Population & Resources: Malthus and the Environment

The Malthusian model may seem of interest today only for the light that it sheds on how the world worked before the Industrial Revolution ushered in an era of growth and increasing prosperity. Recall, however, that Malthus's views on how rising population reduced living standards focused on how increasing numbers of people placed pressures on the allocation of scarce resources, particularly food. In a world in which global population has just passed 7 billion, up from 4 billion in 1960 and 2 billion in 1927, it is reasonable to ask whether important global resources, such as energy sources, agricultural land and the global resource of a stable climate, can continue to withstand the strain of increasing population.

In these notes, we will study a model that combines a Malthusian approach to population dynamics with an approach to modelling changes in a renewable resource base, which can expand or contract. The model was first presented by James A. Brander and M. Scott Taylor in their 1998 *American Economic Review* paper "The Simple Economics of Easter Island: A Ricardo-Malthus Model of Renewable Resource Use."

Easter Island

On Easter Sunday 1722, a Dutch explorer called Jacob Roggeveen came across a Pacific island that is believed to be the most remote inhabitable place in the world. Situated over two thousand miles west of Chile (see Figure 1) it is about 1300 miles east of its nearest inhabited neighbour, Pitcairn Island. Known as Easter Island since Roggeveen's brief visit, the island its inhabitants called Rapa Nui has had a long and fascinating history.

There is no written history of events at Easter Island prior to Roggeveen's visit so we are relying on the interpretation of archeological evidence to reconstruct what happened a long time ago. The interpretation I'm passing on in these brief notes comes from my reading of a chapter in Jared Diamond's book, *Collapse*, but there are archeologists and scientists who disagree with some aspects of this story.

Easter Island was probably first populated sometime around 900 AD. That it was ever populated, given its remoteness, is somewhat extraordinary. It seems likely that, once populated, it had little (and possibly no) contact with the outside world. The most remarkable feature of the island is its collection of hundreds of carved ceremonial statues featuring torsos and heads (see Figures 2 and 3) which were mainly built between 1100 and 1500. The natives most likely erected the statues as a form of religious worship. Evidence suggests that the island was divided into twelve tribes and they competed with each other (perhaps for local pride, perhaps for favour with the gods) by building larger and larger statues over time.

The statues were enormous. On average, they were 12 feet high and weighed 14 tons, while the largest weighs 82 tons. There is plenty of evidence to show that the statues required huge resources and that at least some of these resources were organised on a shared basis by a centralised leadership. Large teams of carvers were needed to create the statues and as many as 250 people were required to spend days transporting the statues around the island. When first populated, the island had large amounts of palm trees which supplied the resources for canoes, for tools for hunting and for materials used to transport the statues (sleds, rope, levers etc.) Estimates of peak population vary but it appears that the population peaked at about 15,000 in the early 1600s.

By the time Europeans began to visit the island one hundred years later, however, the island was largely deforested and population seemed to be as low as 3,000. Without palm trees, the islanders no longer had materials with which to build good canoes and this limited their abilities to catch fish. Without forests, the island lost most of its land birds, which had been an important source of meat. By the 1700s, the population survived mainly on farming, with chickens the main source of protein, but deforestation had also reduced water retention in the soil and lead to soil erosion (the island is quite windy) so agricultural yields also declined.

Statue building had ceased by the early 1600s: Many of the statues remain today in various states of completion at the quarry at Rano Raraku where they were carved. Archeological evidence shows increasing numbers of spears and daggers appearing around this time, as well as evidence of people starting to live in caves and fortified dwellings. By the time Europeans arrived in the following century, tensions over food shortages had spilled over into intra-tribal rivalries with tribes knocking over the statues of their rivals. By the mid-1800s, all the statues had been toppled, so today's standing statues have been put in place in modern times.

There are many gaps in our understanding of what happened at Easter Island but the basic story appears to be that the population expanded to the point where the island's resources began to diminish and once population started to decline, the island went into a downward spiral. By the time Europeans visited in the seventeenth century, both population and resources had been greatly diminished from their peak levels.

The model laid out over the next few pages provides a description of how this can happen. We conclude with some thoughts about why it was allowed to happen and the potential implications for current global environmental problems.

