Analysing the IS-MP-PC Model

In the previous set of notes, we introduced the IS-MP-PC model. We will move on now to examining its properties.

Inflation Expectations and the Inflation Target

Let's start by repeating a graph from the last time. Figure 1 shows the simplest possible example of the model. This is the case where both the temporary shocks, ϵ_t^π and ϵ_t^y equal zero and the public's expectation of inflation equals the central bank's inflation target. Specifically, the graph shows a case where the public's expectation of inflation $\pi_t^e = \pi_1$ and the central bank's inflation target is $\pi^* = \pi_1$. With no temporary shocks, the value of output consistent with $\pi_t = \pi_1$ for the IS-MP curve is y_t^*. Similarly, the value of output consistent with $\pi_t = \pi_1$ for the PC curve is also y_t^*. So the model generates an outcome where $\pi_t = \pi_1$ and $y_t = y_t^*$.

Now consider a case in which the public's inflation expectations shift to being higher than the central bank's target rate. Figure 2 illustrates this case. It shows the PC curve shifting upwards to the red line. This position of this red line is determined by the new higher level of expected inflation. Specifically, the public's inflation expectations are now determined by $\pi_t^e = \bar{\pi}$. Note that $\bar{\pi}$ is the higher level of inflation noted on the y-axis and that this level is consistent with $y_t = y_t^*$ in the new Phillips curve described by the red line.

The outcomes for inflation and output of the increase in inflation expectations are described by the intersection of the new red PC line and the old unchanged IS-MP curve. The actual outcome for inflation (denoted as π_2 on the graph) ends up being higher than the central bank's inflation target but lower than the public's inflationary expectations. Output ends up being lower than its natural rate (consistent with a slump or perhaps a full-blown recession) because the higher level of inflation leads the central bank to raise real interest rates which reduces output.

When studying this graph, it's important to understand the various markings on the curves and the axes. If I ask you on the final exam to illustrate the impact of an increase in inflation expectations using this model, my preference would be to see the various assumptions about inflation targets and inflation expectations explicitly marked out, rather than just a graph that shows one curve has shifted upwards.

Can We Learn More?

Figure 2 is a good example of how we can use graphs to illustrate a model's properties. It gets the basic story across as to what happens when inflation expectations rise above target when the central bank is pursuing a monetary policy rule that increases real rates in response to higher inflation.

Still, one could look to dig a bit deeper. The inflation outcome as drawn in Figure 2 is slightly more than halfway towards the public's inflation expectations relative to the central bank's inflation target. But what actually determines this outcome? In other words, what determines how far from away target inflation will move when the public's inflation expectations change? How much does it depend on the monetary policy rule? How much does it depend on other aspects of the model, like the impact of real interest rates on output and the impact of output on inflation? It would be tricky to get these answers from a graph. However, using the equations underlying the model, we can get a full solution that fully answers all these questions.

Figure 1: The IS-MP-PC Model When Expected Inflation Equals the Inflation Target

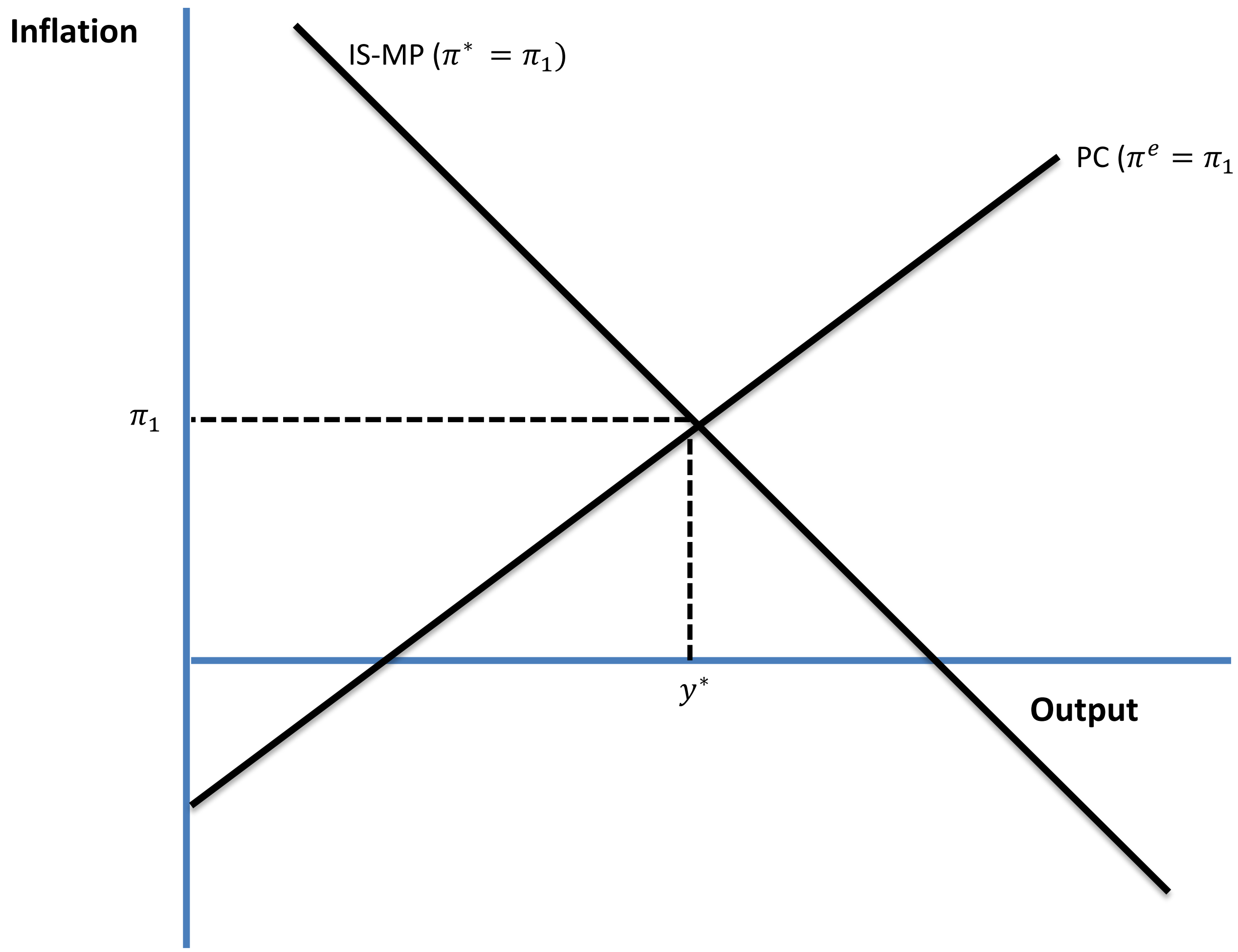
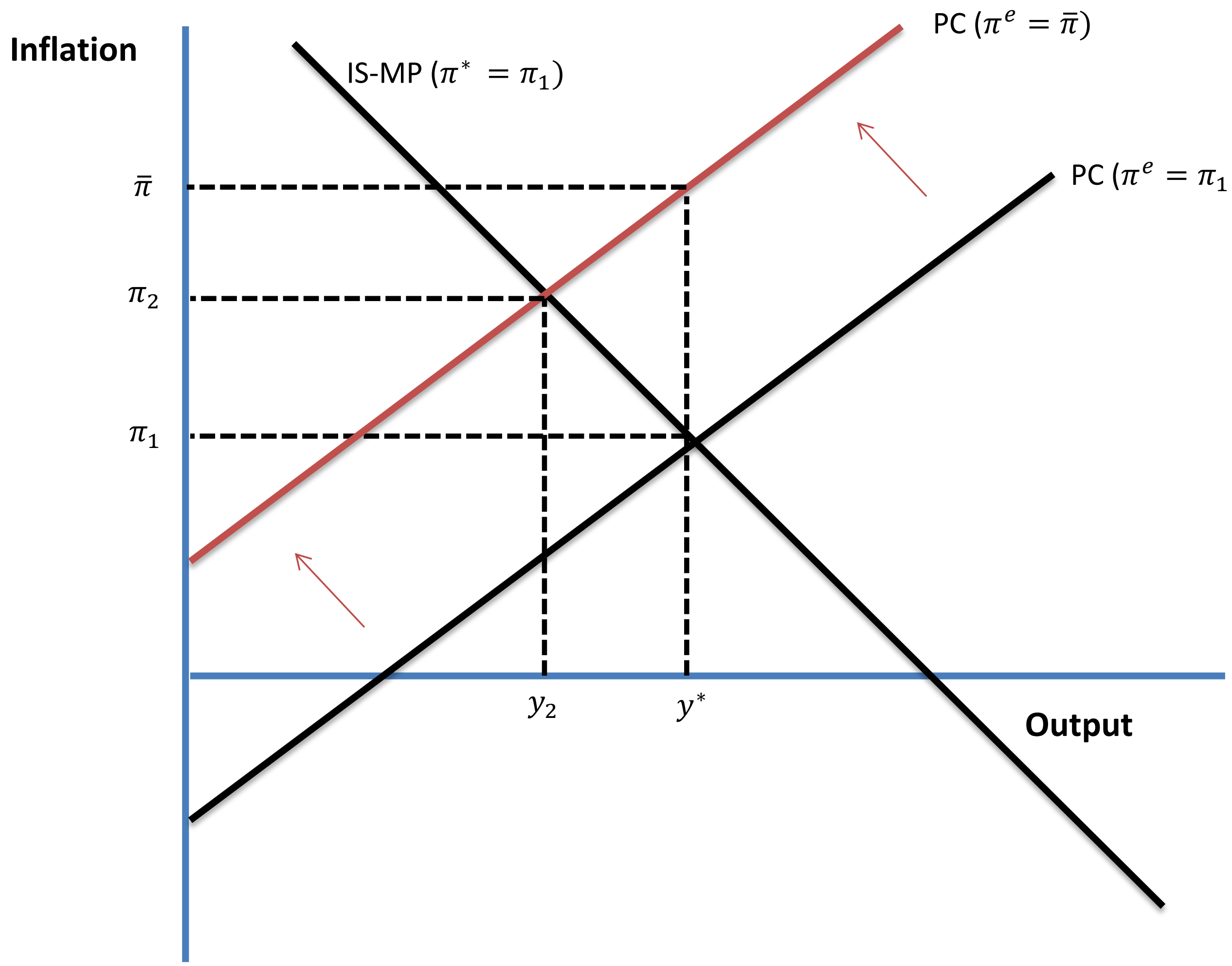


