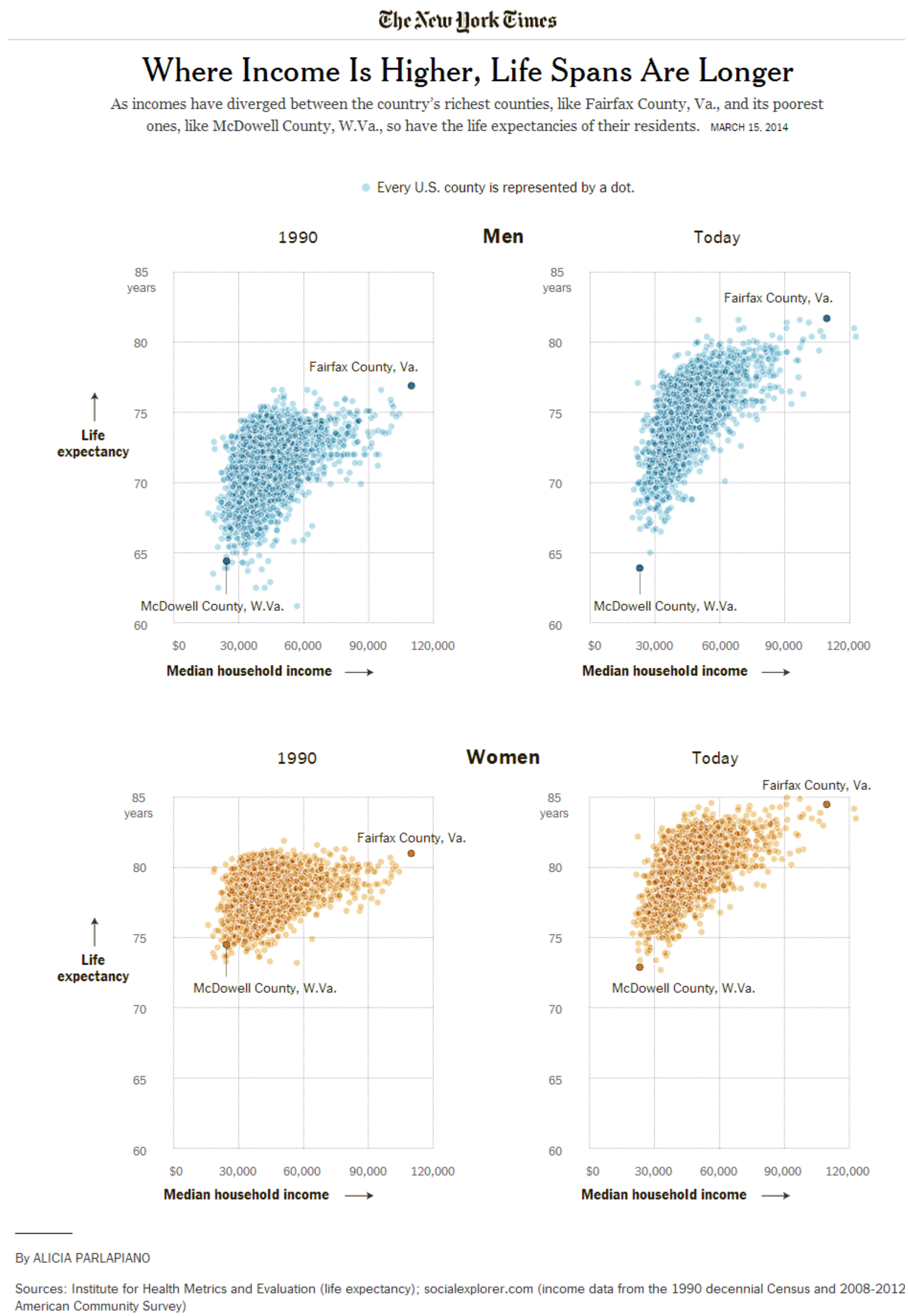
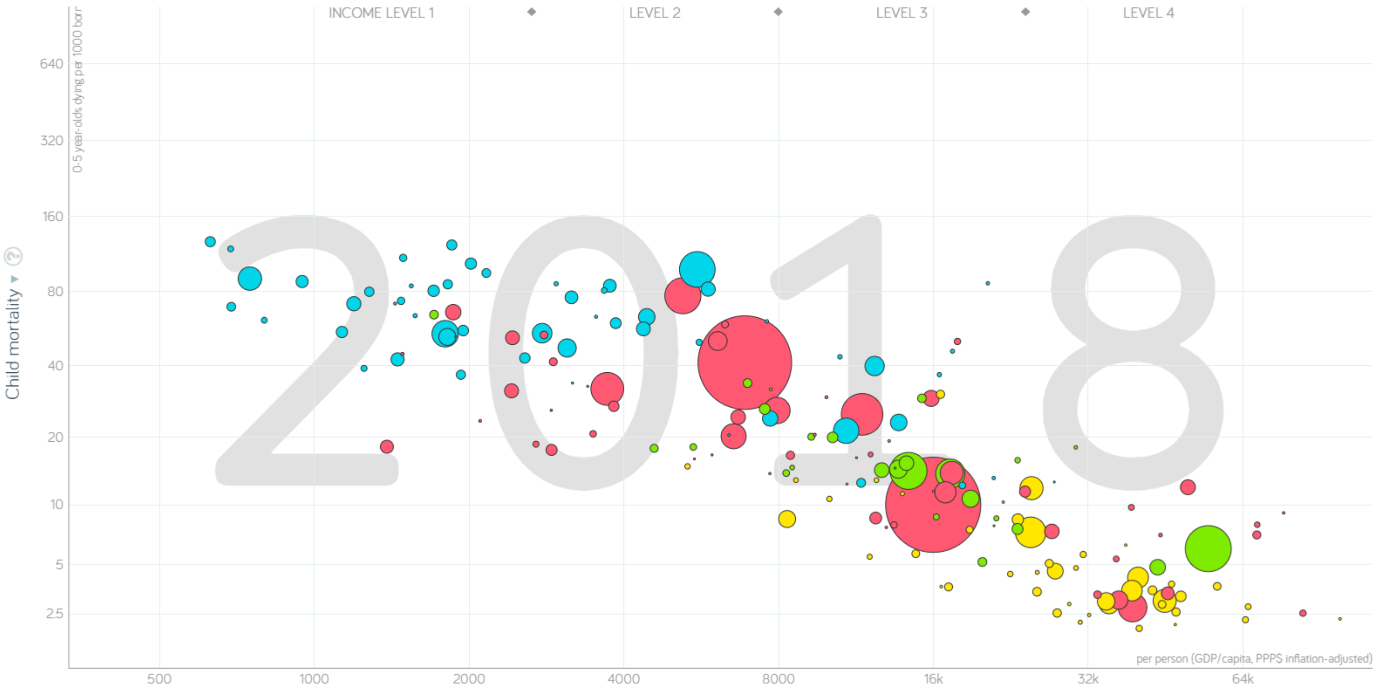