Figure 1: The World's Most Remote Place

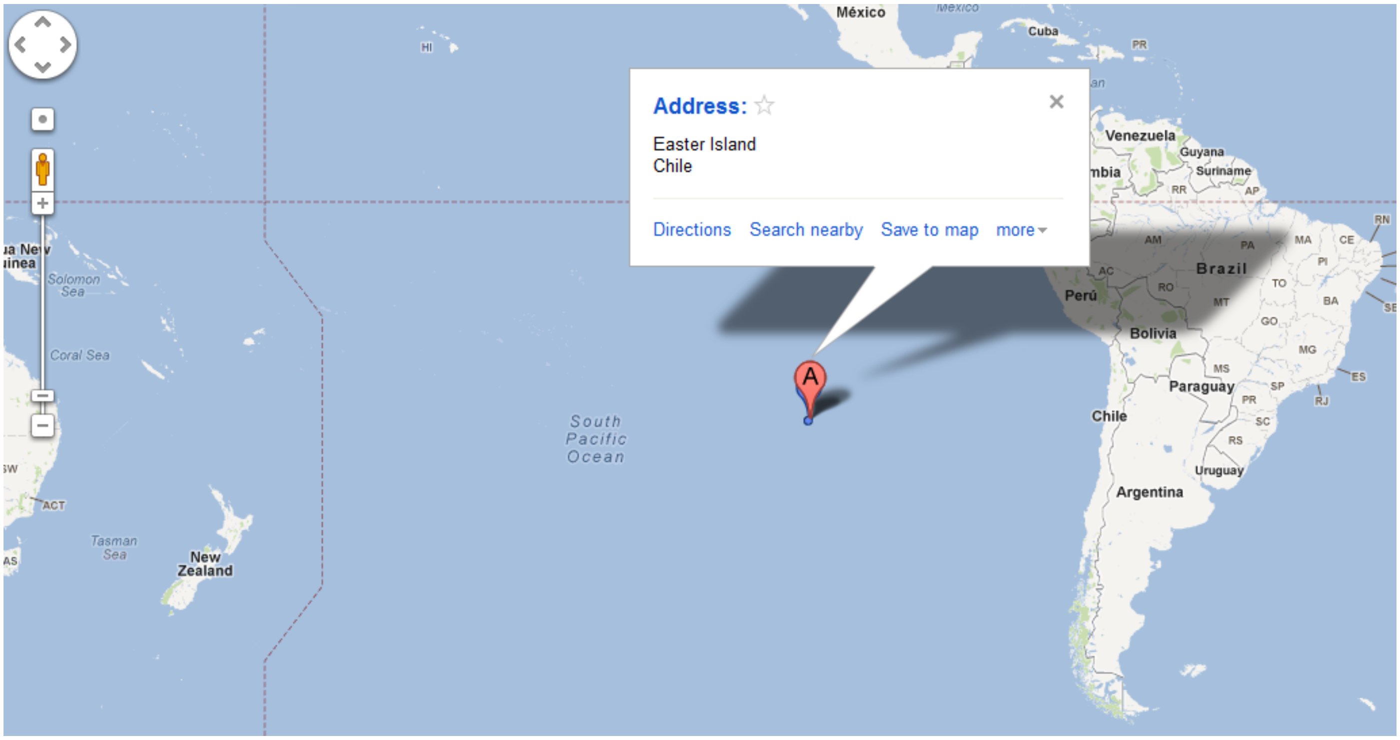


Figure 2: Easter Island Statues

Figure 3: Some Standing, Some Toppled

The Model

The model economy consists of population of N_t people at time t, who sustain themselves by collecting a harvest, H_t from a renewable resource stock denoted by S_t. Think of S_t as equivalent to a forest, or a herd of animals, or a stock of fishes; more realistically, think of it as the combination of a set of different resources of this type.

The model consists of three elements. The first element describes the change in population: This depends positively on the size of the harvest (a bigger harvest means less deaths and perhaps more births) and on an exogenous factor $d > 0$ such that without a harvest, there is a certain percentage reduction in population.

$$\frac{dN_t}{dt} = -dN_t + \theta H_t \tag{1}$$

The next element describes the harvest. We assume that the harvest reaped per person is a positive function of the size of the resource stock.

$$\frac{H_t}{N_t} = \gamma S_t \tag{2}$$

The final element, describing the change in the resource stock, is perhaps the most important. We are describing a resource stock that is renewable. It doesn't simply decline when harvested until it is all gone. Instead, it has its own capacity to increase. For example, stocks of fish can be depleted but will increase naturally again if fishing is cut back. So, our equation for the change in resources is

$$\frac{dS_t}{dt} = G(S_t) - H_t \tag{3}$$

The second term on the right-hand-side captures that the resource stock is reduced by the amount that is harvested. The first element is more interesting. It describes the ability of the resource to grow. Brander and Taylor use a logistic function to describe how the resource stock renews itself

$$G(S_t) = rS_t(1 - S_t) \tag{4}$$

This equation can be interpreted as follows. The maximum level of resources is $S_t = 1$: At this level, there can no further increase in S_t. Also, if $S_t = 0$ so the resource base has disappeared, then it cannot be regenerated. For all levels in between zero and one, we can note that

$$\frac{G(S_t)}{S_t} = r(1 - S_t) \tag{5}$$

So the amount of natural renewal as a fraction of the stock decreases steadily as the stock reaches its maximum value of one. This means that if the stock gets very low, it can grow at a fast rate if there is limited harvesting. However, if the stock is starting from a low base, the absolute size of this increase may still be small.

Dynamics of Population

We are going to describe the dynamics of this model using what is known as a *phase diagram*, which is a diagram that shows the direction in which variables are moving depending upon the values that they take. In our case, we are going to describe the joint dynamics of N_t and S_t.

Inserting the equation for the harvest, equation (2), into equation (1) for population growth, we get

$$\frac{dN_t}{dt} = -dN_t + \theta\gamma S_t N_t \tag{6}$$

This equations shows us that the change in population is a positive function of the resource stock. This means there is a particular value of the resource stock, S^*, for which population growth is zero. When resources are higher than S^* population increases and when it is lower than S^* population declines. The value of S^* can be calculated as the value for which the change in population is zero meaning

$$-dN_t + \theta\gamma S^* N_t = 0 \Rightarrow S^* = \frac{d}{\theta\gamma} \tag{7}$$

The resource stock consistent with an unchanged population depends positively on the exogenous death rate of the population, d, and negatively on the sensitivity of the population to the size of the harvest, θ, and on γ which describes the productivity of the harvesting technology.

Figure 4 shows how we illustrate the dynamics with a phase diagram. We put population on the x-axis and the stock of resources on the y-axis. Unchanged population corresponds to a straight line at S^*. For all values of resources above S^* population is increasing: Thus in the area above the line, we show an arrow pointing right, meaning population is increasing. In the area below this line, there is an arrow pointing left, meaning population is falling.

Dynamics of Resources

The dynamics of resources are derived by substituting in the logistic resource renewal function, equation (4), and the equation for the harvest, equation (2), into equation (3) to get

$$\frac{dS_t}{dt} = rS_t\left(1 - S_t\right) - \gamma N_t S_t \tag{8}$$

The stock of resources will be unchanged for all combinations of S_t and N_t that satisfy

$$rS_t\left(1 - S_t\right) - \gamma N_t S_t = 0 \Rightarrow N_t = \frac{r\left(1 - S_t\right)}{\gamma} \tag{9}$$

This means that there is downward sloping line in $N - S$ space along which each point is a point such that the change in resources is zero. This line is shown on Figure 5. The upper point crossing the S axis corresponds to no change because $S = 1$ and there are no people; as we move down the line we get points that correspond to no change in the stock of resources because while there are progressively larger numbers of people, the stock gets smaller and so can renew itself at a faster pace.

