

The Zero Lower Bound and the Liquidity Trap

Up to now, we have assumed that the central bank in our model economy sets its interest rate according to a specific policy rule. Whatever the rule says the interest rate should be, the central bank sets that interest rate. But what if the rule predicts the central bank should set interest rates equal to a negative value? Will they?

In the past, the economics profession had a simple answer to this question. There should be a lower bound on interest rates of zero. If I loan you \$100 and only get \$101 back next period, I haven't earned much interest but at least I earned some. A negative interest rate would mean me loaning you \$100 and getting back less than that next year. Why would I do that? Since money maintains its nominal face value, I'd be better off just keep the money in my bank account or under a mattress.

In practice, however, we have seen in recent years that negative interest rates can occur. For example, in the Euro Area, the ECB has charged banks for depositing money with it. This has essentially set the relevant marginal interest rate for these banks to a negative value and they are willing to make loans to other banks or purchase securities that have a negative interest rate, provided it is less negative than the deposit rate paid by the ECB.

There are, however, limits to how negative rates could get. At some point, banks would be better off to withdraw all of their money from their central bank deposit account and hold it in warehouses. This means there is effectively a lower bound on the interest rates set by monetary policy, though exactly what that lower limit might be is a bit unclear.

With these considerations in mind, we are going to adapt our model to take into account that there are times when the central bank would like to set i_t below zero but is not able to do so. We stick with zero rather than specifying a particular negative value for the lower bound:

We could specify a specific non-zero value for the lower bound but this would just introduce an extra parameter into the model without gaining us much additional insight.

The Zero Lower Bound

When will the “zero lower bound” become a problem for a central bank? In our IS-MP-PC model, it depends on the form of the monetary policy rule. Up to now, we have been considering a monetary policy rule of the form

$$i_t = r^* + \pi^* + \beta_\pi (\pi_t - \pi^*) \quad (1)$$

This rule sees the nominal interest rate adjusted upwards and downwards as inflation changes. So the lower bound problem occurs when inflation goes below some critical value. This value might be negative, so it may occur when there is deflation, meaning prices are falling. Amending our model to remove the possibility that interest rates could become negative, our new monetary policy rule is

$$i_t = \text{Maximum} [r^* + \pi^* + \beta_\pi (\pi_t - \pi^*), 0] \quad (2)$$

Because the intended interest rate of the central bank declines with inflation, this means that there is a particular inflation rate, π^{ZLB} , such that if $\pi_t < \pi^{ZLB}$ then the interest rate will equal zero. So what determines this specific value, π^{ZLB} that triggers the zero lower bound?

Algebraically, we can characterise π^{ZLB} as satisfying

$$r^* + \pi^* + \beta_\pi (\pi^{ZLB} - \pi^*) = 0 \quad (3)$$

This can be re-arranged as

$$\beta_\pi \pi^{ZLB} = \beta_\pi \pi^* - r^* - \pi^* \quad (4)$$

which can be solved to give

$$\pi^{ZLB} = \left(\frac{\beta_\pi - 1}{\beta_\pi} \right) \pi^* - \frac{r^*}{\beta_\pi} \quad (5)$$

Equation (5) tells us that three factors determine the value of inflation at which the central bank sets interest rates equal to zero.

1. **The inflation target:** The higher the inflation target π^* is, then the higher is the level of inflation at which a central bank will be willing to set interest rates equal to zero.
2. **The natural rate of interest:** A higher value of r^* , the “natural” real interest rate, lowers the level of inflation at which a central bank will be willing to set interest rates equal to zero. An increase in this rate makes central bank raise interest rates and so they will wait until inflation goes lower than previously to set interest rates to zero.
3. **The responsiveness of monetary policy to inflation:** Increases in β_π raise the coefficient on π^* in this formula, increasing the first term and it makes the second term (which has a negative sign) smaller. Both effects mean a higher β_π translates into a higher value for π^{ZLB} . Central banks that react more aggressively against inflation will wait for inflation to reach lower values before they are willing to set interest rates to zero.

The IS-MP Curve and the Zero Lower Bound

Given this characterisation of when the zero lower bound kicks in, we need to re-formulate the IS-MP curve. Once inflation falls below π^{ZLB} , the central bank cannot keep cutting interest rates in line with its monetary policy rule. Recalling that the IS curve

$$y_t = y_t^* - \alpha (i_t - \pi_t - r^*) + \epsilon_t^y \quad (6)$$

We had previously derived the IS-MP curve by substituting in the monetary policy rule formula (1) for i_t term. This gave us the IS-MP curve as:

$$y_t = y_t^* - \alpha (\beta_\pi - 1) (\pi_t - \pi^*) + \epsilon_t^y \quad (7)$$

However, when $\pi_t \leq \pi^{ZLB}$ we need to substitute in zero instead of the negative value that the monetary policy rule would predict. So the IS-MP curve becomes

$$y_t = \begin{cases} y_t^* - \alpha (\beta_\pi - 1) (\pi_t - \pi^*) + \epsilon_t^y & \text{when } \pi_t > \pi^{ZLB} \\ y_t^* + \alpha r^* + \alpha \pi_t + \epsilon_t^y & \text{when } \pi_t \leq \pi^{ZLB} \end{cases} \quad (8)$$

The effect of inflation on output in this revised IS-MP curve changes when inflation moves below π^{ZLB} . Above π^{ZLB} , higher values of inflation are associated with lower values of output. Below π^{ZLB} , higher values of inflation are associated with *higher* values of output. Graphically, this means the IS-MP curve shifts from being downward-sloping to being upward-sloping when inflation falls below π^{ZLB} . Figure 1 provides an example of how this looks.