Figure 5: Child Mortality and Real GDP Per Capita Around the World in 2018



Population and Income Levels

The second element of the Malthusian model is a negative relationship between income levels and the level of population. Before discussing Malthus's thoughts on this issue, it's worth using the language of modern economics to describe this relationship.

Consider an economy in which aggregate output is determined by a Cobb-Douglas production function

$$Y_t = AK^{\alpha} L_t^{1-\alpha} \tag{1}$$

Here, I've assumed that both capital and technology are fixed (and so have no time subscript), so that labour input is the only factor that produces changes in output. We can figure out the demand for labour by assuming that the firms in the economy maximise profits in a competitive manner. Thus, firms are maximising

$$\pi = pAK^{\alpha} L_t^{1-\alpha} - wL - rK \tag{2}$$

where p is the price of output, w is the wage rate and r is an implicit rental rate for capital. The first-order condition for labour is

$$(1-\alpha)\, pAK^{\alpha} L^{-\alpha} - w = 0 \tag{3}$$

This can be re-arranged as

$$\frac{w}{p} = (1-\alpha)\, A \left(\frac{K}{L}\right)^{\alpha} \tag{4}$$

Assuming that a constant fraction θ of the population is working

$$L = \theta N \tag{5}$$

we get

$$\frac{w}{p} = (1-\alpha)\, A \left(\frac{K}{\theta N}\right)^{\alpha} \tag{6}$$

The higher the population, the lower will be the real wage. This is because of diminishing marginal returns to labour and the fact that workers are being paid their marginal wage product.

Now note that the direct link between higher population and lower wage rates (and thus lower living standards) works here because technology and capital are held constant. In the Solow growth model, there is both rising population and increasing wages because technology improvements and capital accumulation offset the negative effects on wages of rising population. In this example, we have assumed something quite different, i.e. no technological progress. We will return, however, to the question of what happens when there is a slow but steady rate of technological improvement.

Malthus (1798)

Thomas Malthus's 1798 book *An Essay on the Principle of Population* put together the two ideas that we have just discussed. He noted that rising living standards can lead to higher population growth but the famously-gloomy Malthus believed that this increase in population would ultimately undo the original increase in living standards.

Malthus placed a somewhat different emphasis on the various links than in our discussion. In relation to the link between demographics and living standards, Malthus focused on two mechanisms ("checks on living standards") that would cause population growth to increase as living standards rose and thus ultimately see the increase in living standards reversed.

The first mechanism, which Malthus labelled "the preventative check" was the tendency to see more births when real wages are high. In pre-Industrial Revolution Britain, the tradition was for people to marry relatively late as they waited to accumulate the wealth to be able to support a family. This tended to keep fertility rates relatively low. In practice, as discussed in Greg Clark's book on the Malthusian model, the evidence for a link between living standards and birth rates prior to the Industrial Revolution is fairly weak and I will assume a constant birth rate in the model treatment below (though the logic of the model is unchanged if you assume a positive relationship between birth rates and living standards.)