Remembering that equation (8) tells us that the change in the stock resources depends negatively on the size of the population, note now that every point that lies to the right of the downward-sloping $\dot{S}_t = \frac{dS}{dt} = 0$ line has a higher level of population than the points on line. That means that the stock of resources is declining for every point to the right the line and increasing for every point to the left of it. Thus, in the area above the downward-sloping line on Figure 5, we show an arrow pointing down, meaning the stock of resources is falling. In the area below this line, there is an arrow pointing up, meaning the stock of resources is increasing.

The Joint Dynamics of Population and Resources

In Figure 6, we put together the four arrows drawn in Figures 4 and 5. This phase diagram shows that the joint dynamics of population and resources can be divided up into four different quadrants.

We can also see that there is one point at which both population and resources are unchanged, and thus the model stays at this point if it is reached. We know already from equation (7) that the level of the resource stock at this point is $S^* = \frac{d}{\theta\gamma}$. We can calculate the level of population associated with this point by inserting this formula into equation (9):

$$N^* = \frac{r\left(1 - \frac{d}{\theta\gamma}\right)}{\gamma} = \frac{r\left(\theta\gamma - d\right)}{\theta\gamma^2} \tag{10}$$

This level of population depends positively on r (so faster resource renewal raises population) and on θ (the sensitivity of population growth to the harvest) and negatively on d (the exogenous death rate coefficient).

This point is clearly some kind of "equilibrium" in the sense that once the economy reaches this point, it tends to stay there. But is the economy actually likely to end up at this point? The answer is yes: From any interior point (i.e. a point in which there is a non-zero population and resource stock) the economy eventually ends up at (N^*, S^*). It's beyond the scope of this class to prove formally that this is the case (the Brander-Taylor paper goes through all the gory details) but I can note that, after messing around with the equations, one can show that

$$\frac{1}{N_t}\frac{dN_t}{dt} = \theta\gamma\left(S_t - S^*\right) \tag{11}$$

$$\frac{1}{S_t}\frac{dS_t}{dt} = \gamma\left(N_t - N^*\right) + r\left(S_t - S^*\right) \tag{12}$$

so the dynamics of both population and the resource stock are both driven by how far the economy is from this equilibrium point.

Harvesting and Long-Run Population

What does changing the parameter γ (which determines the fraction of the resources that is harvested) do to the equilibrium level of population? There are two different effects. On the one hand, a higher γ means a smaller amount of people consume the natural growth in resources that occurs in steady-state — this would tend to reduce the sustainable level of population. On the other hand, the smaller stock of resouces associated with the higher value of γ implies a higher harvest which could sustain more people. We can calculated the derivative of the equilibrium level of population with respect to γ as follows

$$\frac{dN^*}{d\gamma} = -\frac{r}{\gamma^2} + \frac{2rd}{\theta\gamma^3} \tag{13}$$

$$= \frac{1}{\gamma^2}\left(\frac{2d}{\theta\gamma} - 1\right) \tag{14}$$

$$= \frac{r}{\gamma^2}(2S^* - 1) \tag{15}$$

This shows that whether an increase in γ raises or reduces the equilibrium population depends on the size of the equilibrium level of resoures. If the equilibrium level of resources is over half the original maximum amount (which we have set equal to one) then we have $2S^* - 1 > 0$ and a more intensive rate of harvesting raises the population even though it reduces the total amount of resources. On the other hand, if the equilibrium level of resources is less than half the original maximum amount (which we have set equal to one) then we have $2S^* - 1 > 0$ and a more intensive rate of harvesting reduces the population.

An economy like Easter Island, where the economy ended up with a hugely diminished amount of resources, likely corresponds to the latter case, so it was an example of an economy that would have had a higher long-run population if they had harvested less.

Back to Easter Island

Let's go back to Easter Island and imagine the island in its early days with a full stock of resources and very few residents. What happens next? Figure 7 provides an illustration.

The economy starts out in what we can call "the happy quadrant" with resources above the long-run equilibrium and an expanding population. How do we know the dynamics take the "curved" form displayed in Figure 7? Well, when the economy crosses into the bottom right quadrant, in which population is now falling, the economy doesn't suddenly jump off in a different direction; the model's equations don't allow for any sudden jumps. Thus, the turnaround from increasing population to falling population must occur gradually over time.

So what happens to our theoretical Easter Island?

- For many years, the population expands and resources decline.
- Then, when it moves into the bottom right quadrant, population falls and resources keep declining.
- Then the economy moves into the bottom left quadrant where population keeps falling but resources finally start to recover.
- Then the economy moves into the quadrant in the triangle under the two curves and population starts to recover and resources increase.
- Finally, the economy moves back into the quadrant where it started but with less population and lower resources. The process is repeated with smaller fluctuations until it ends up at equilibrium with $S = S^*$ and $N = N^*$

Our theoretical Easter Island sees its population far overshoot its long-run equilibrium level before collapsing below this level and then oscillating around the long-run level and then finally settling down.