Figure 2: The IS-MP-PC Model When Expected Inflation Rises Above the Inflation Target



The IS-MP-PC Model Solution for Inflation

Let's repeat the equations describing our two curves as presented in our last set of notes. The PC curve is

$$\pi_t = \pi_t^e + \gamma \left(y_t - y_t^*\right) + \epsilon_t^\pi \tag{1}$$

And the IS-MP curve is

$$y_t = y_t^* - \alpha \left(\beta_\pi - 1\right) \left(\pi_t - \pi^*\right) + \epsilon_t^y \tag{2}$$

Taking all the other elements of the model as given, we can view this as two linear equations in the two variables π_t and y_t. These equations can be solved to give solutions that describe how these two variables depend on all the other elements of the model.

This can be done as follows. First, we will derive a complete expression for the behaviour of inflation and then derive an expression for output. We derive the expression for inflation by starting with the Phillips curve and replacing the output gap term $y_t - y_t^*$ with the variables that the IS-MP curve tells us determines this gap. This gives us the following equation:

$$\pi_t = \pi_t^e + \gamma \left[-\alpha \left(\beta_\pi - 1\right) \left(\pi_t - \pi^*\right) + \epsilon_t^y\right] + \epsilon_t^\pi \tag{3}$$

Adding the term $\alpha\gamma \left(\beta_\pi - 1\right) \pi_t$ to both sides we get

$$\left[1 + \alpha\gamma \left(\beta_\pi - 1\right)\right] \pi_t = \pi_t^e + \alpha\gamma \left(\beta_\pi - 1\right) \pi^* + \gamma\epsilon_t^y + \epsilon_t^\pi \tag{4}$$

Now dividing each side by $1 + \alpha\gamma \left(\beta_\pi - 1\right)$, we get that inflation is determined by

$$\pi_t = \left(\frac{1}{1 + \alpha\gamma \left(\beta_\pi - 1\right)}\right) \pi_t^e + \left(\frac{\alpha\gamma \left(\beta_\pi - 1\right)}{1 + \alpha\gamma \left(\beta_\pi - 1\right)}\right) \pi^* + \frac{\gamma\epsilon_t^y + \epsilon_t^\pi}{1 + \alpha\gamma \left(\beta_\pi - 1\right)} \tag{5}$$

There are a lot of symbols in this equation, which make it a bit hard to read. One way to simplify it is to take the term multiplying inflation expectations and denote it by a single symbol. In this case, we will denote it by the Greek letter θ (theta, pronounced "thay-ta"). So, we define this as

$$\theta = \frac{1}{1 + \alpha\gamma\left(\beta_\pi - 1\right)} \tag{6}$$

Having done this, we can re-write the equation for inflation as

$$\pi_t = \theta\pi_t^e + (1 - \theta)\,\pi^* + \theta\left(\gamma\epsilon_t^y + \epsilon_t^\pi\right) \tag{7}$$

This equation shows that, apart from the shocks to output and inflation (the $\theta\left(\gamma\epsilon_t^y + \epsilon_t^\pi\right)$ terms) inflation is a weighted average of the public's inflation expectations and the central bank's inflation target i.e. it must lie between these two values as long as $0 < \theta < 1$ (which it should be). What determines whether inflation depends more on the public's expectations or the central bank's target? In other words, what determines the value of θ? Three different factors determine this value.

1. γ: This is the parameter that determines how inflation changes when output changes. The central bank can only influence inflation by influencing output. If the effect of output on inflation gets bigger, then the central bank's inflation target will have more influence on the outcome for inflation.

2. α: This is the parameter that determines how output changes when real interest rates change. If the effect of interest rates on output gets bigger, then the central bank's inflation target will have more influence on the outcome for inflation.

3. β_π: Let's continue to assume $\beta_\pi > 1$ (we'll return to this in the next set of notes). Then as β_π gets bigger, the central bank is reacting more to inflation being above its target level. So this parameter getting bigger means less weight on inflation expectations in

determining the outcome for inflation and more weight on the central bank's inflation target.

While the calculations here may seem difficult, they illustrate that a formal mathematical solution can sometimes give us a much more complete insight into the properties of a model than graphs. While graphs are often useful at illustrating a particular feature of a model, they also often fall short of explaining the full properties of a model.

The IS-MP-PC Model Solution for Output

Next we provide an expression for output. Looking at the IS-MP curve, we see that the output gap depends on how far inflation is from the central bank's target as well as the "supply shock" term ϵ_t^π. We can use the equation determining inflation, equation (7), to get an expression for the gap between inflation and the target level. Subtract π^* from either side of equation (7) and you get

$$\pi_t - \pi^* = \theta \left(\pi_t^e - \pi^* + \epsilon_t^\pi + \gamma \epsilon_t^y\right) \tag{8}$$

We can now replace the $\pi_t - \pi^*$ on the right-hand-side of the IS-MP curve, equation (2), with the right-hand-side of the equation above. This gives

$$y_t = y_t^* - \theta\alpha \left(\beta_\pi - 1\right) \left(\pi_t^e - \pi^* + \epsilon_t^\pi + \gamma \epsilon_t^y\right) + \epsilon_t^y \tag{9}$$

which can be simplified to

$$y_t = y_t^* - \theta\alpha \left(\beta_\pi - 1\right) \left(\pi_t^e - \pi^* + \epsilon_t^\pi\right) + \left(1 - \theta\alpha\gamma \left(\beta_\pi - 1\right)\right) \epsilon_t^y \tag{10}$$

This equation tells us that whether output is above or below target depends upon the gap between expected inflation and the inflation target as well as on the two temporary shocks

ϵ_t^y and ϵ_t^π. Provided we have the usual condition that $\beta_\pi > 1$, the combined coefficient $-\theta\alpha\left(\beta_\pi - 1\right)$ is negative. This means that increases in the public's inflation expectations relative to the inflation target end up having a negative effect on output. Inflationary supply shocks (positive values for ϵ_t^π) also have a negative effect on output while positive aggregate demand shocks ($\epsilon_t^y > 0$) have a positive effect on output.

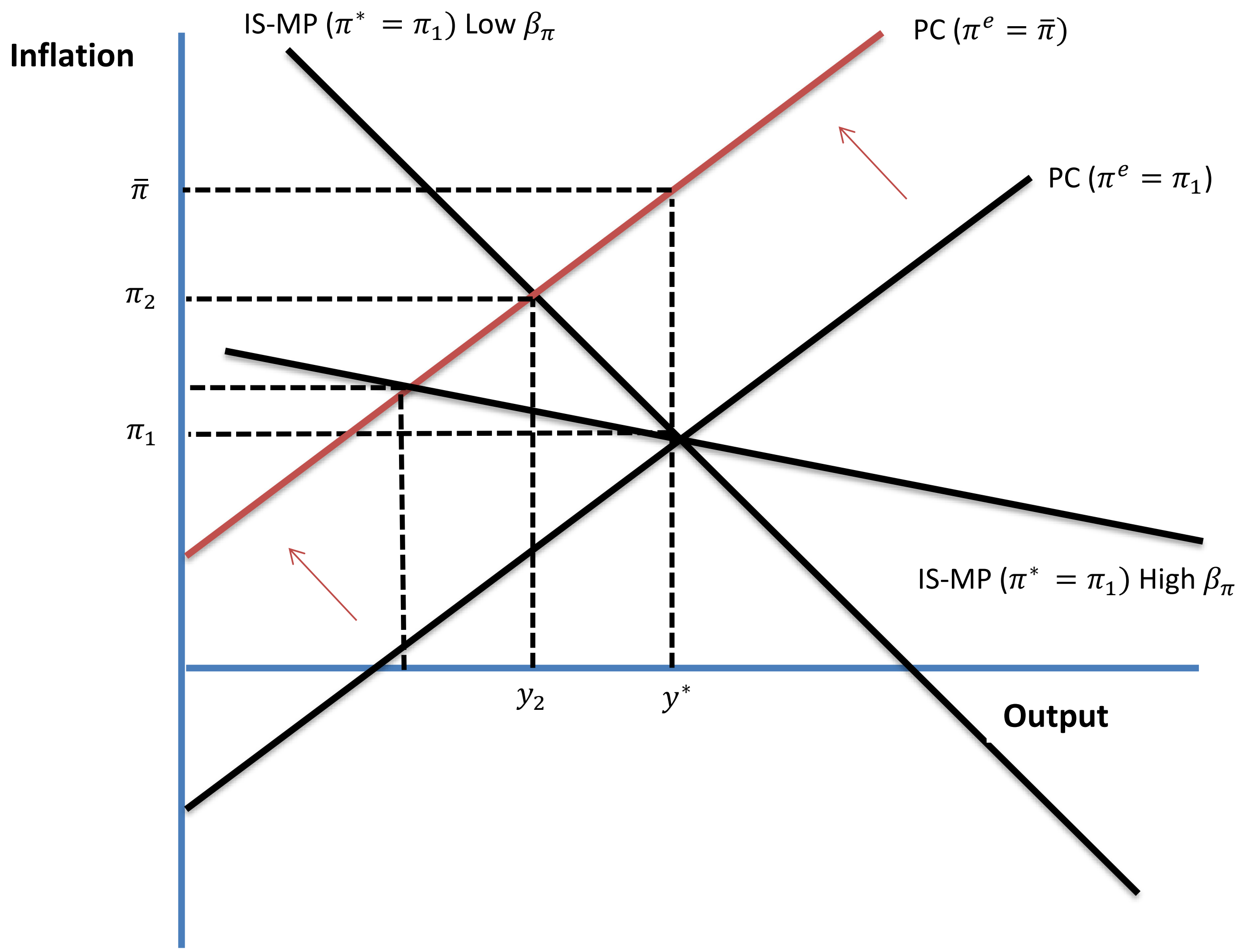
How far does output fall short of its natural rate when inflation expectations rise above the central bank's target? The coefficient determining this is $-\theta\alpha\left(\beta_\pi - 1\right)$. This can be re-expressed as

$$-\theta\alpha\left(\beta_\pi - 1\right) = \frac{\alpha\left(\beta_\pi - 1\right)}{1 + \alpha\gamma\left(\beta_\pi - 1\right)} \tag{11}$$