Equation (8) also explains the conditions under which the zero lower bound is likely to be relevant. If there are no aggregate demand shocks, so $\epsilon_t^y = 0$, then the zero lower bound is likely to kick in at a point where output is above its natural rate; this is the case illustrated in Figure 1. But this combination of high output and low inflation is unlikely to be an equilibrium in the model unless the public expects very low inflation or deflation so the Phillips curve intersects the IS-MP curve along the section that has output above its natural rate and inflation below π^{ZLB} . However, if we have a large negative aggregate demand shock, so that $\epsilon_t^y < 0$, then it is possible to have output below its natural rate and inflation falling below π^{ZLB} . As illustrated in Figure 2, this situation is more likely to be an equilibrium (i.e. this position for the IS-MP curve is more likely to intersect with the Phillips curve) even if inflation expectations are close to the inflation target.

Figure 1: The IS-MP Curve with the Zero Lower Bound

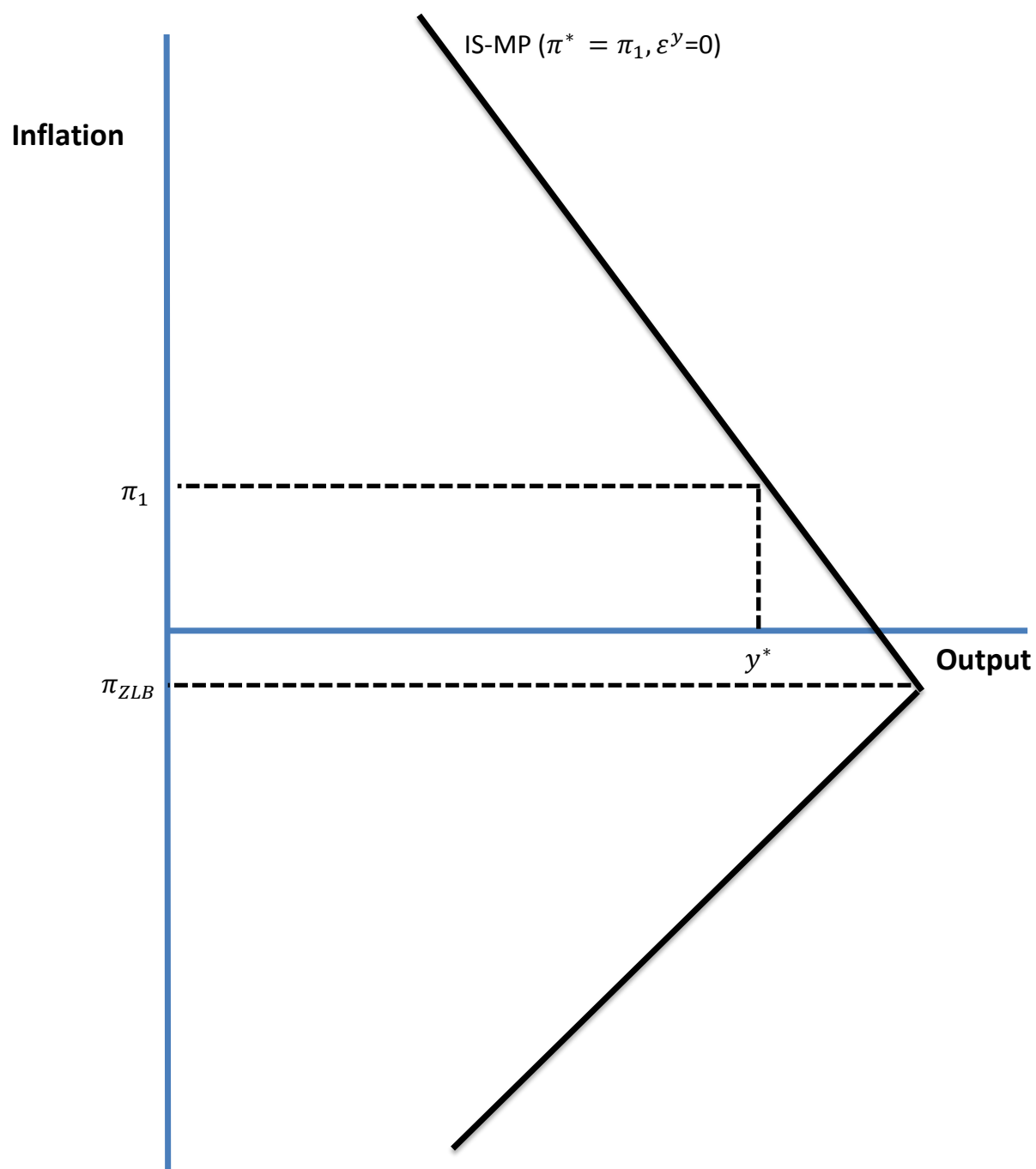
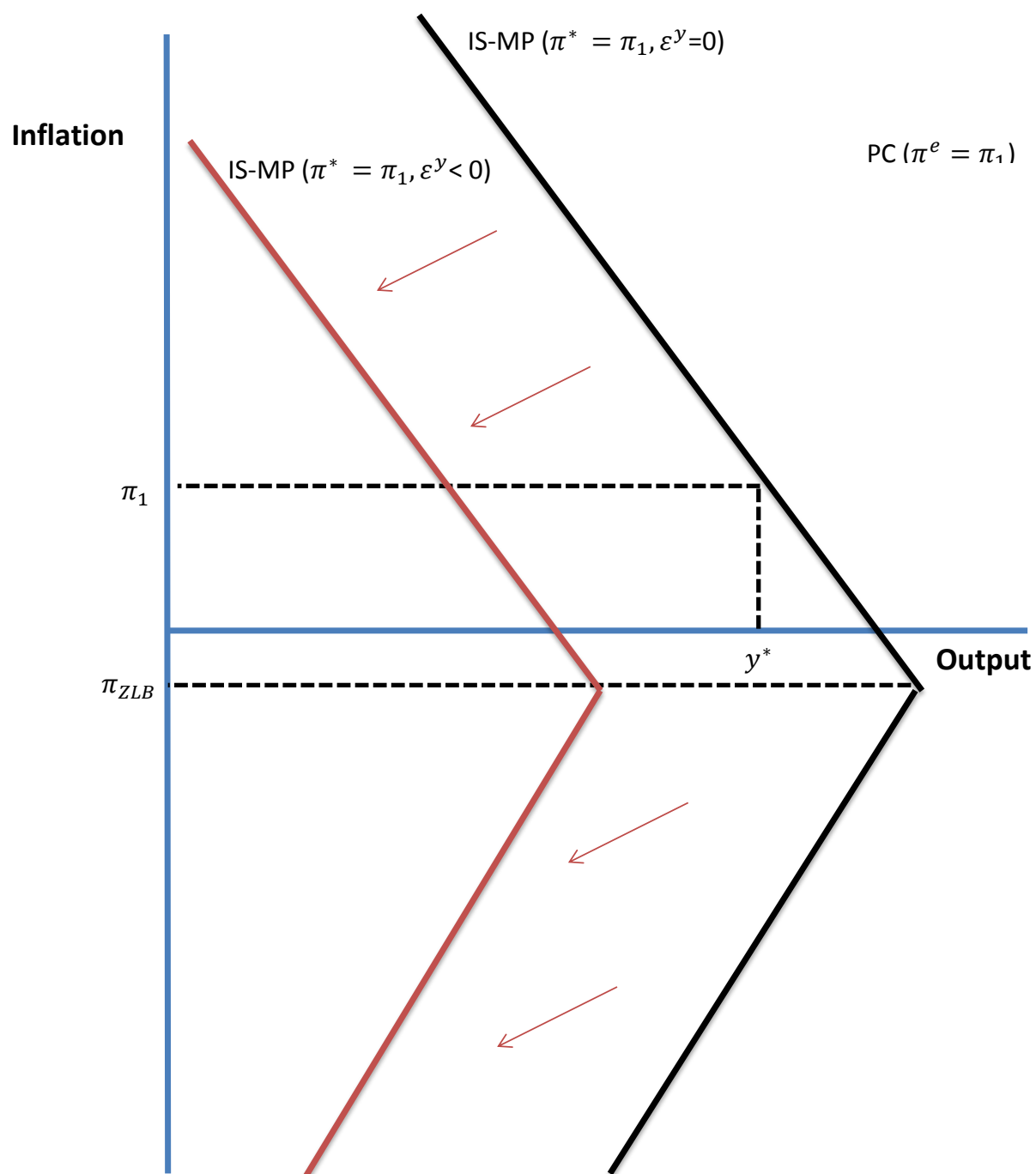


Figure 2: A Negative Aggregate Demand Shock



The Liquidity Trap

When inflation falls below the lower bound, output is determined by

$$y_t = y_t^* + \alpha r^* + \alpha \pi_t + \epsilon_t^y \quad (9)$$