Figure 4: Population Dynamics

Figure 5: Resource Stock Dynamics

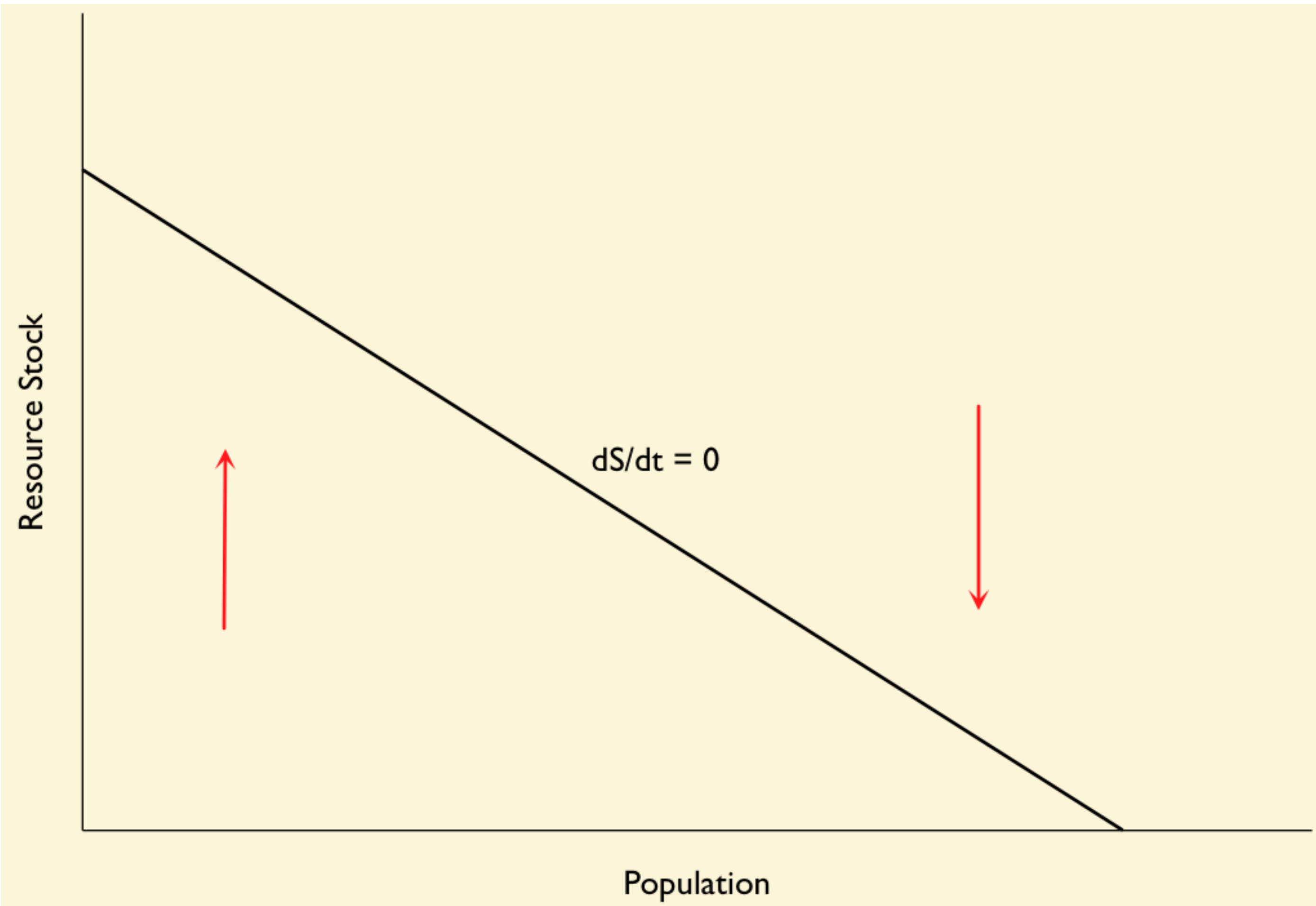


Figure 6: Dynamics Differ In Four Quadrants

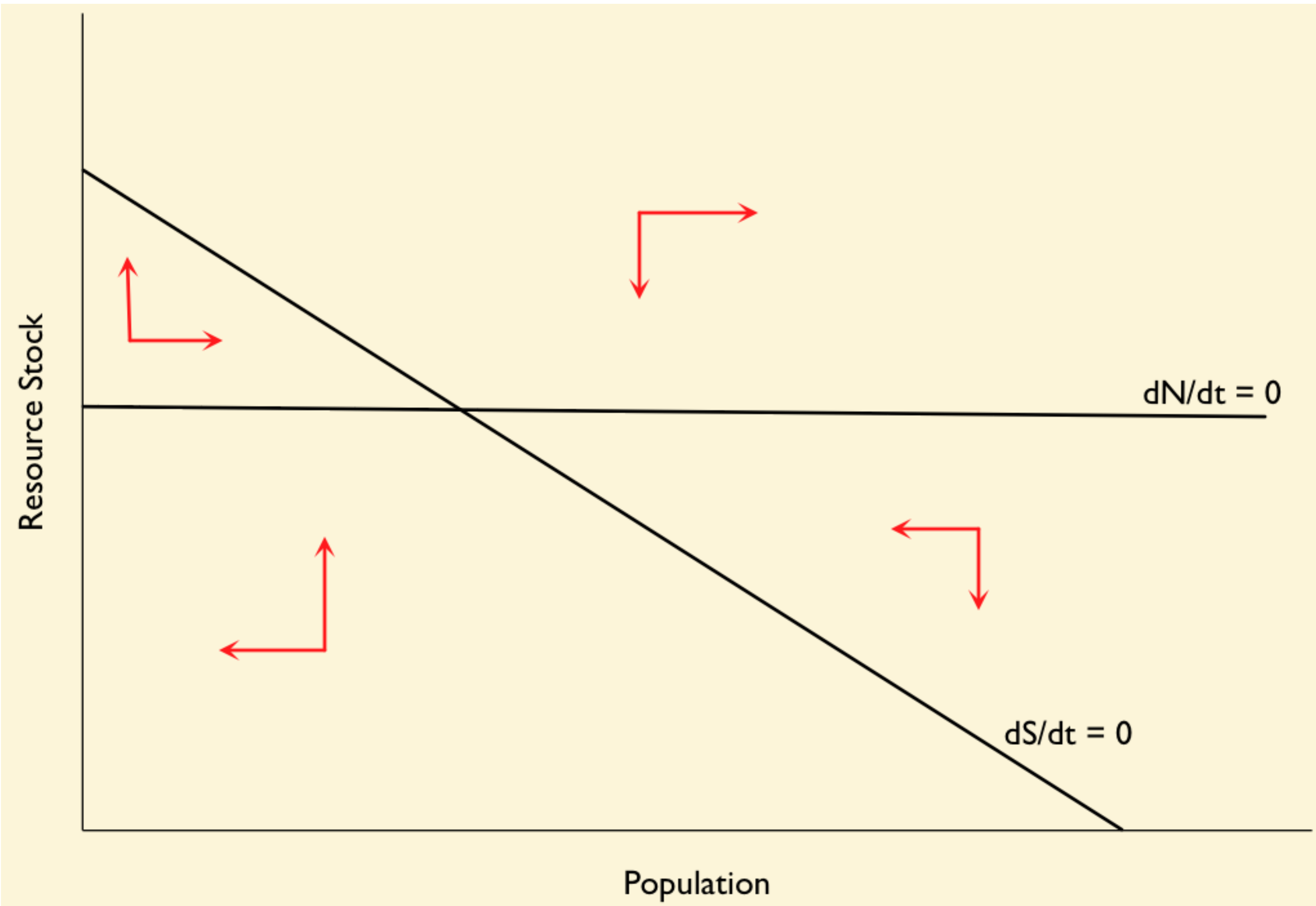
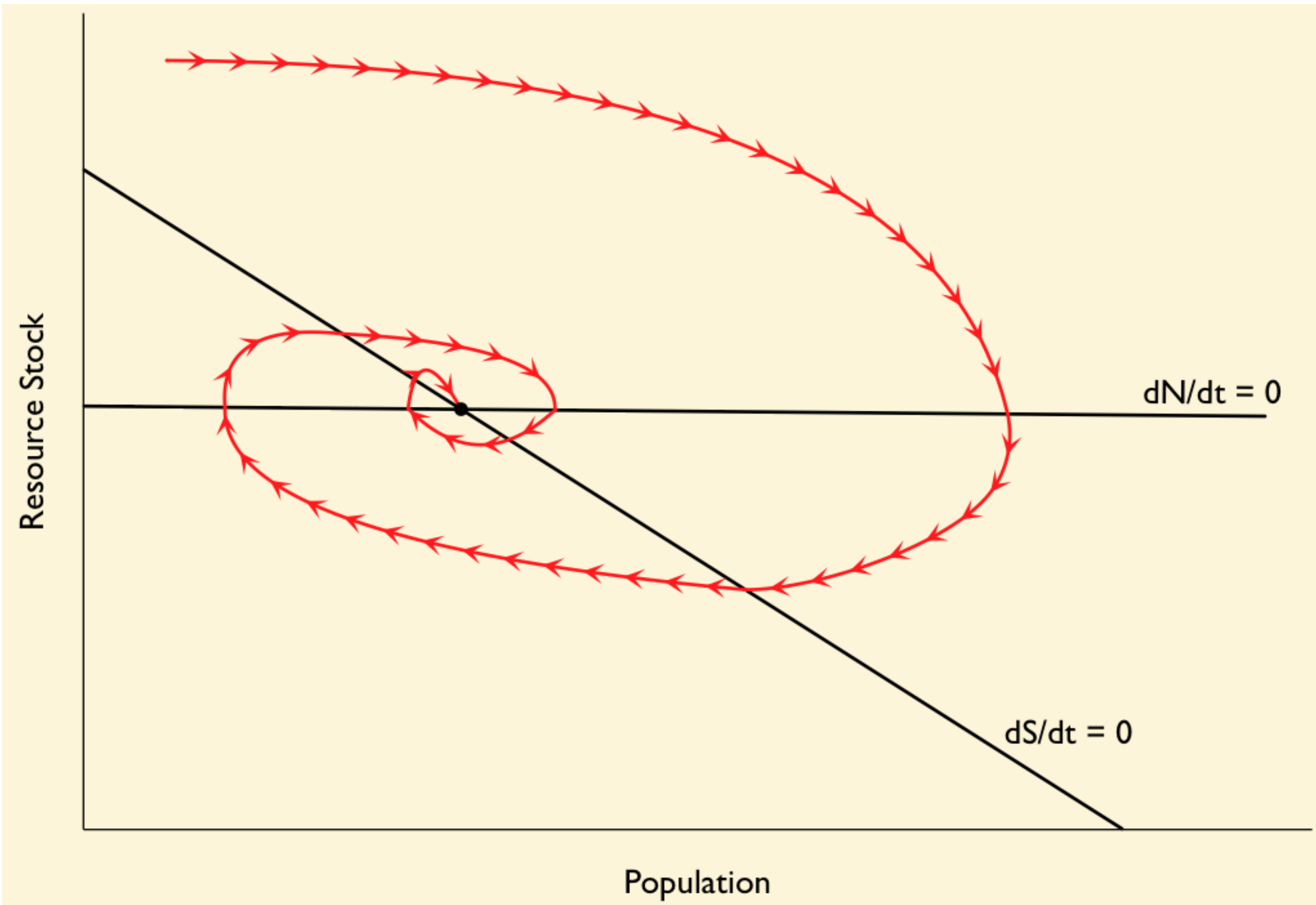


Figure 7: Illustrative Dynamics Starting from Low Population and High Resources



Numerical Example: A Lower Harvesting Rate

One of the ways to explore the properties of models like this one is to use software like Excel or more "'programming-oriented" econometric software like RATS to simulate discrete-time versions of the model. Figures 8 and 9 show time series for resources and population generated from a RATS programme that simulates a discrete-time adaptation of the model. The programme is shown at the back of the notes in an appendix. It implements a version of the model with parameter values $d = 0.075, r = 0.075, \gamma = 2, \theta = 0.1$ and an initial population of $N_1 = 0.0001$. The parameter values are set so that the equilibrium level of resources is $S^* = \frac{d}{\theta\gamma} = \frac{0.075}{0.2} = 0.375$ while the equilibrium level of population is $N^* = \frac{r(1-S^*)}{\gamma} = \frac{(0.075)(1-0.375)}{2} = 0.023475$.