Calculating partial derivatives, you find that the size of the short fall in output depends positively on α and β_π. In other words, the larger the impact of interest rates on output and the larger the central bank's interest rate response to inflation, the larger the shortfall in output will be when inflation expectations rise above the central bank's target. In contrast, the output shortfall depends negatively on γ, the parameter determining the effect of output on inflation: As γ gets bigger, the central bank requires a smaller shortfall in output to implement its policy of getting inflation back to target.

The calculations here tell us that the more aggressive a central bank is in its response to inflation—the higher the value of β_π—then the smaller the rise in inflation will be and the larger the drop in output will be. We can illustrate this graphically by comparing Figure 2 with what would have happened if the IS-MP curve had been flatter: A higher value of β_π means a flatter IS-MP curve, meaning each unit increase in inflation is associated with a more aggressive policy response from the central bank and thus a larger fall in output. Figure 3 overlays a second, flatter, IS-MP curve on top of Figure 2. As with the original IS-MP curve, this curve generated by a higher β_π also intersects with the original curve so that $\pi_t = \pi^*$ and $y_t = y_t^*$ but after the Phillips curve shifts up, it generates a smaller increase in inflation and a larger decrease in output.

Figure 3: A Rise in Expected Inflation For Two Values Of $\beta\pi$



How Do Inflation Expectations Change?

Let's go back to Figure 2 now. We have seen that after the public's inflation expectations rise, the result is a fall in output below its natural rate and in increase inflation, though this increase is smaller than had been expected by the public. What happens next? How does the public's expectation of inflation change at this point?

Friedman's 1968 paper *The Role of Monetary Policy* suggested that people gradually adapt their expectations based on past outcomes for inflation. Consider now a simple model of this idea of "adaptive expectations" by assuming that, each period, the expected level of inflation is simply equal to the level that prevailed last period. Formally, this can be written as

$$\pi_t^e = \pi_{t-1} \tag{12}$$

Under this formulation of expectations, the Phillips curve becomes

$$\pi_t = \pi_{t-1} + \gamma \left(y_t - y_t^* \right) + \epsilon_t^\pi \tag{13}$$

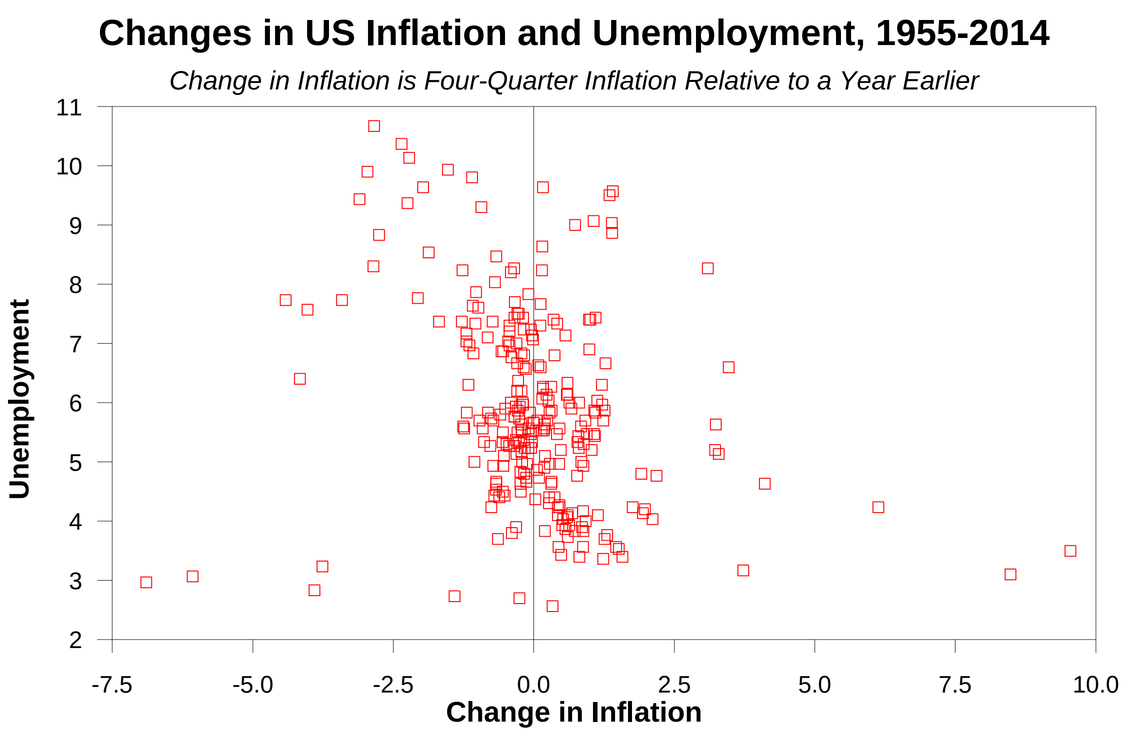
Note that if we subtract π_{t-1} from both sides of this equation, it becomes

$$\pi_t - \pi_{t-1} = \gamma \left(y_t - y_t^* \right) + \epsilon_t^\pi \tag{14}$$

In other words, there should be a positive relationship between the *change* in inflation and the output gap. There are various methods for measuring output gaps but one quick and easy method is to use the unemployment rate as an indication of what the output might be. If unemployment is high, then output is likely to be below its natural rate so the output gap is negative. In contrast, a low unemployment rate is an indicator that the output gap is likely to be positive. So if the adaptive expectations formulation of the Phillips curve was correct, then we would expect to see a negative relationship in the data between the change in inflation and the unemployment rate.

Figure 4 uses the same US quarterly data that we used for Figure 4 in the last set of notes. That figure showed that there was very little relationship between the level of the unemployment rate and the level of inflation. Figure 1 shows a scatter plot of datapoints on the change in inflation (measured as the four quarter percentage change in the price level minus the percentage change in the price level over the preceding four quarters) and the unemployment rate. In contrast to the basic Phillips curve, this adaptive-expectations-style Phillips curve shows a clear and strong negative relationship between the change in inflation and the unemployment rate.

Figure 4: Evidence for Adaptive Inflation Expectations



These results suggest that the adaptive expectations approach appears to provide a reasonable model of how people formulate inflation expectations. That said, people are unlikely to simply use mechanical formulae to arrive at their expectations and one can imagine conditions in which people's inflation expectations could radically depart from what had happened in the past e.g. the appointment of a new central bank governor with a different approach to inflation, the adoption of a new currency or other major events. Let's examine for now, however, how the IS-MP-PC model behaves when people have adaptive inflationary expectations.

Adjustment of Inflation Expectations

After inflation expectations moved up to $\bar{\pi}$, the outcome was that inflation moved from π_1 (which is the central bank's inflation target) to π_2. If people follow adaptive expectations then the next period, they will set $\pi^e = \pi_2$. Figure 5 shows what happens after this. The PC curve moves back downwards and inflation moves down to a lower level, denoted on this graph by π_3. Figure 6 indicates how the process plays out. If the public has adaptive expectations, then inflation and output gradually converge back to the point where output is at its natural rate and inflation equals the central bank's target rate.

Here we have illustrated the implications of an increase in inflation expectations away from the central bank's inflation target. But if the public has adaptive expectations, how could inflation expectations just jump upwards? Rather than a random unexplained increase in inflation expectations, the more likely explanation for the Phillips curve shifting upwards because is temporary supply shocks, i.e. ϵ_t^π is positive for a number of periods. Under adaptive expectations, the public becomes used to higher inflation and so the Phillips curve will remain above its long-run position even after the temporary supply shock has been reversed.