The second mechanism, which Malthus labelled "the positive check", was the negative effect of living standards on death rates. Evidence for this mechanism is stronger and still exists today. Malthus describes it as follows:

> *the actual distresses of some of the lower classes, by which they are disabled from giving the proper food and attention to their children, act as a positive check to the natural increase of population.*

This is the mechanism that we will focus on in our description of the model.

In relation to the negative effect of population on living standards, I've used a production function approach and emphasised the role played by the assumption of technology increases failing to offset the effect of increased population. Malthus focused more the idea of increased numbers of people putting a strain on food resources:

> "*An increase of population without a proportional increase of food will evidently have the same effect in lowering the value of each mans patent. The food must necessarily be distributed in smaller quantities, and consequently a days labour will purchase a smaller quantity of provisions. An increase in the price of provisions would arise either from an increase of population faster than the means of subsistence, or from a different distribution of the money of the society.*

The Model and its Convergent Dynamics

We will now describe a Malthusian model in somewhat more formal terms than Malthus did. Basically, I'm following Greg Clark's version of the model as described in Chapter 2 of his book, though I'm using a constant birth rate rather than one that depends on income levels.

The model has four equations. First, there is the definition of the change in the population, which just states that population equals last period's population plus last period's level of births minus deaths. (There are lots of different possible timing conventions here. I have in mind that the population level is measured at the start of each period, while births and deaths occur over the course of the period, but the particular timing convention adopted isn't important):

$$N_t = N_{t-1} + B_{t-1} - D_{t-1} \tag{7}$$

Births are a constant fraction of the population

$$\frac{B_t}{N_t} = b \tag{8}$$

While deaths are a decreasing function of real income per person

$$\frac{D_t}{N_t} = d_0 - d_1 Y_t \tag{9}$$

Finally, real income per person is a negative function of the population size:

$$Y_t = a_0 - a_1 N_t \tag{10}$$

Figure 6 shows how the death and birth rate equations combine together to make population dynamics a function of income per person. The death rate depends negatively on income per person, so at sufficiently high income levels—in this case, levels above Y^*—births are greater than deaths and population is growing, while population is falling at income levels below Y^*.

Figure 7 then shows that the economy tends to return to this equilibrium level of income. When income is above Y^*, population is growing. But Figure 7 shows that growing population means income levels are falling. So income levels tend to fall when income is above Y^* and increase when it is below Y^*. Similarly population tends to fall when it is above the level of population associated with Y^*, call this N^*, and rise when it is below this level. This means that both income and population display what we have called *convergent dynamics* in our discussion of the Solow model: Wherever the economy starts out, it tends to converge towards these specific levels of income and population. Because the economy tends to revert back to the same levels of income and population, this phenomenon is often called *The Malthusian Trap*.

Figure 8 shows how the birth and death schedules, on the one hand, and the income-population schedule on the other, combine to determine the model's properties. Perhaps surprisingly, it is the birth and death schedules and not the income-population schedule that determines the long-run level of real income per person in the model. The income-population schedule then determines how many people are alive, given that level of income.