Inflation is still determined by the Phillips curve

$$\pi_t = \pi_t^e + \gamma (y_t - y_t^*) + \epsilon_t^\pi \quad (10)$$

Using the expression for the output gap when the zero lower bound limit has been reached from equation (9) we get an expression for inflation under these conditions as follows

$$\pi_t = \pi_t^e + \gamma (\alpha r^* + \alpha \pi_t + \epsilon_t^y) + \epsilon_t^\pi \quad (11)$$

This can be re-arranged to give

$$\pi_t = \frac{1}{1 - \alpha\gamma} \pi_t^e + \frac{\alpha\gamma}{1 - \alpha\gamma} r^* + \frac{\gamma}{1 - \alpha\gamma} \epsilon_t^y + \frac{1}{1 - \alpha\gamma} \epsilon_t^\pi \quad (12)$$

The coefficient on expected inflation, $\frac{1}{1 - \alpha\gamma}$ is greater than one. So, just as with the Taylor principle example from the last notes, changes in expected inflation translate into even bigger changes in actual inflation. As we discussed the last time, this leads to unstable dynamics. Because these dynamics take place only when inflation has fallen below the zero lower bound, the instability here relates to falling inflation expectations, leading to further declines in inflation and further declines in inflation expectations. Because output depends positively on inflation when the zero-bound constraint binds, these dynamics mean falling inflation (or increasing deflation) and falling output.