Figure 5: Inflation Expectations Adjusting Back Downwards

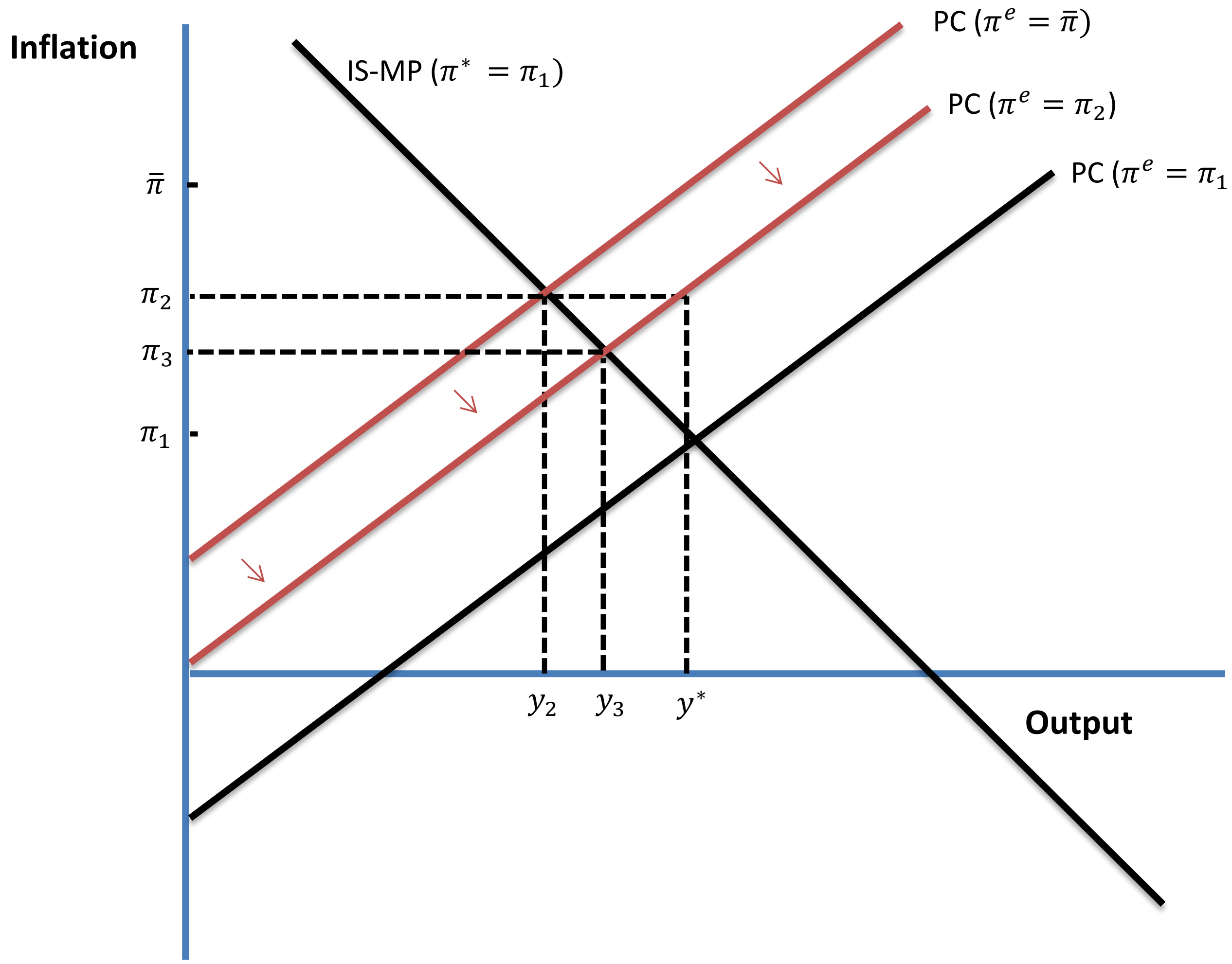
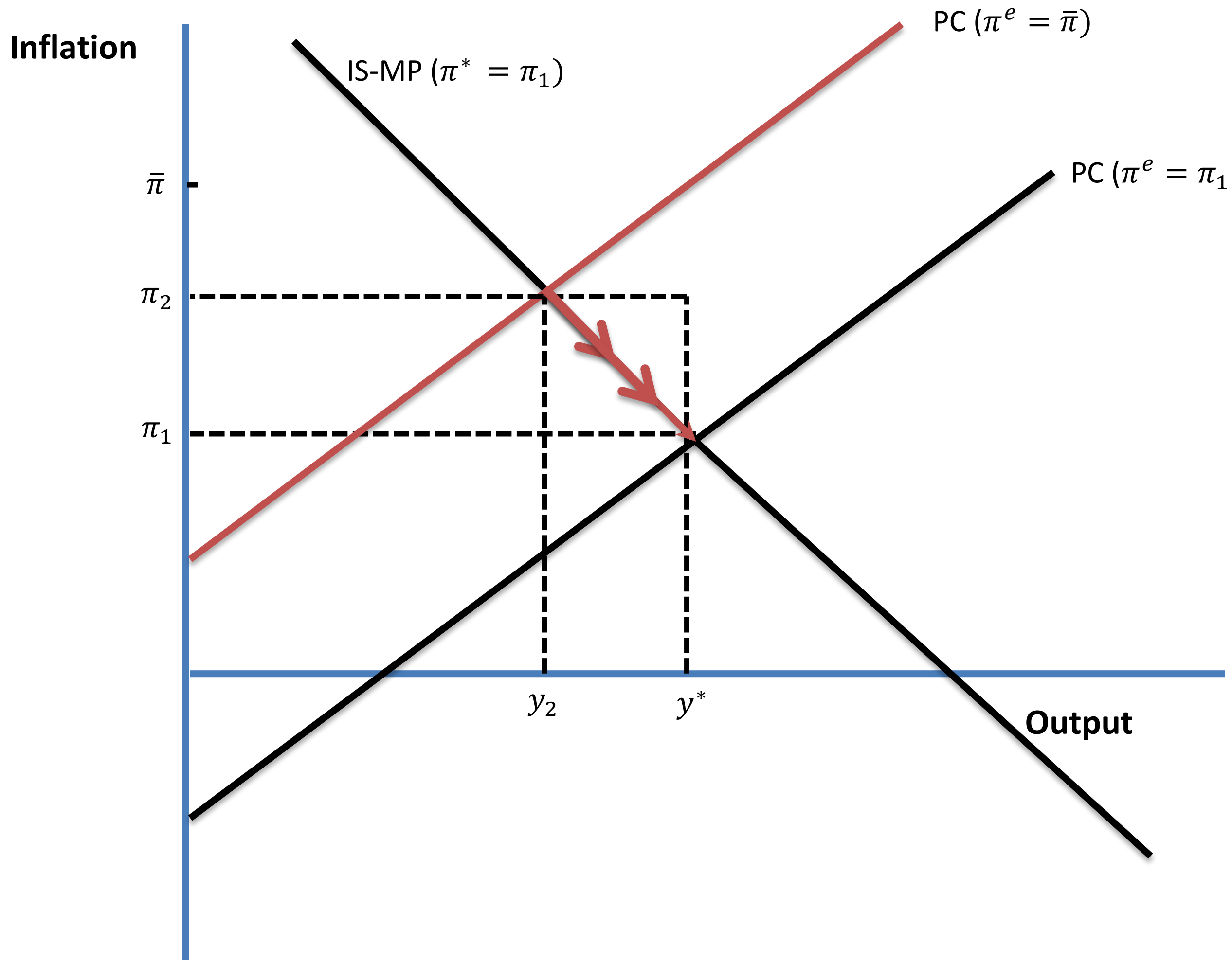


Figure 6: Inflation and Output Adjust Back to Starting Position



Inflation and Output Dynamics for Soft and Tough Central Banks

Do we want a "soft" central bank that limits the increase in real interest rates when inflation rises to protect the economy and which isn't too concerned about getting inflation back to target quickly? Or do we want a "tough" central bank that raises interest rates aggressively and is very concerned about getting inflation back to target?

The model doesn't give a clear answer between these two options. Both have positive and negative aspects. If the public's inflation expectations behave in an adaptive fashion, then central banks have a choice between different types of adjustments. We showed above in Figure 3 that a central bank that acts more aggressively to inflation—that has a greater β_π—produces a smaller increase in inflation but a larger decline in output. However, with adaptive expectations, this larger reduction in output is short-lasting than when β_π is smaller. This is because the initial increase in inflation is smaller, so the central bank is able to return real interest rates to their natural rate faster.

We can illustrate the differences between the two scenarios by simulating the model on a computer. Figures 7 and 8 show the results of a computer simulation of the model in which it is assumed that $\pi^* = 2$, that y_t^* is constant at 100 and the other parameters are $\alpha = 1$ and $\gamma = 1$. The model is simulated with two different values for β_π. One version has $\beta_\pi = 1.5$ (this is the "soft" central bank) and other has $\beta_\pi = 3$ (this is the "tough" central bank). Figure 7 shows the rise in inflation is smaller and disappears quicker when there is a tough central bank. Figure 8 shows that the tough central bank engineers a much larger recession but this ends much quicker. The total average value of output over the whole sample is the same for the two scenarios. This isn't an accident but rather is a feature of the model: A certain amount of cumulative output below its natural rate is required to lower the inflation rate back to the central bank's target.

This suggests central banks face a choice when dealing with high inflation: They can go for the "cold turkey" option and have a sharp but short recession or they can take a softer approach which ends up taking more time to get output and inflation back to target.