Figure 6: Birth and Death Rate Schedules

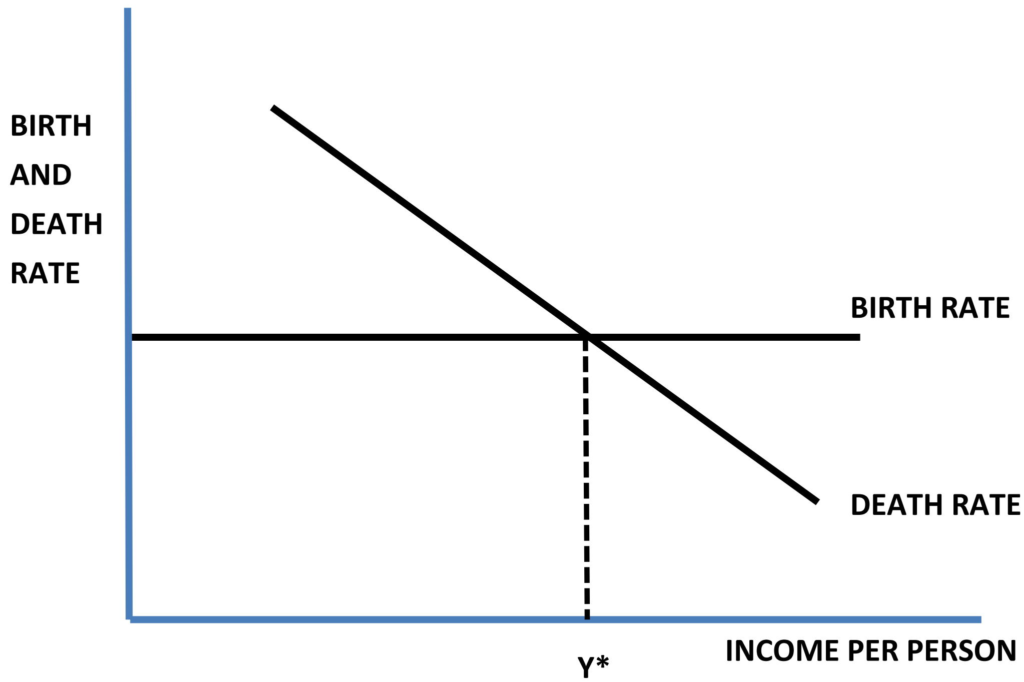


Figure 7: The Income-Population Schedule

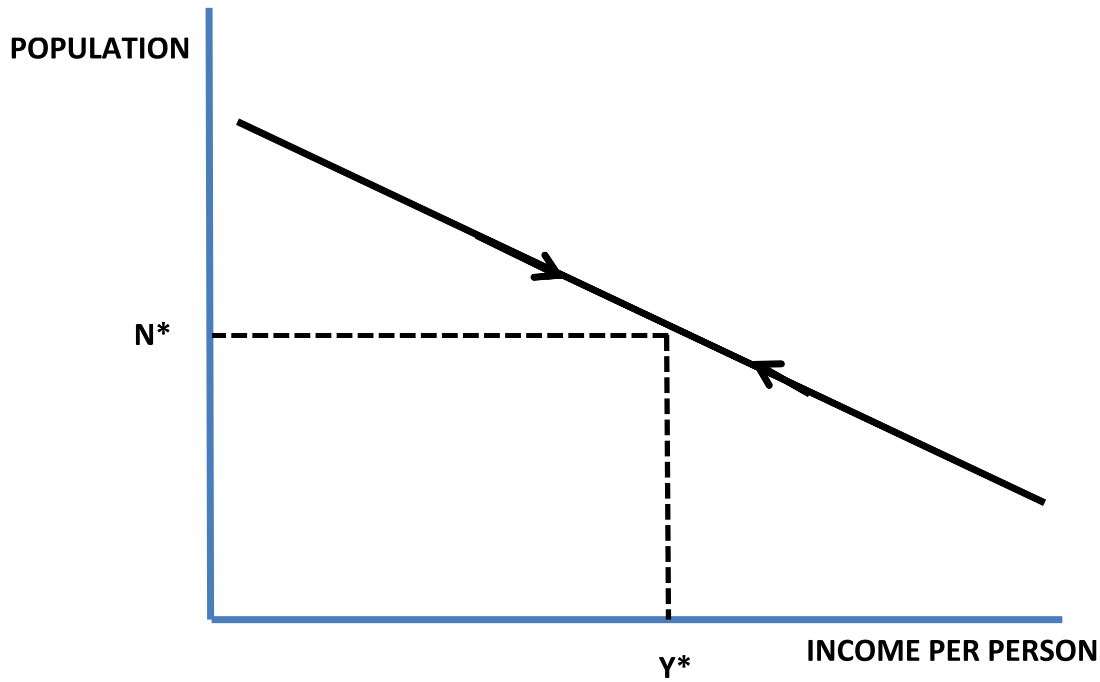
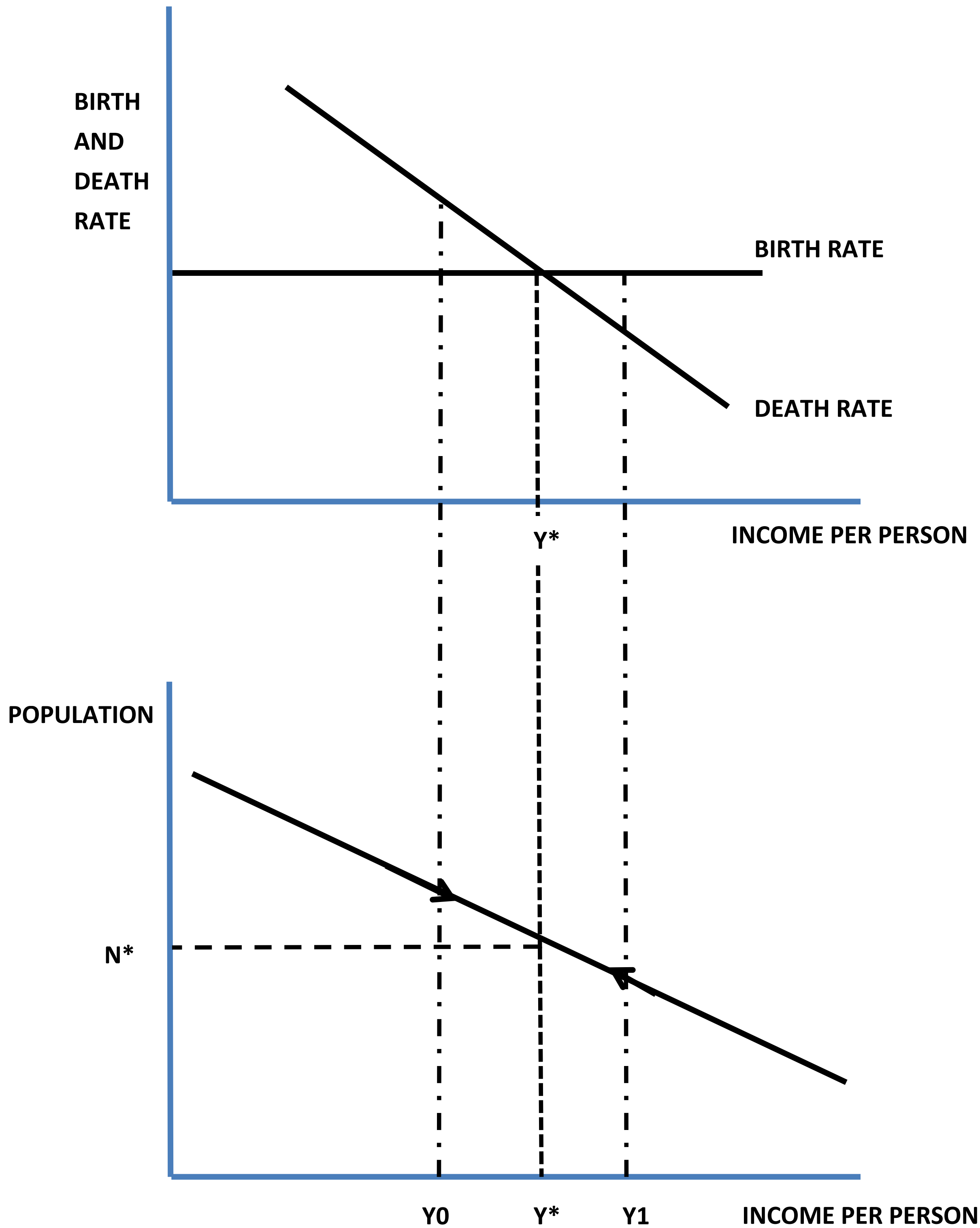