This position in which nominal interest rates are zero and the economy falls into a deflationary spiral is known as *the liquidity trap*. Figures 3 and 4 illustrate how the liquidity trap

operates in our model. Figure 3 shows how a large negative aggregate demand shock can lead to interest rates hitting the zero bound even when expected inflation is positive.

Figure 4 illustrates how expected inflation has a completely different effect when the zero lower bound has been hit. It shows a fall in expected inflation after the negative demand shock (this example isn't adaptive expectations because I haven't drawn inflation expectations falling all the way to the deflationary outcome graphed in Figure 3). In our usual model set-up, a fall in expected inflation raises output. However, once at the zero bound, a fall in expected inflation reduces output, which further reduces inflation.

Figure 3: Equilibrium At the Lower Bound

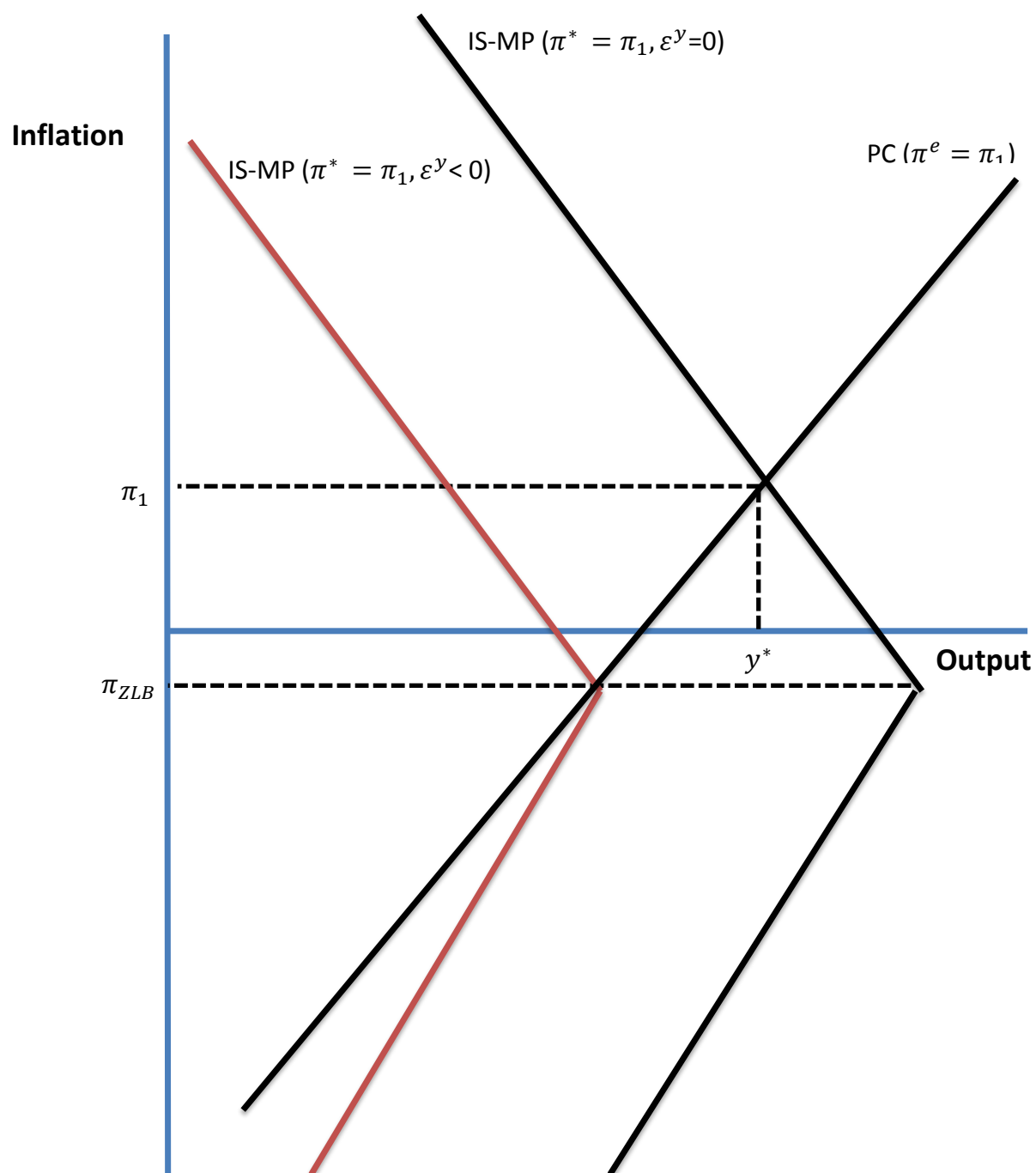
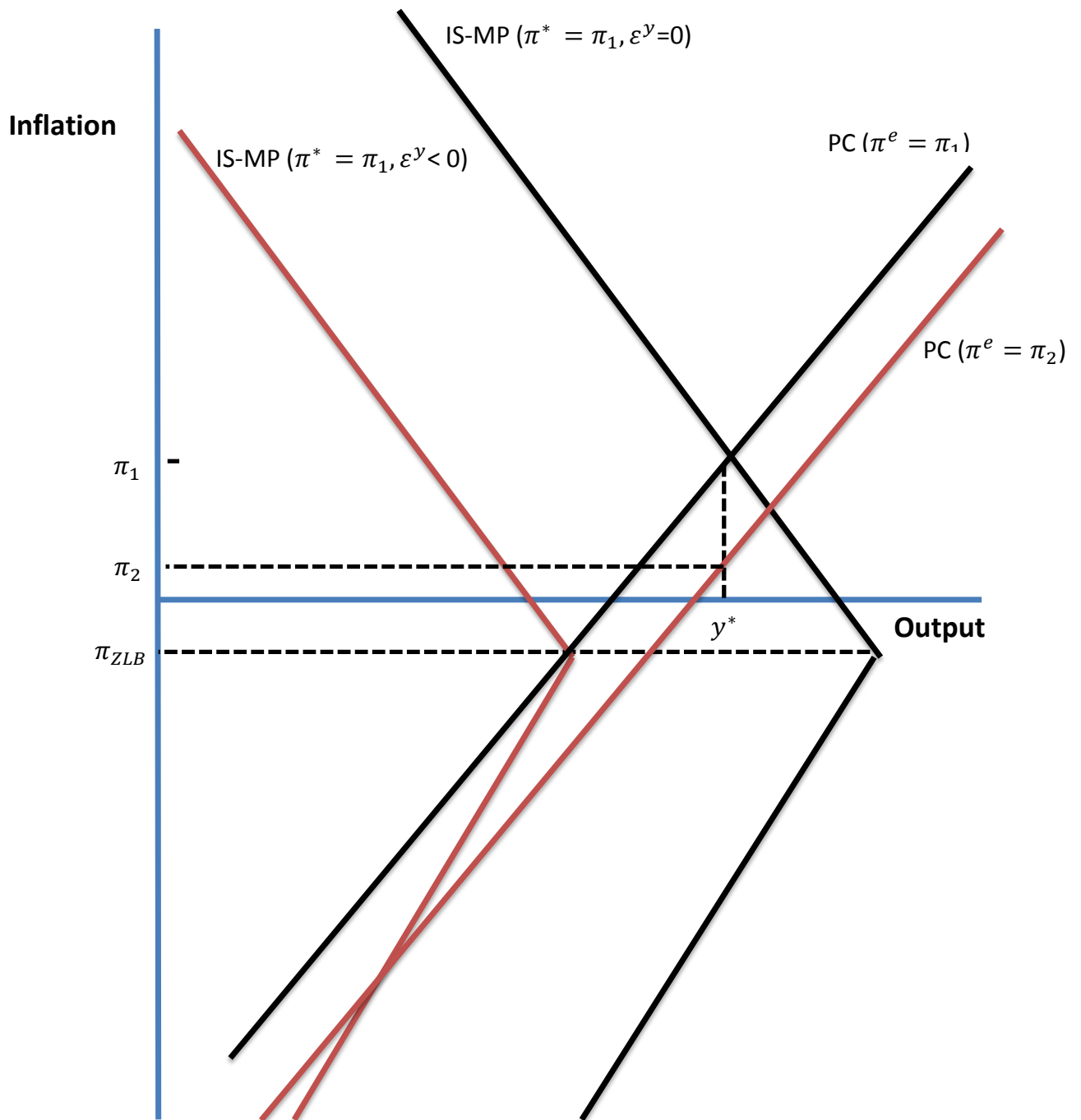


Figure 4: Falling Expected Inflation Worsens Slump



The Liquidity Trap with a Taylor Rule

For the simple monetary policy rule that we have been using, the zero lower bound is hit for a particular trigger level of inflation. Plugging in reasonable parameter values into equation (5) this