Figure 7: Inflation Dynamics for High and Low Values of β_π

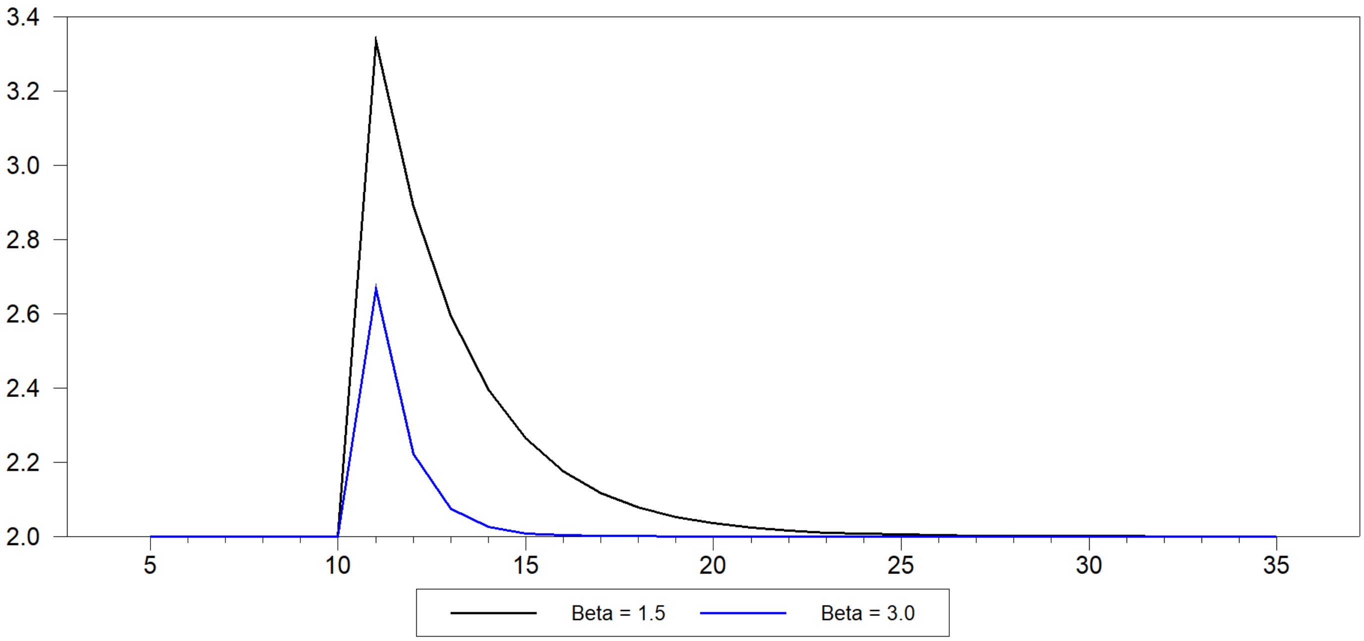
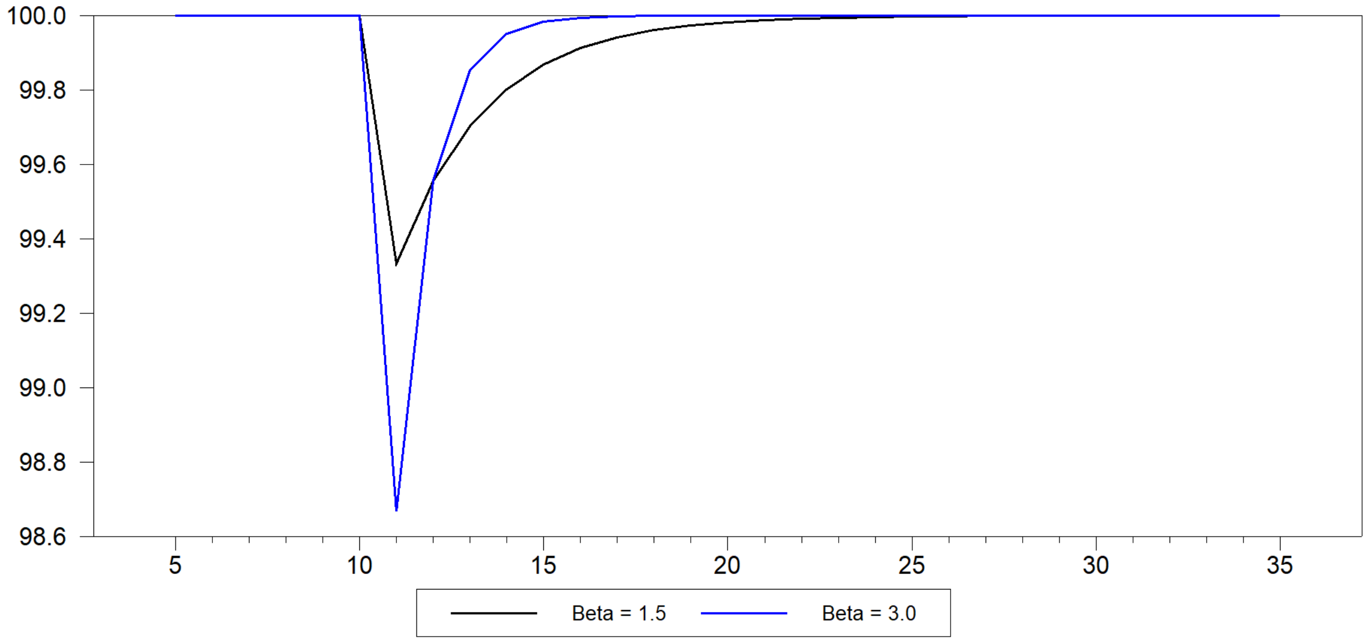


Figure 8: Output Dynamics for High and Low Values of β_π



A Temporary Aggregate Demand Shock

Having looked at what happens under adaptive expectations when the Phillips curve shifts, let's consider what happens when we have a temporary shock to aggregate demand, so ϵ_t^y takes a different value from zero, which means a shift in the IS-MP curve. In Figures 7 to 10, we illustrate a case where there is a shift towards a positive value of ϵ_t^y for a couple of periods but then it shifts back to zero.

Figure 9 shows the immediate impact of a positive aggregate demand shock. Output and inflation both go up with inflation reaching the point denoted as π_2 in the figure. If the public has adaptive expectations, then this results in an increase in inflation expectations the following period. Figure 10 shows what happens when the aggregate demand shock persists but inflation expectations move up to match the previous period's inflation rate. The inflation rate now rises again to π_3. Figure 11 shows how this triggers a further increase in inflation in the next period as inflation expectations move up from π_2 to π_3.

Figure 12 shows what happens if the aggregate demand shock then reverses itself in the next period. The IS-MP curve shifts back to its original position but the Phillips curve remains elevated. The result is a nasty combination of high inflation and output below its natural rate. Figure 12 contains arrows showing the full set of movements generated by this aggregate demand shock:

- An increase in output and inflation as the shock hits.
- A further increase in inflation as inflation expectations adjust upwards, accompanied by a decline in output.
- A decline in output and inflation as the shock disappears.

- A further decline in inflation accompanied by an increase in output as inflationary expectations gradually return to the central bank's target.

This chart shows that when the public has adaptive inflation expectations, temporary positive aggregate demand shocks lead to counter-clockwise loops on graphs that have output on the x-axis and inflation on the y-axis.

It turns out that much of the data on inflation and output correspond to these kinds of movements. Figure 13 is borrowed from notes on Stanford economist Charles I. Jones's website. They show the data from US on inflation and an estimated output gap from 1960 to 1983. The figure shows a number of periods of clear counter-cyclical movements. Figure 14 shows the same data from 1983-2009. This figure also shows some evidence counter-cylical loops, thought the movements are smaller than the for the pre-1983 period.

Figure 9: A Temporary Aggregate Demand Shock ($\epsilon_t^y > 0$)