Figure 8 shows that, for these parameter values, the stock of resources falls to about half of its long-run equilibrium value, then rises and overshoots this value and then oscillates before settling down at this equilibrium level. Figure 9 shows the associated movements in population. We see population surge to levels that are over twice the long-run sustainable level, then dramatically drop to undershoot this level before eventually settling down.

Because we have chosen a base case in which $S^* < 0.5$, this is a case where there would be higher resources and population in the long-run if we had a somewhat lower rate of harvesting. Indeed, you can pick a rate of harvesting that avoids a collapse scenario altogether. Figures 10 and 11 compare the base case we have just looked at with a case in which the rate of harvesting was 40 percent lower, so $\gamma = 1.2$. In this case, the resource stock only slightly undershoots its long-run level and the population only slightly-overshoots. The economy ends up a similar level of population but arrives there in a less dramatic fashion.

Figure 8: Resources in a Simulated Easter Island Economy

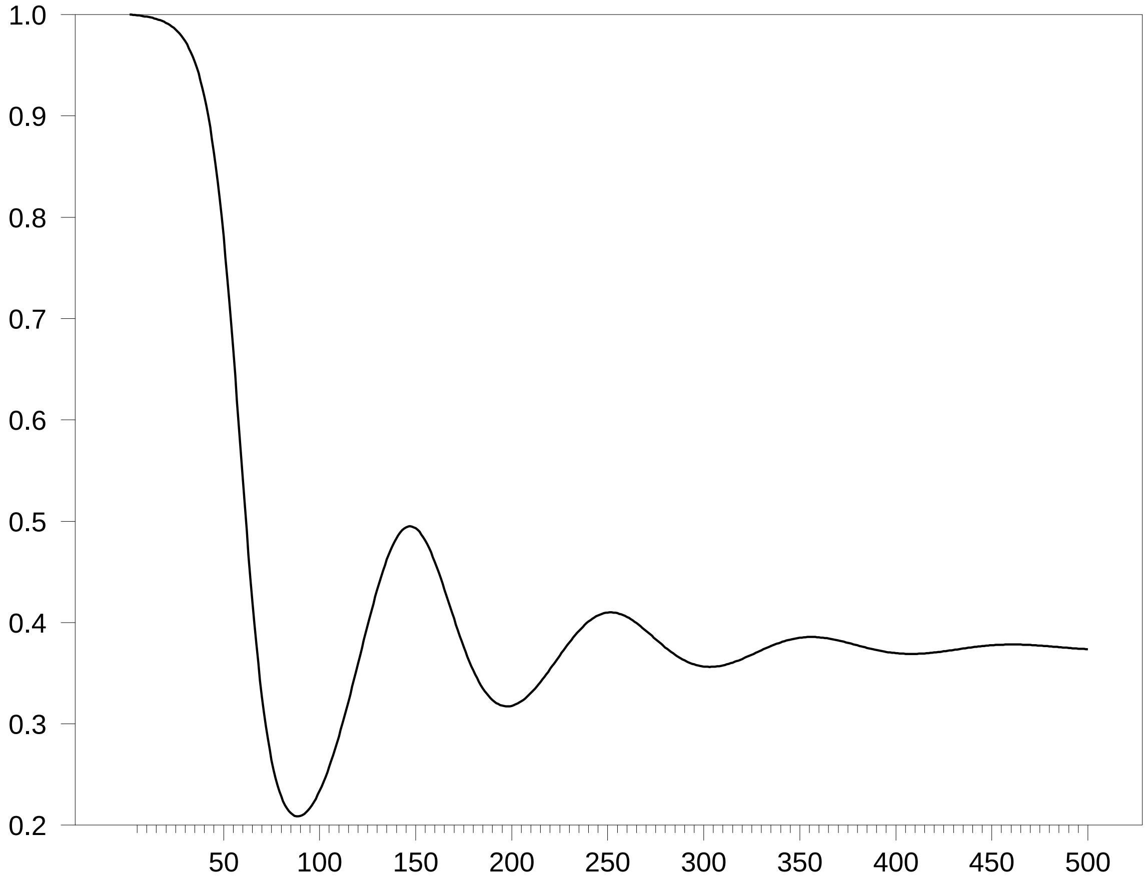


Figure 9: Population in a Simulated Easter Island Economy

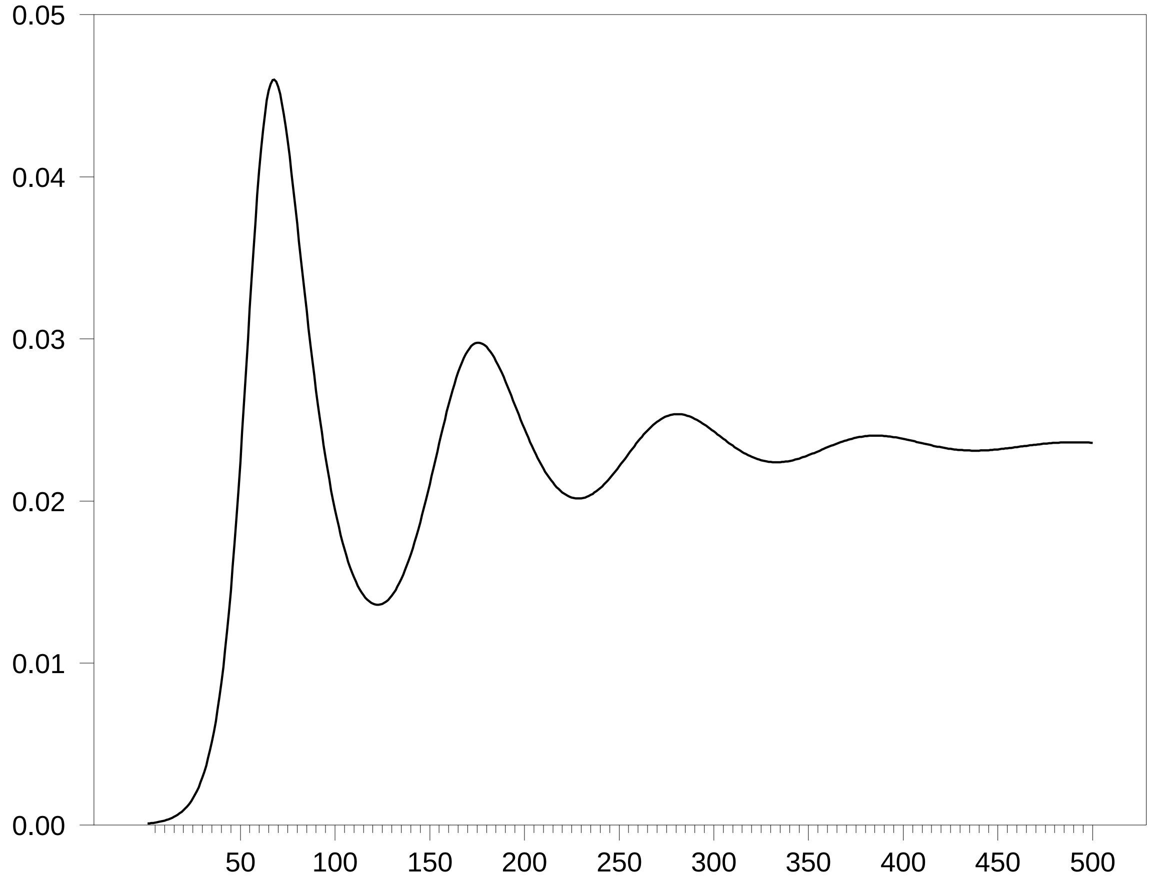


Figure 10: Resource Stock with Less Harvesting

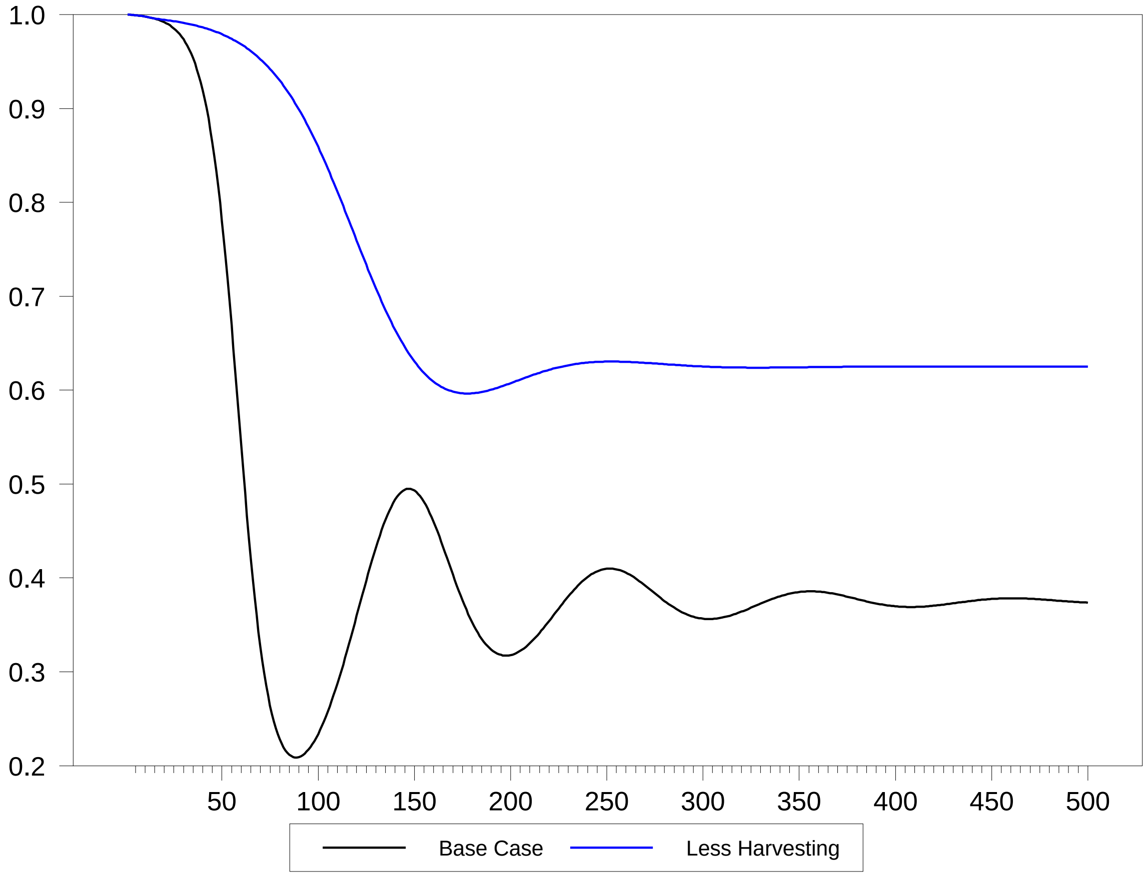
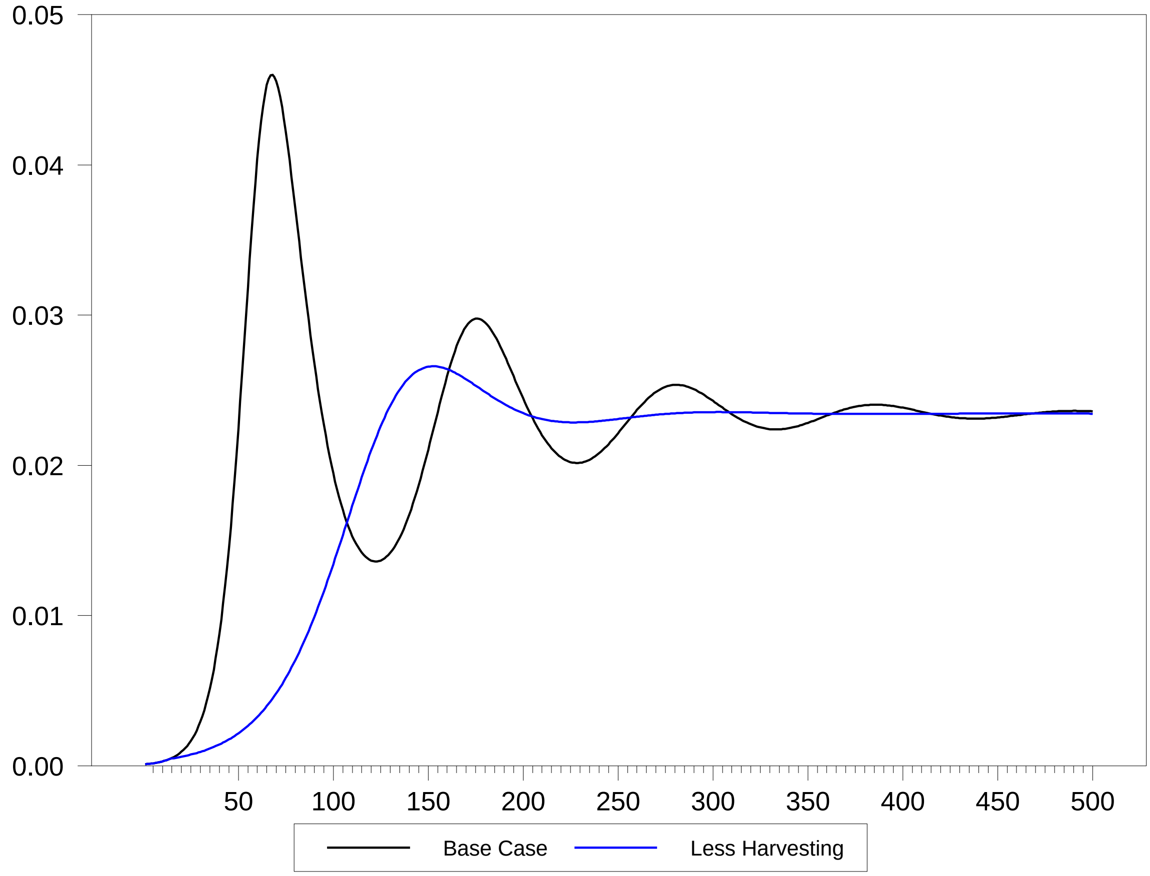