trigger value will most likely be negative. In other words, with the monetary policy rule that we have been using, the zero lower bound will only be hit when there is deflation. However, if we have a different monetary policy rule this result can be overturned. For example, remember the Taylor-type rule we considered in the first set of notes

$$i_t = r^* + \pi^* + \beta_\pi (\pi_t - \pi^*) + \beta_y (y_t - y_t^*) \quad (13)$$

Incorporating the zero lower bound, this would be adapted to be

$$i_t = \text{Maximum} [r^* + \pi^* + \beta_\pi (\pi_t - \pi^*) + \beta_y (y_t - y_t^*), 0] \quad (14)$$

For this rule, the zero lower bound is hit when

$$r^* + \pi^* + \beta_\pi (\pi_t - \pi^*) + \beta_y (y_t - y_t^*) = 0 \quad (15)$$

This condition can be re-written as

$$\beta_\pi (\pi_t - \pi^*) + \beta_y (y_t - y_t^*) = -r^* - \pi^* \quad (16)$$

In other words, there are a series of different combinations of inflation gaps and output gaps that can lead to monetary policy hitting the zero lower bound. For example, if $y_t = y_t^*$ the lower bound will be hit at the value of inflation given by equation (5), i.e. the level we have defined as π^{ZLB} . In contrast, inflation could equal its target level but policy would hit the zero bound if output fell as low as $y_t^* - \frac{r^* + \pi^*}{\beta_y}$.