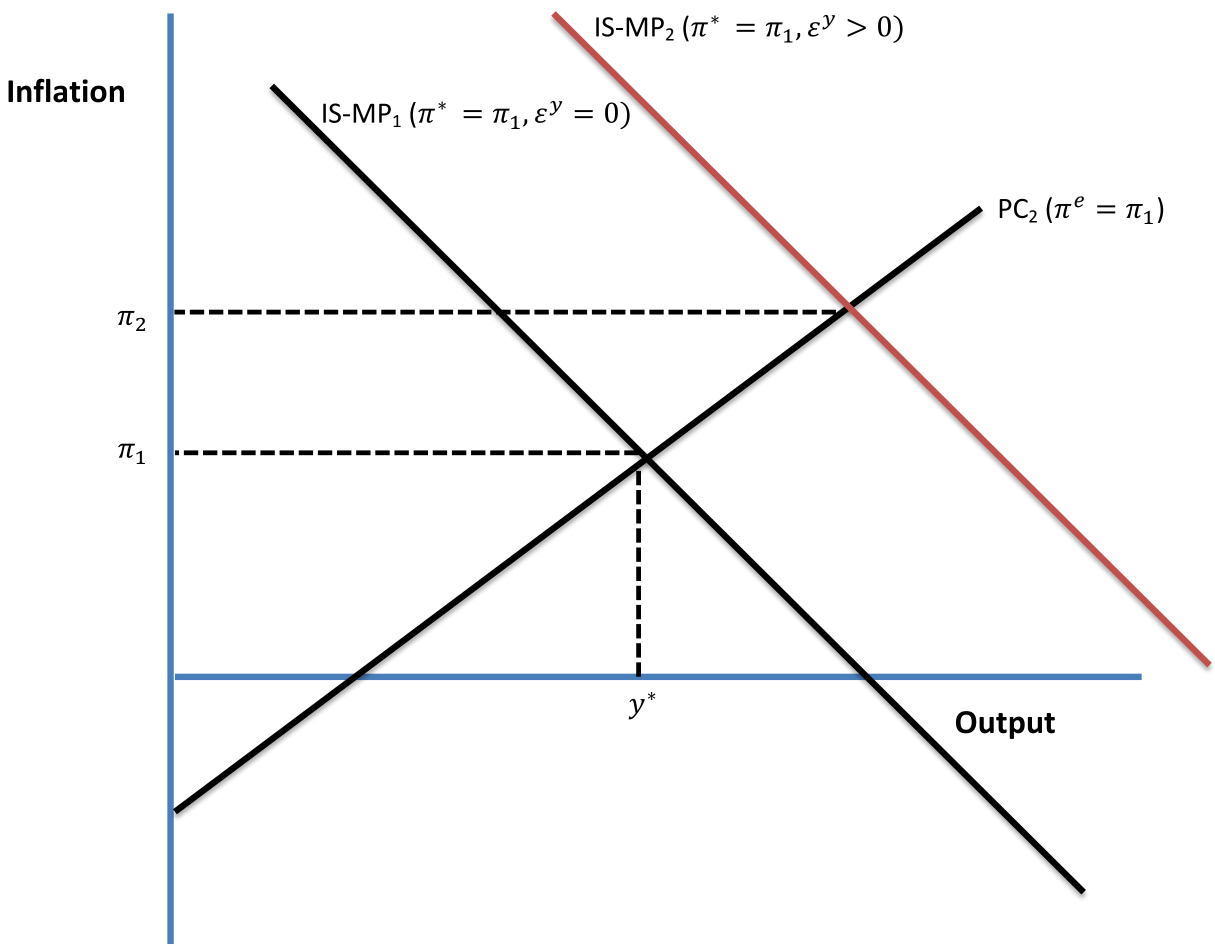


Figure 10: Inflation Expectations Adjust Upwards to π_2

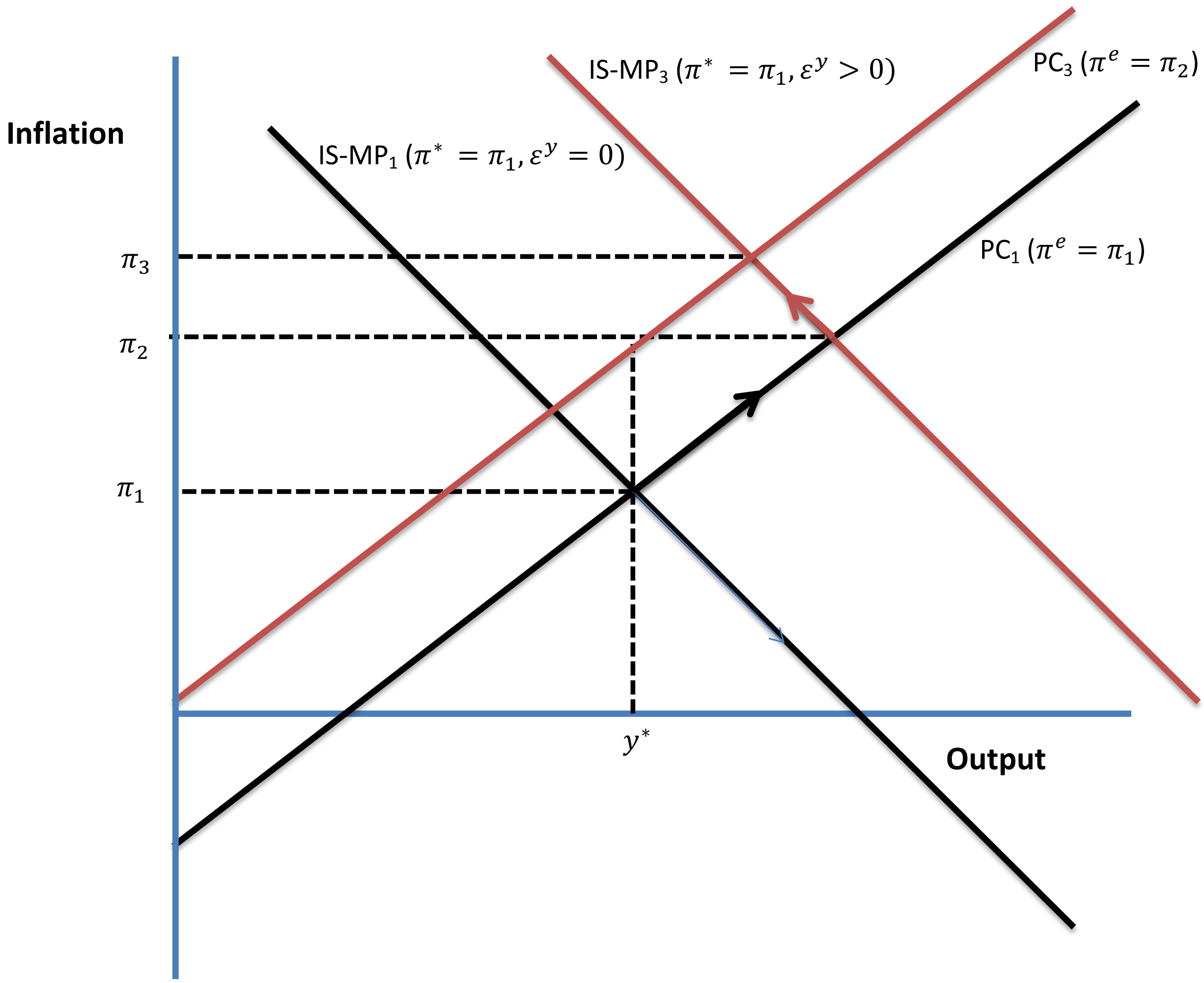


Figure 11: Inflation Expectations Adjust Upwards Further to π_3

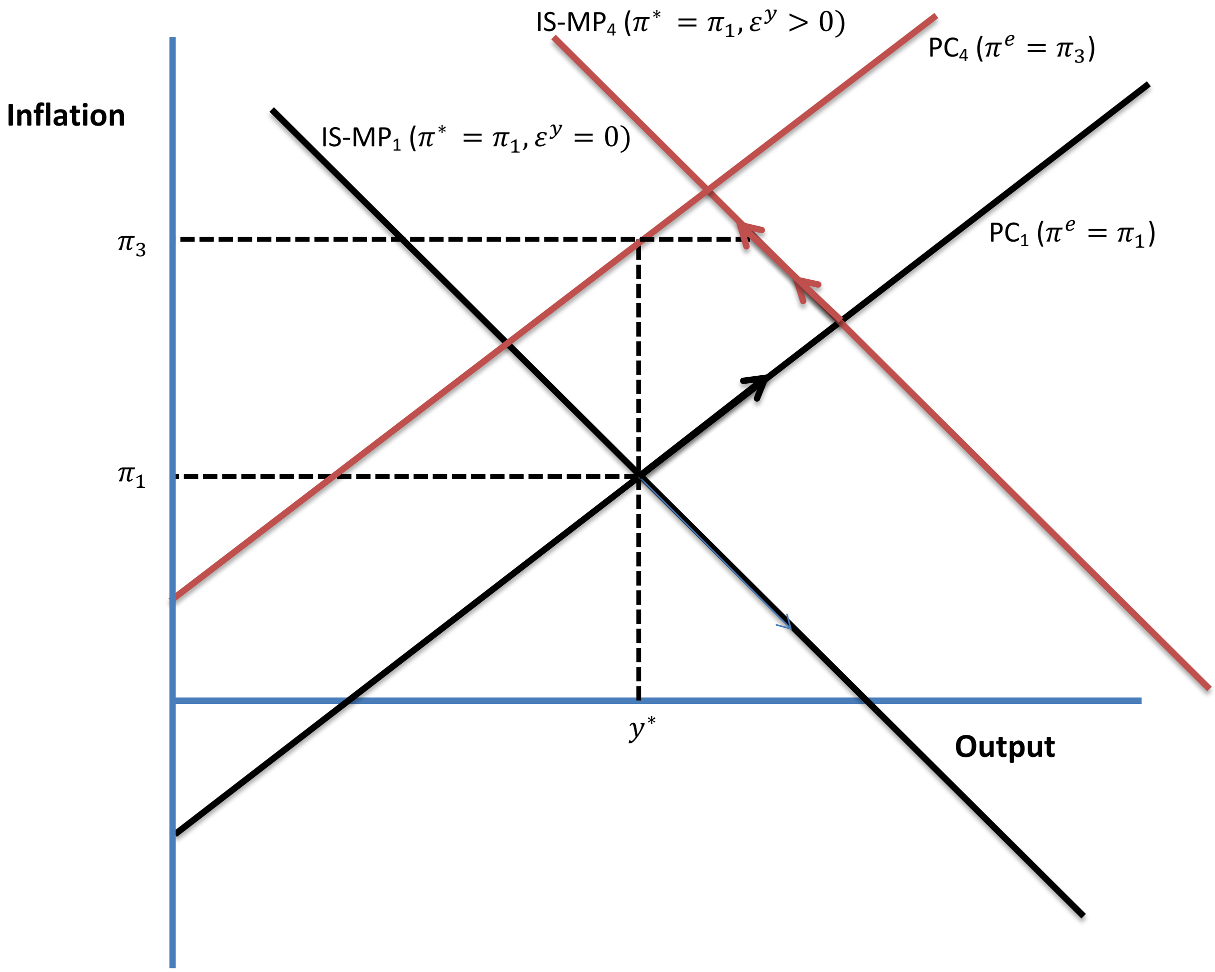


Figure 12: Reversal of Aggregate Demand Shock Leads to Recession With High Inflation