Figure 11: Population with Less Harvesting



Why Doesn't Someone Shout Stop?

The pattern demonstrated in the model—in which the economy far overshoots its long-run level before collapsing to an equilibrium with lower population and depleted resources—may seem to fit what happened at Easter Island. But it raises plenty of questions: Why did the residents of the island allow this to happen? Why didn't they establish better governance rules to prevent the deforestation that proved so devestating? And could this model possibly be a warning that today's global economy could represent an overshooting with a significant collapse awaiting us all?

In his book, *Collapse*, Jared Diamond discusses Easter Island and a number of other cases in which societies saw dramatic collapses, many triggered by long-term environmental damage. Diamond points to a number of potential explanations for why societies can let environmental damage occur up to the point where they trigger disasters.

- *The Tragedy of the Commons*: It may simply never be in anyone's interests at any point in time to prevent environmental degradation. A fisherman may acknowledge that excess fishing will eventually put him out of business but there may be little he can do to prevent others fishing and today he needs to earn an income. Some societies can put in place centralised political institutions to prevent environmental disasters and some cannot. At present, the society called The Earth is not known for its efficient centralised political decision making.

- *Failure to Anticipate*: Societies may not realise exactly how much damage they are doing to their environment or what its long-term consequences will be. Up until the point at which Easter Island's environment failed to support a growing population, there was probably a limited realisation among the population of the damage being done. Once the population began to shrink and the tribes turned against each other (there's some evidence of cannibalism during this period) the likelihood of a common negotiated solution to cut down less trees to preserve the environment was unlikely. Similarly today, the future effects of climate change are unpredictable and the costs (and even potential benefits) may be unevenly distributed.

- *Failure to Perceive, Until Too Late*: Diamond notes that environmental change often occurs at such a slow pace that people fail to notice it and plan to deal with it. The Easter Islanders of 1500 probably couldn't remember (and certainly had no written record of) their island being covered in palm trees. The islander who eventually cut down the last tree probably had little idea that these trees had once been the mainstay of the local economy. Similarly, global climate change has occurred at such a slow pace that, despite the mountain of scientific evidence that it is real, many simply choose to deny it.

Things to Understand from these Notes

Here's a brief summary of the things that you need to understand from these notes.

1. A rough idea of the facts about the history of Easter Island prior to the arrival of Europeans.
2. The structure of the Brander-Taylor model.
3. How to derive the dynamics of population.
4. How to derive the dynamics of resources.
5. The long-run impacts of more intensive harvesting.
6. How to draw the phase diagram.
7. The likely dynamics starting from low population and high resources.
8. Why environmental disasters are not prevented.

Appendix: Programme For Easter Island Simulation

Figures 8 and 9 were produced using the programme below. The programme is written for the econometric package RATS but a programme of this sort could be written for lots of different types of software including Excel.

```
allocate 10000

set d = 0.075
set r = 0.075
set gamma = 2
set theta = 0.1

set s = 1
set n = 0.0001
set h = 0

do k = 2,10000
comp s(k) = s(k-1) +  r(k)*s(k-1)*(1-s(k-1) ) - h(k-1)
comp h(k) = gamma(k)*s(k-1)*n(k-1)
comp n(k) = (1-d(k))*n(k-1) +  theta(k)*h(k)
end do k

graph 1
# s 1 500

graph 1
# n  1 500
```