Figure 8: The Full Model



Calculating the Long-Run Equilibrium

We can figure N^* and Y^* out algebraically as follows. Combining the birth and death schedules with the equation for population change, we get

$$\frac{N_t - N_{t-1}}{N_{t-1}} = b - d_0 + d_1 Y_{t-1} \tag{11}$$

Inserting the dependence of income levels on wages, we get

$$\frac{N_t - N_{t-1}}{N_{t-1}} = b - d_0 + d_1 a_0 - d_1 a_1 N_{t-1} \tag{12}$$

This shows that the growth rate of population depends negatively on last period's level of population: This is what determines the convergent dynamics. The level of N such that population stays unchanged, shown in the Figure 7 as N^*, is given by

$$b - d_0 + d_1 a_0 - d_1 a_1 N^* = 0 \tag{13}$$

which solves to give

$$N^* = \frac{b - d_0 + d_1 a_0}{d_1 a_1} \tag{14}$$

The long-run equilibrium level of population depends positively on the birth rate, b, and on a_0, which effectively measures the level of technology in the model (if this increases it can offset the negative effect of higher population on income levels). The equilibrium level of population depends negatively on the exogenous element of the death rate (d_0), on the sensitivity of the death rate to income levels (d_1), and on the sensitivity of income levels to population (a_1).

The long-run equilibrium level of real income per person can be derived as the income level that gives a growth rate of population of zero

$$\frac{N_t - N_{t-1}}{N_{t-1}} = b - d_0 + d_1 Y^* = 0 \Rightarrow Y^* = \frac{d_0 - b}{d_1} \tag{15}$$

This level of income, as we noted above from the graphical illustration of the model, depends only on the parameters of the birth and death schedule and not at all on the parameters of the income-population schedule. So, for example, even if there was an increase in a_0 so that people could be paid more wages for each level of population, this would result, over time, only in higher population rather than higher income levels. Income levels depend negatively on birth rates, positively on death rates and negatively on the sensitivity of death rates to income levels.

A final way of illustrating the convergent dynamics of the model is to note that equation (12) for population growth can be re-written as

$$\frac{N_t - N_{t-1}}{N_{t-1}} = (d_1 a_1)\left(\frac{b - d_0 + d_1 a_0}{d_1 a_1} - N_{t-1}\right) \tag{16}$$

Using the formula for N^* in equation (14), this becomes

$$\frac{N_t - N_{t-1}}{N_{t-1}} = (d_1 a_1)\left(N^* - N_{t-1}\right) \tag{17}$$

In other words, the growth rate of population is determined by how far population is from its equilibrium level, with the speed of adjustment to this equilibrium, $d_1 a_1$, determined by the sensitivity of income levels to population and the sensitivity of the death rate to income levels.

How the Malthusian Economy Responds to Shocks

Finally, we consider three kinds of shocks to the Malthusian economy. In each case, we assume the economy starts at an equilbrium with population of N_0 and income levels Y_0. First, consider an increase in d_0 which shifts the death rate schedule up. Figure 9 illustrates what happens: At the starting level of income, Y_0, death rates now start to exceed birth rates.

Population falls and income rises until we reach the new higher equilibrium level of income Y_1 with its corresponding lower level of population N_1.