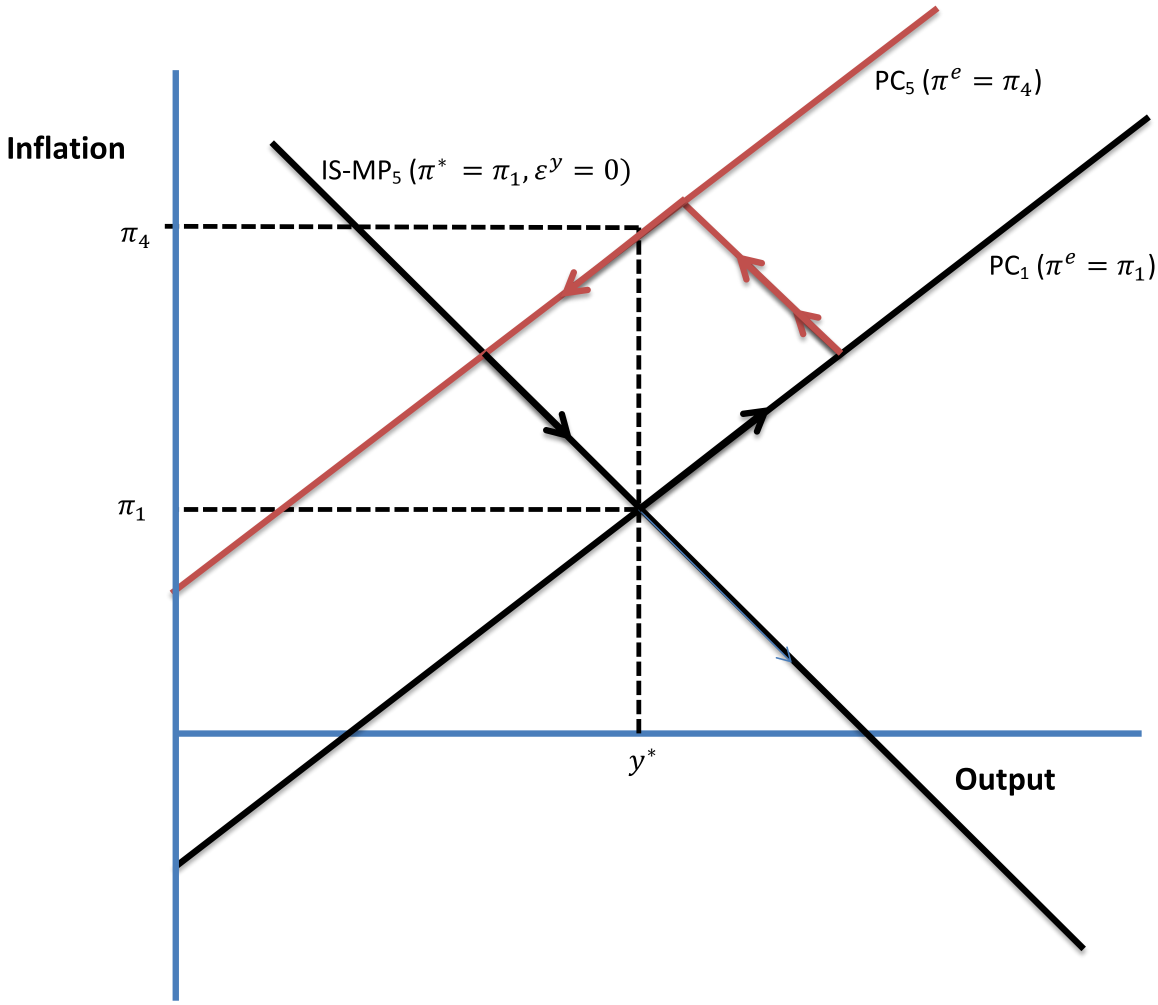


Figure 13: From Chad Jones's Notes: US Inflation-Output Loops 1960-1983

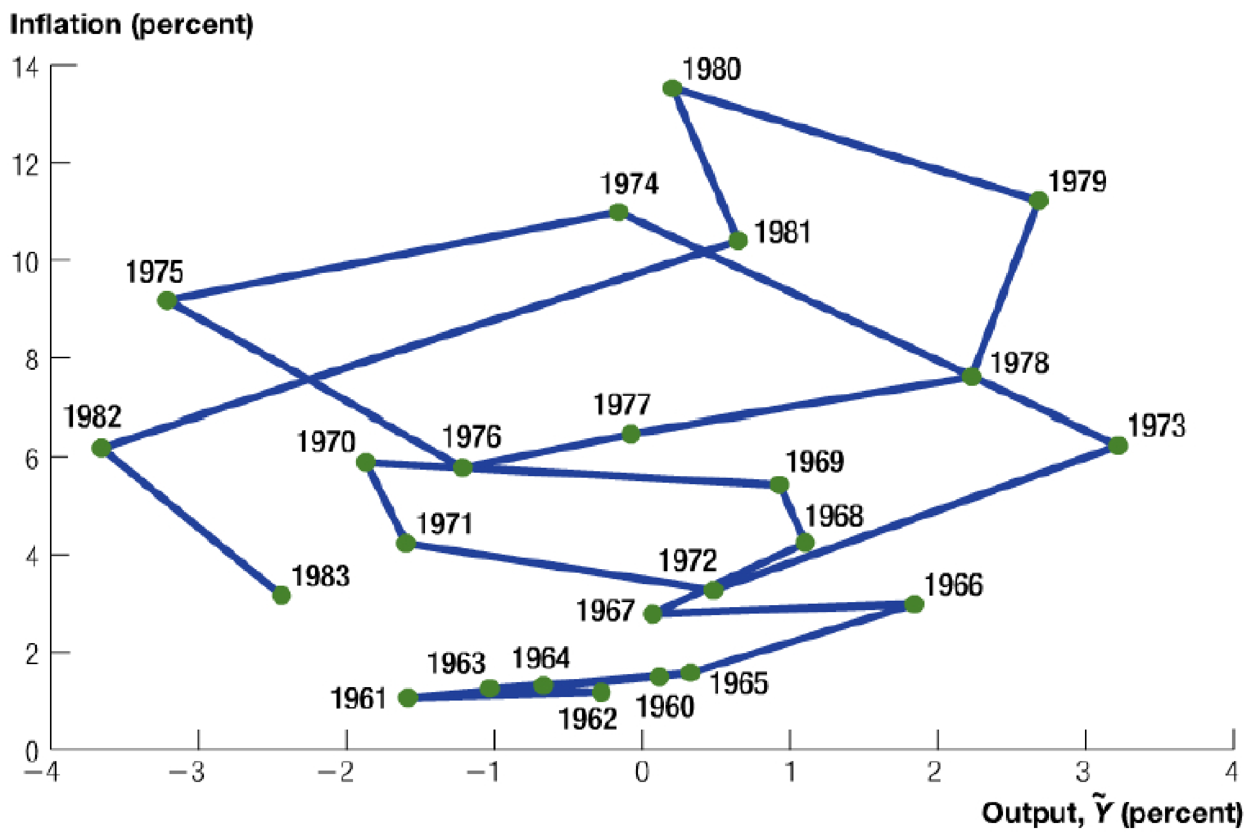
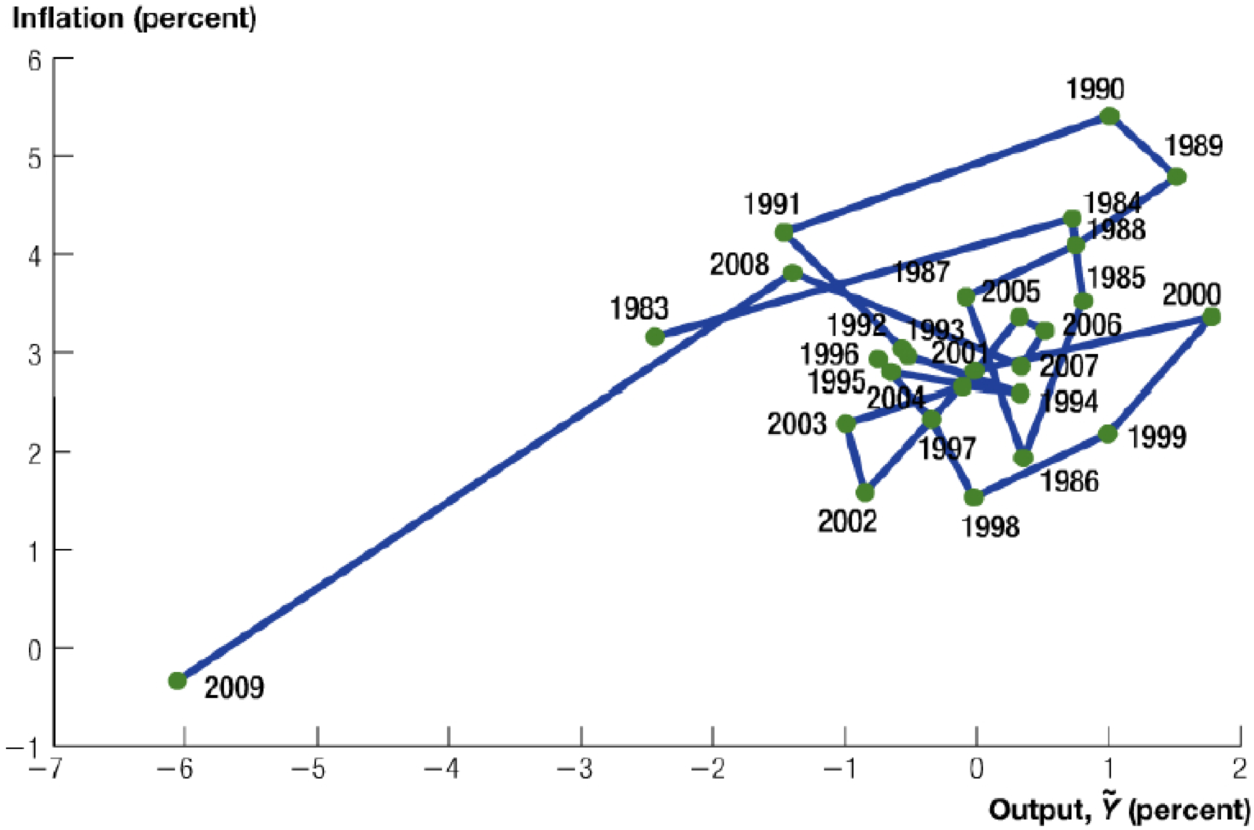