Graphically, we can represent all the combinations of output and inflation that produce zero interest rates under the Taylor rule as the area under a downward-sloping line in Inflation-Output space. Figure 5 gives an illustration of what this area would look like. We showed

in the first set of notes that when we are above the zero bound, the IS-MP curve under the Taylor rule is of the same downward-sloping form as under our simple inflation targeting rule. At the zero bound, the arguments we've already presented here also apply so that the IS-MP curve becomes upward sloping.

Figure 6 illustrates two different cases of IS-MP curves when monetary policy follows a Taylor rule. The right-hand curve corresponds to the case $\epsilon_t^y = 0$ (no aggregate demand shocks) and this curve only intersects with the zero bound area when there is a substantial deflation. In contrast, the left-hand curve corresponds to the case in which ϵ_t^y is highly negative (a large negative aggregate demand shocks) and this curve intersects with the zero bound area even at levels of inflation that are positive and aren't much below the central bank's target.

Figure 5: Zero Bound is Binding in Blue Triangle Area

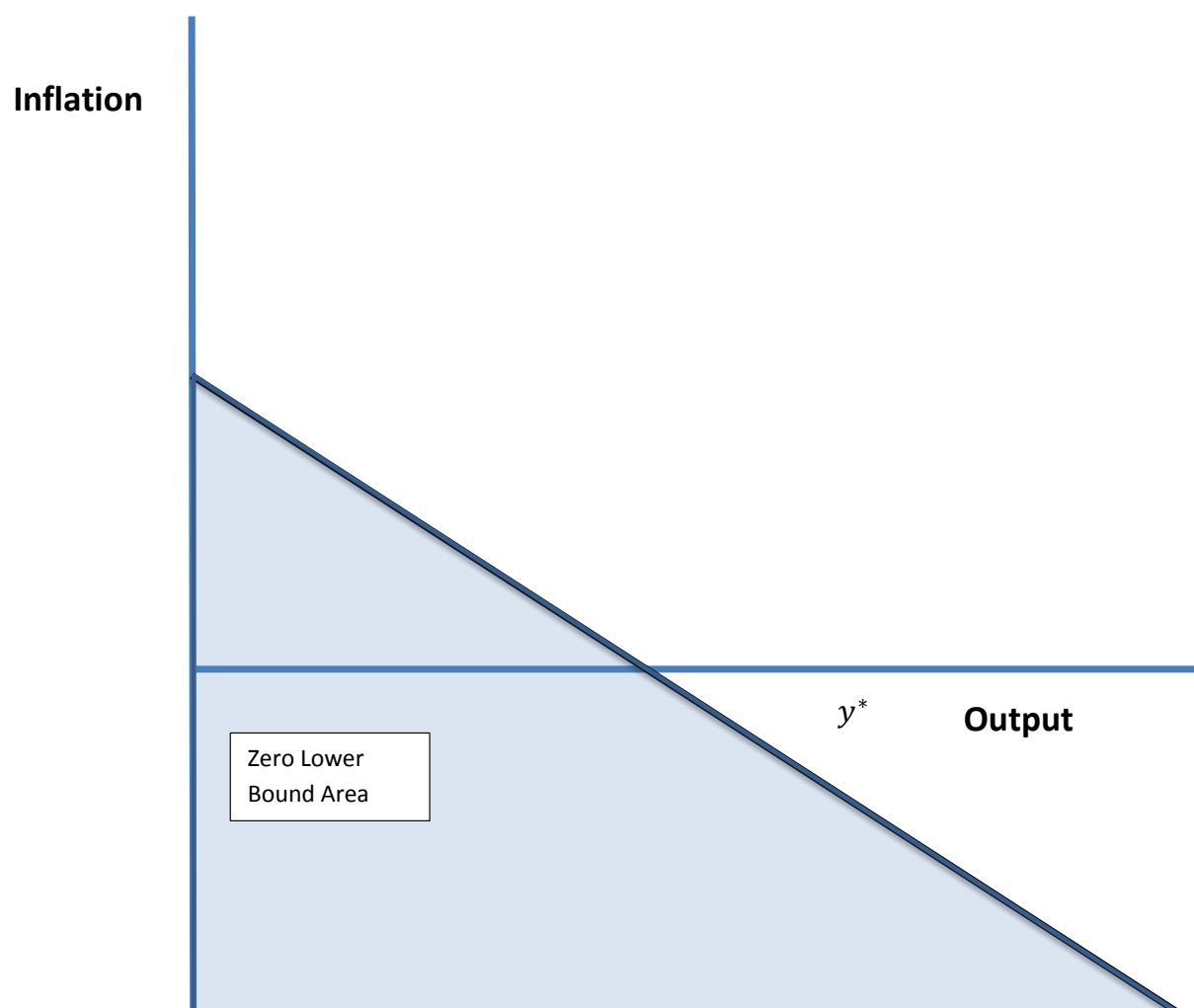