Figure 10 illustrates the consequences of an increase in the birth rate, b. This shock works in the opposite fashion to the death rate shock.

Finally, Figure 11 illustrates the consequences of a once-off increase in technology, i.e. an increase in a_0 so that people are able to earn more money at each level of population. The initial response to this shock is higher income levels. However, these higher income levels reduce the death rate and, over time, income levels return to their original equilibrium level. While income levels return to their original level, population is permanently higher because the new level of productivity permits a higher level of population than the old level.

There is an interesting contrast here between what happens when there is technological progress in the Solow model and when technology improves in the Malthusian model. The difference relates to the assumption in the Solow model that there is a consistent and non-trivial pace of technology increase. In the Malthusian model, the instantaneous effect of an increase in efficiency is an improvement of living standards. But this is offset over time by population increases if there aren't any further increases in technology.

In the Solow model, technology keeps increasing and keeps pushing up incomes every period, so the population can steadily increase without pushing income levels down. Greg Clark argues that while, cumulatively, there was a large increase in technology from ancient times to 1800, the pace of this increase was never fast enough to prevent population growth eroding its effects on living standards, so that prior to the Industrial Revolution, improvements in productive efficiency only translated into higher population.

Figure 9: A Shift in the Death Rate Schedule

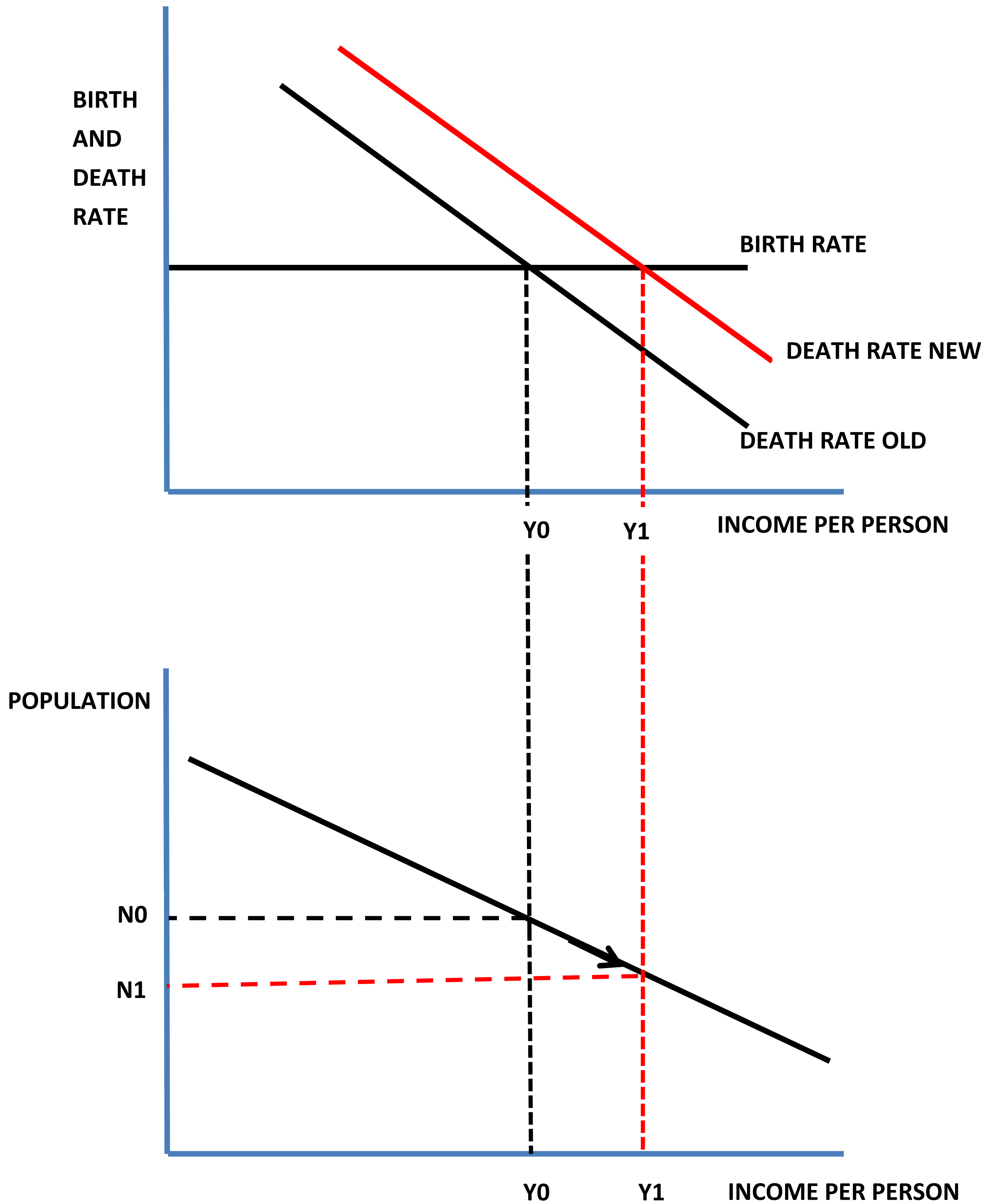


Figure 10: A Shift in the Birth Rate Schedule

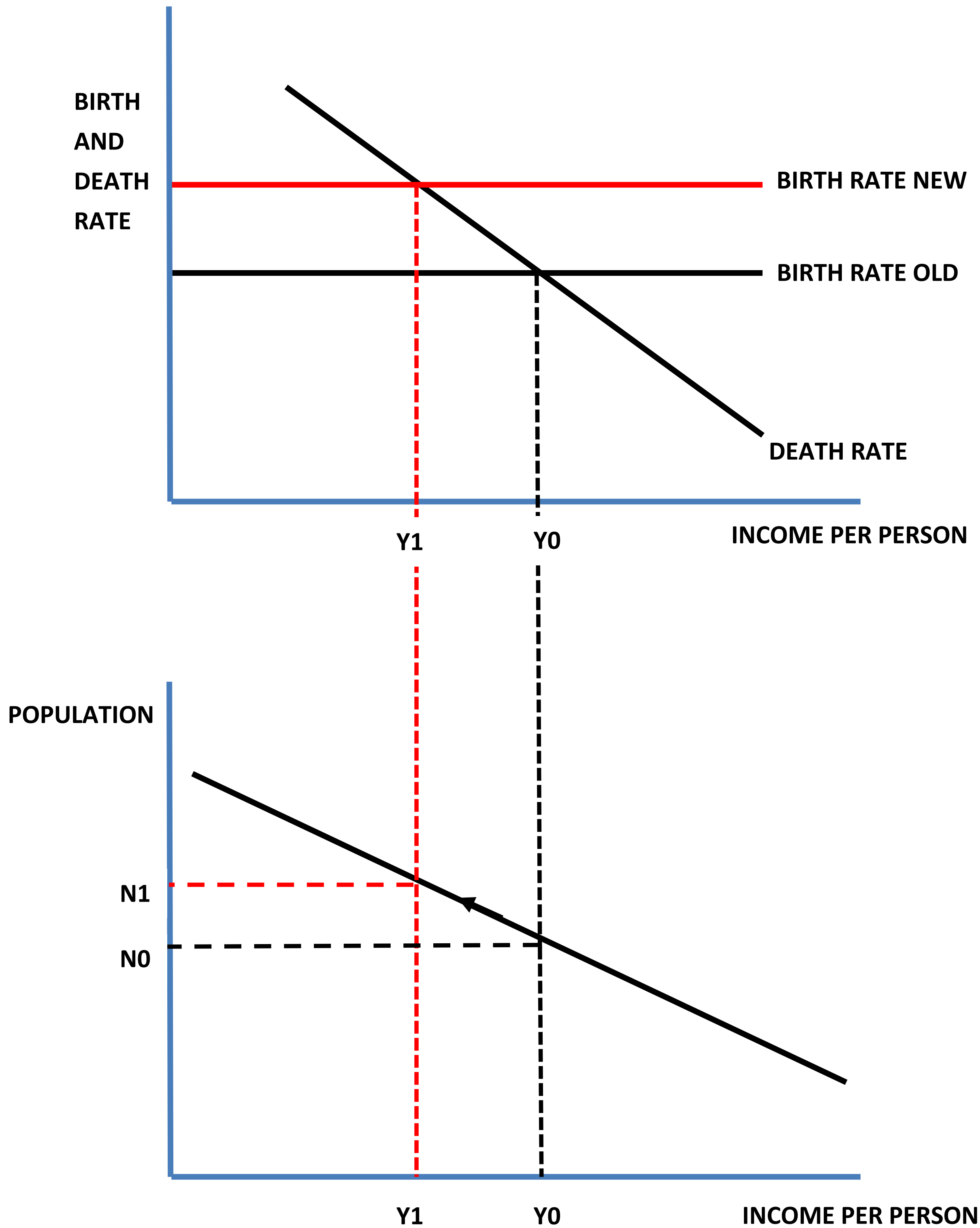
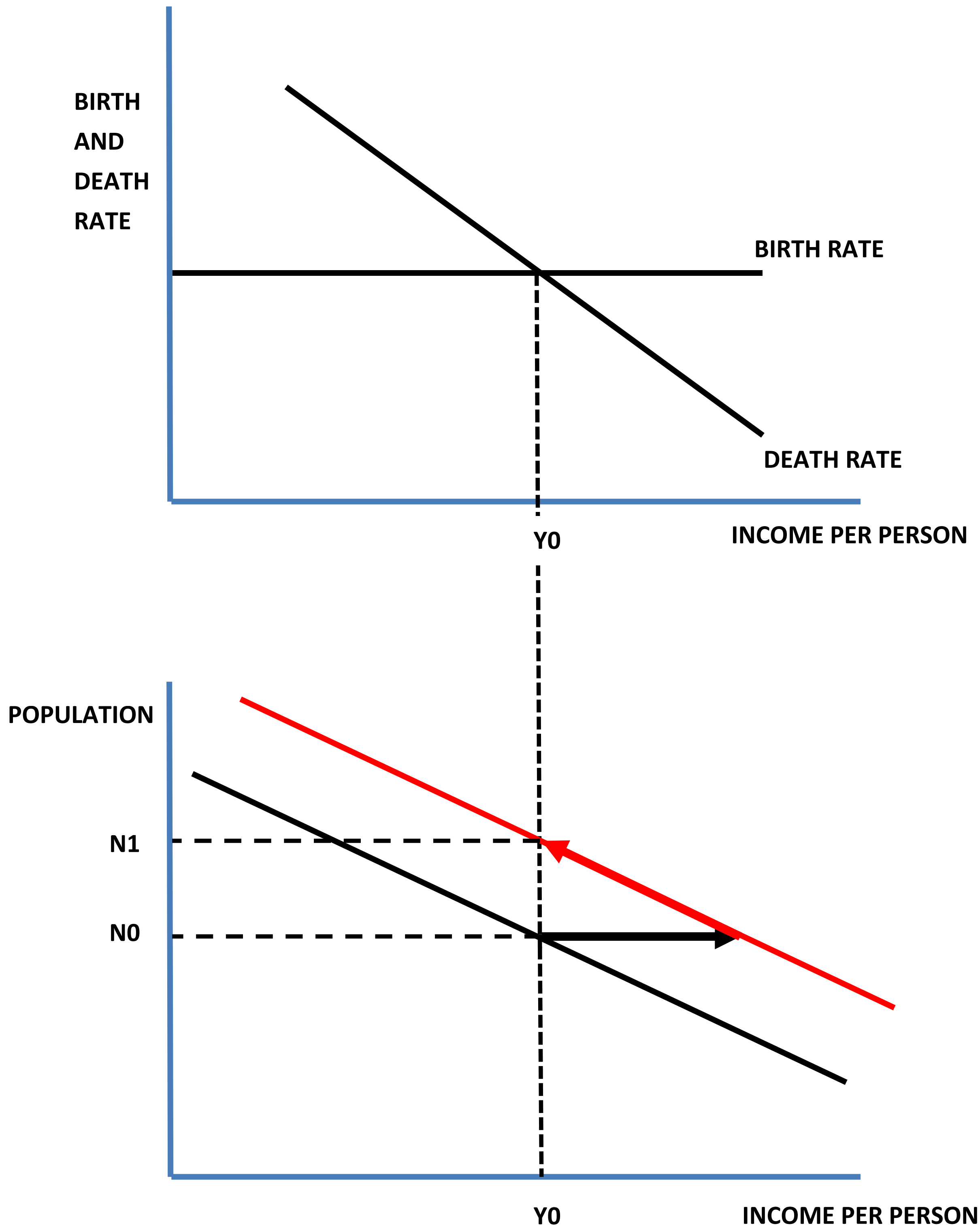