Figure 14: From Chad Jones's Notes: US Inflation-Output Loops 1983-2009



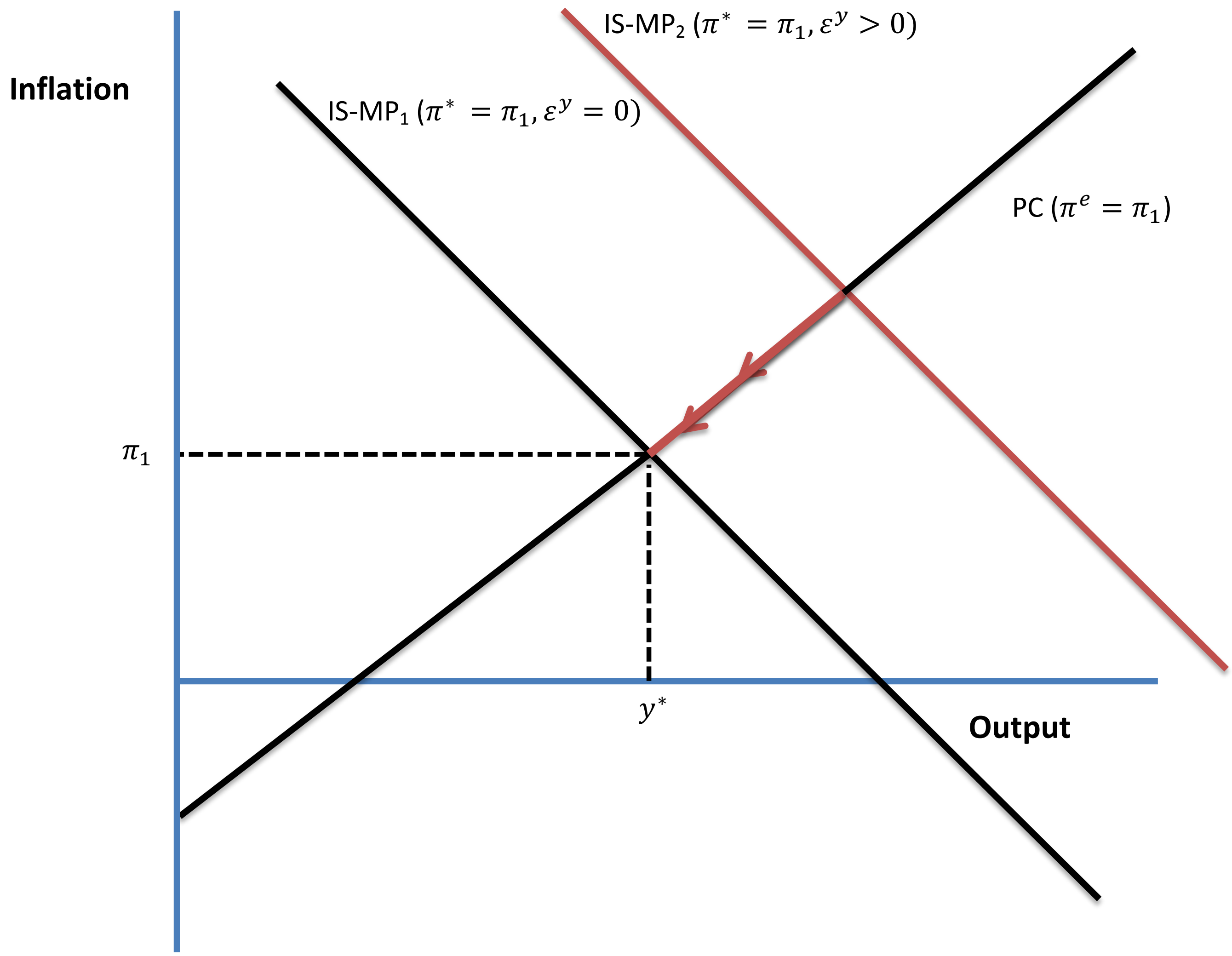
What If Inflation Expectations Don't Adjust?

The evidence presented in Figure 4 suggests that adaptive expectations seems to be a reasonable model for how people have formulated their expectations of inflation. And it can be argued that it is a fairly convincing model of how people behave: Most people don't have the time or knowledge to fully understand exactly what's going in the economy and anticipating that last year's conditions provide a guide to what will happen this year probably works well enough for most people. Indeed, if the value of θ is relatively high, then inflation will only change slowly under adaptive expectations, making the adaptive expectations assumption more accurate.

All that said, it is also possible to imagine situations in which the public's inflation expectations are not formed adaptively. For example, if the public believes that the central bank will always act to return inflation quickly towards its target, then they may assume that deviations from the target will be temporary.

Figure 15 shows how the economy reacts to a temporary positive demand shock when inflation expectations don't change. The outcome here is much nicer than the counter-clockwise cycle described in Figure 12. There is no recession at any point, just a short period of output being above its natural rate and inflation being above its target, followed by a return to their starting levels.

Figure 15: Adjustment if Inflation Expectations Don't Change



The Importance of Anchoring Inflation Expectations

The previous examples provide further food for thought about what kind of monetary policy we would like a central bank to implement. The more people believe that a central bank is maintaining its low inflation target, the less likely the economy is to go through boom-bust cycles. We can see this by comparing the dynamics from Figure 12 where inflation expectations shift over time (perhaps because the public believes the central bank is willing to be flexible about its target) and Figure 15, which shows what happens when inflation expectations do not change after an expansionary shock.

These results predict that we get better outcomes if we have a "tough" central bank which the public believes is committed to keeping the economy near its inflation target. How can this outcome be achieved? The academic literature on this topic has suggested a number of different ways:

1. **Political Independence**: A central bank that plans for the long-term (and does not worry about economic performance during election years) is more likely to stick to a commitment to low inflation. So, independence from political control is an important way to reassure the public about the bank's credibility.

2. **Conservative Central Bankers**: If the central banker is known to really dislike inflation—and the public believes this, the economy gets closer to the ideal low inflation outcome even without commitment. So the government may choose to appoint a central banker who is more inflation-averse than they are.

3. **Consequence for Bad Inflation Outcomes**: Introducing laws so that bad things happen to the central bankers when inflation is high is one way to make the public believe the they will commit to a low inflation rate.

These ideas have had a considerable influence on the legal structure of central banks around the world over the past few decades:

1. **Political Independence**: There has been a substantial move around the world towards making central banks more independent. The Bank of England was made independent in 1997 (previously the Chancellor of the Exchequer had set interest rates) and the ECB/Eurosystem is highly independent from political control.

2. **Conservative Central Bankers**: All around the world, central bankers talk much more now about the evils of inflation and the benefits of price stability. This may be because they believe this to be the case. But there is also a marketing element. Perhaps they can face a better macroeconomic tradeoff if the public believes the central bank's commitment to low inflation.

3. **Consequence for Bad Inflation Outcomes**: In tandem with the move towards increased independence, many central banks now have legally imposed inflation targets and various types of bad things happen to the chief central banker when the inflation target is not met. For instance, the Governor of the Bank of England has to write a letter to the Chancellor explaining why the target was not met. The Bank of England's 2012 "inflation targeting handbook" (linked to on the website) provides lots of information on the inflation targeting regimes adopted around the world over the past 30 years.

Things to Understand from these Notes

Here's a brief summary of the things that you need to understand from these notes.

1. What happens when inflation expectations rise above the central bank's target.
2. The IS-MP-PC solution for inflation and how to derive it.
3. The IS-MP-PC solution for output and how to derive it.
4. Adaptive expectations.
5. Inflation and output dynamics with tough and soft central banks under adaptive expectations
6. Evidence for the adaptive expectations version of the Phillips curve.
7. Effects of a temporary demand shock under adaptive expectations.
8. Effects of a temporary demand shock when inflation expectations don't change.
9. Implications for central bank design and practice.

Appendix: Programme For IS-MP-PC Model Simulations

Figures 7 and 8 were produced using the programme below. The programme is written for the econometric package RATS but a programme of this sort could be written for lots of different types of software including Excel.

```
allocate 50
set pi = 2.0
set y  = 100
set ystar = 100
set pistarcb = 2.0
set pie = 2.0
comp alpha = 1
comp gamma = 1

*** BETA_PI = 1.5
comp betapi = 1.5
comp kappa  = alpha*gamma*(betapi - 1)
comp theta  = (1 / (1+kappa) )

set pie 11 11 = 4
comp pi(11) = theta*pie(11) + (1-theta)*pistarcb(11)
comp y(11) = ystar(11) - theta*alpha*(betapi - 1)*(pie(11) - pistarcb(11))


do j= 12, 50
comp pie(j) = pi(j-1)
comp pi(j) = theta*pie(j) + (1-theta)*pistarcb(j)
comp y(j) = ystar(j) - theta*alpha*(betapi - 1)*(pie(j) - pistarcb(j))
end do j

print 1 50 pi y pie

set y1 = y
set pi1 = pi


*** BETA_PI = 3
comp betapi = 3
comp kappa  = alpha*gamma*(betapi - 1)
comp theta  = (1 / (1+kappa) )

set pie 11 11 = 4
comp pi(11) = theta*pie(11) + (1-theta)*pistarcb(11)
comp y(11) = ystar(11) - theta*alpha*(betapi - 1)*(pie(11) - pistarcb(11))

do j= 12, 50
comp pie(j) = pi(j-1)
comp pi(j) = theta*pie(j) + (1-theta)*pistarcb(j)
comp y(j) = ystar(j) - theta*alpha*(betapi - 1)*(pie(j) - pistarcb(j))
```

```
end do j


set y2 = y
set pi2 = pi


labels y1 y2
# 'Beta = 1.5' 'Beta = 3.0'

labels pi1 pi2
# 'Beta = 1.5' 'Beta = 3.0'

graph(key=below) 2
# pi1 5 35
# pi2 5 35

graph(key=below)  2
# y1 5 35
# y2 5 35
```