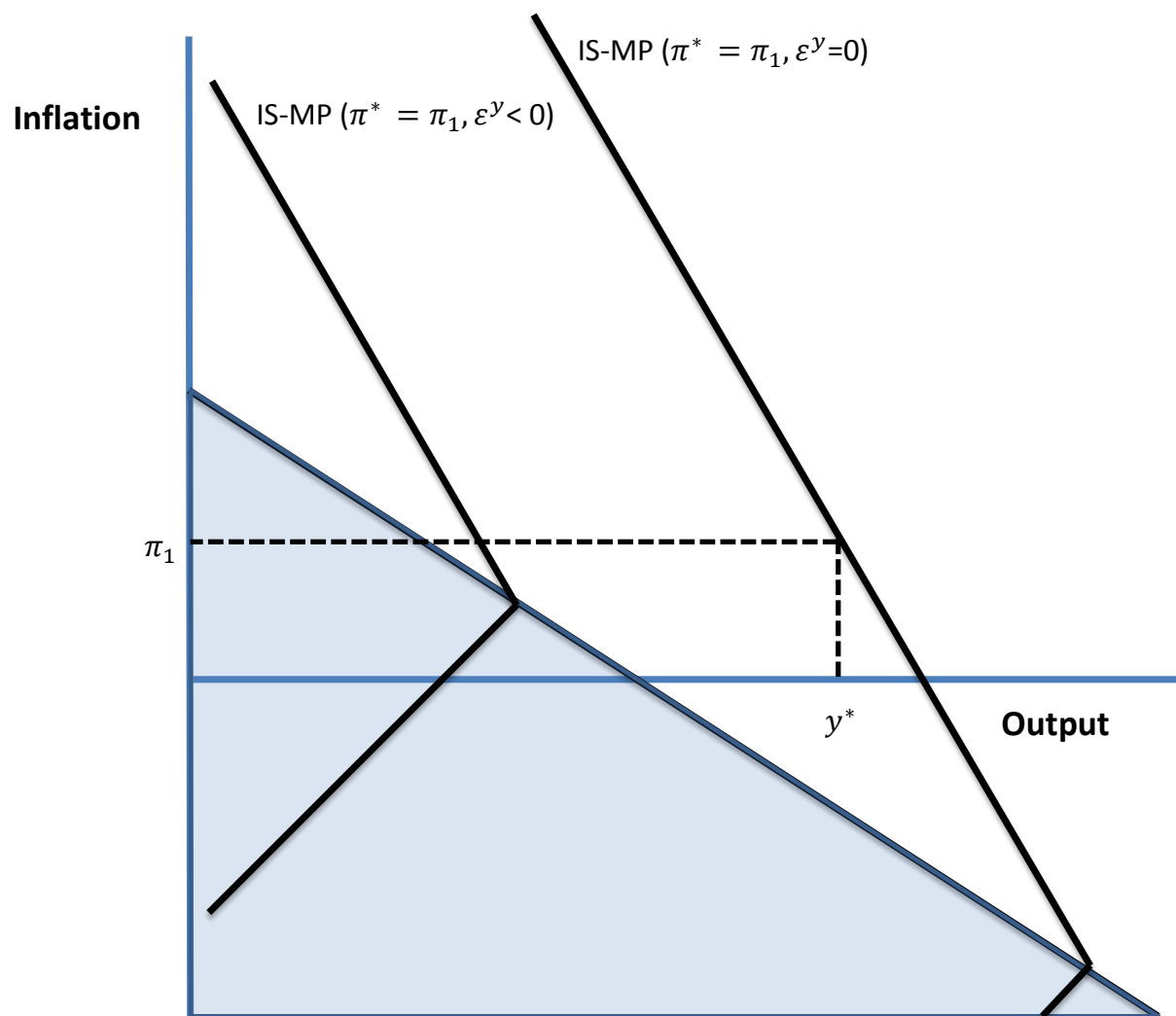


Figure 6: Zero Bound Can Be Hit With Positive Inflation



The Liquidity Trap: Reversing Conventional Wisdom

An important aspect of this model of the liquidity trap is it shows that some of the predictions that our model made (and which are now part of the conventional wisdom among monetary policy makers) do not hold when the economy is in a liquidity trap.

Up to now we have seen that as long as the central bank maintains its inflation targets, then the model with adaptive expectations predicts that deviations of the public's inflation expectations from this target will be temporary and the economy will tend to converge back towards its natural level of output. However, once interest rates have hit the zero bound, this is no longer the case. Instead, the adaptive expectations model predicts the economy can spiral into an ever-declining slump.

Similarly, our earlier model predicted that a strong belief from the public that the central bank would keep inflation at target was helpful in stabilising the economy. However, once you reach the zero bound, convincing the public to raise its inflation expectations (perhaps by announcing a higher target for inflation) is helpful.