Figure 11: An Increase in Technological Efficiency



Malthus on the Poor Laws

The Malthusian model is one in which our usual understanding of what is good and what is bad is turned on its head. Things that we think are good, such as people living londer, turn out to be bad for average living standards, and things that we think are bad, like plagues and diseases, have a positive effect on those who survive. This non-intuitive worldview translated into Malthus's own policy recommendations. For example, he argued strongly against "poor laws" that provided assistance to the poor:

> *The poor laws of England tend to depress the general condition of the poor in these two ways. Their first obvious tendency is to increase population without increasing the food for its support. A poor man may marry with little or no prospect of being able to support a family in independence. They may be said therefore in some measure to create the poor which they maintain, and as the provisions of the country must, in consequence of the increased population, be distributed to every man in smaller proportions, it is evident that the labour of those who are not supported by parish assistance will purchase a smaller quantity of provisions than before and consequently more of them must be driven to ask for support.*
>
> *Secondly, the quantity of provisions consumed in workhouses upon a part of the society that cannot in general be considered as the most valuable part diminishes the shares that would otherwise belong to more industrious and more worthy members, and thus in the same manner forces more to become dependent. If the poor in the workhouses were to live better than they now do, this new distribution of the money of the society would tend more conspicuously to depress the condition of those out of the workhouses by occasioning a rise in the price of provisions.*

Over the years, Malthus has often been criticised for being overly-pessimistic about the fate of mankind and for opposing socially-progressive policies. However, it is worth noting the date that he wrote his famous essay—1798. Up until the time that he wrote his essay, his version of how the world worked actually described the economy remarkably well. It was only after his book was written that technological progress became fast enough to render his analysis less relevant.

Things to Understand from these Notes

Here's a brief summary of the things that you need to understand from these notes.

1. Facts about income levels and population before and after 1800.
2. Facts about life expectancy and child mortality around the world.
3. The elements that make up the Malthusian model.
4. The properties of the long-run equilibrium of the Malthusian model.
5. How the Malthusian economy responded to shocks.
6. Why the Solow and Malthusian models deliver such different outcomes.
7. Why Malthus opposed helping the poor.