How to Get Out of the Liquidity Trap?

The most obvious way that a liquidity trap can end is if there is a positive aggregate demand shock that shifts the IS-MP curve back upwards so that the intersection with the Phillips curve occurs at levels of output and inflation that gets the economy out of the liquidity trap.

However, in reality, liquidity traps have often occurred during periods when there are ongoing and persistent slumps in aggregate demand. For example, after decades of strong growth, the Japanese economy went into a slump during the 1990s. Housing prices crashed and businesses and households were hit with serious negative equity problems. This type of

“balance sheet” recession doesn’t necessarily reverse itself quickly. The result in Japan was a long period in which prices were regularly falling and the Bank of Japan setting short-term interest rates close to zero throughout this period.

Given that economies in liquidity traps tend not to self correct with positive aggregate demand shocks from the private sector, governments can try to boost the economy by using fiscal policy to stimulate aggregate demand. Japan has used fiscal stimulus on various occasions during liquidity-trap-like conditions but with only limited success.

What about monetary policy? With its policy interest rates at zero, can a central bank do any more to boost the economy? Debates on this topic have focused on two areas.

The first area relates to the fact that while the short-term interest rates that are controlled by central banks may be zero, that doesn’t mean the longer-term rates that many people borrow at will equal zero. By signalling that they intend to keep short-term rates low for a long period of time and perhaps by directly intervening in the bond market (i.e. quantitative easing) central banks can attempt to lower these longer-term rates.

The second area relates to inflation expectations. Our model tells us that output can be boosted when the economy is in a liquidity trap by raising inflation expectations. This acts to raise inflation (or reduce deflation) and this reduces real interest rates and boost output. Nobel prize-winner, Paul Krugman, recommends that central banks “commit to being irresponsible” as a way out of these slumps. In other words, they should commit to a temporary period of inflation being higher than you would normally like. But central bankers are a conservative crowd and even temporary “irresponsibility” does not come easy to them.

A third area relates to exchange rates. To raise inflation, a central bank could announce

targets for its exchange rate that would see it fall in value relative to its major trading partners. Such a programme could be implemented by the central bank announcing that it is willing to buy and sell unlimited amounts of foreign exchange at an announced exchange rate e.g. The ECB could announce that it is willing to swap a euro for \$1. Even though the market rate may have been higher than this, nobody will now pay more for a euro than the rate available from the ECB. This currency depreciation would make imports more expensive, which would raise inflation. This latter approach has been labelled the “foolproof way to escape from a liquidity trap” by leading monetary policy expert Lars Svensson.¹ However, while this policy might work for a small country, the world’s largest economies have agreed in recent years to refrain from “competitive devaluation” i.e. using depreciation of their exchange rate as a stimulus.

A final area is the level of inflation targeted by central banks. In our model, the stable equilibrium point sees a nominal interest rate of $(r^* + \pi^*)$ percent. If, for example, $\pi^* = 2$ (consistent with the preference of modern central bankers for a 2 percent inflation rate) and $r^* = 3$, then the nominal interest at equilibrium will be 5 percent. This gives quite a lot of room for cutting nominal interest rates before you reach zero. However, around the world, most central bankers think the equilibrium real interest rate (r^*) is not a fixed number and that it has declined a lot in recent years. To give an example, the members of the Federal Reserve’s policy making committee, the Federal Open Market Committee (FOMC), are asked to provide forecasts of major macroeconomic variables over the next few years and also over the long-term. By subtracting their forecasts for long-run inflation (which basically always equal 2 percent) from their estimate of the long-run federal funds rate, you can calculate what

¹Lars Svensson (2003). “Escaping from a Liquidity Trap and Deflation: The Foolproof Way and Others.” *Journal of Economic Perspectives*.

the members think the long-run real interest rate will be. In January 2012, their median estimate of the long-run real federal funds rate was 2.25 percent. By June 2020, this estimate was 0.4 percent.

If the equilibrium real interest rate was only 0.4 percent, then the equilibrium nominal rate with a 2 percent inflation target will be only 2.4 percent, leaving much less room before you hit zero interest rates. One way to address this issue would be to target a higher inflation rate. With an inflation target of 4 percent, then the equilibrium nominal rate would be 4.4 percent and there would be an additional 200 basis points of monetary easing possible before the zero bound was hit. There has been some discussion of this idea in academic circles but most central bankers are generally very much against this idea. See the readings by Ben Bernanke and by Gagnon and Collins for a sense of this debate.

Things to Understand from these Notes

Here's a brief summary of the things that you need to understand from these notes.

1. Why there is a lower bound on interest rates.
2. The factors that influence when the central bank sets zero rates in our model.
3. How the IS-MP curve changes when incorporating the zero lower bound.
4. How changes in inflation expectations affect the economy above and below the zero lower bound.
5. What is meant by the liquidity trap, i.e. why the economy doesn't automatically recover when the zero bound binds.
6. How the IS-MP-PC graphs work when we incorporate the zero bound.
7. Policy options for getting out of the liquidity trap.
8. Arguments for a higher inflation target.