Growth Accounting

The final part of this course will focus on what is known as "growth theory." Unlike most of macroeconomics, which concerns itself with what happens over the course of the business cycle (why unemployment or inflation go up or down during expansions and recessions), this branch of macroeconomics concerns itself with what happens over longer periods of time. In particular, it looks at the question "What determines the growth rate of the economy over the long run and what can policy measures do to affect it?" As we will also discuss, this is related to the even more fundamental question of what makes some countries rich and others poor. We will also examine how economies behaved prior to the modern era of economic growth and discuss the tensions between economic growth and environmental sustainability.

In this set of notes, we will cover what is known as "growth accounting" – a technique for explaining the factors that determine growth.

Production Functions

The usual starting point for growth accounting is the assumption that total real output in an economy is produced using an aggregate production function technology that depends on the total amount of labour and capital used in the economy. For illustration, assume that this takes the form of a Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{\beta} \tag{1}$$

where K_t is capital input and L_t is labour input. Note that an increase in A_t results in higher output without having to raise inputs. Macroeconomists usually call increases in A_t "technological progress" and often refer to this as the "technology" term. As such, it is easy to imagine increases in A_t to be associated with people inventing new technologies that allow firms to be more productive. Ultimately, however, A_t is simply a measure of productive efficiency and it may go up or down for other reasons, e.g. with the imposition or elimination of government regulations. Because an increase in A_t increases the productiveness of the other factors, it is also sometimes known as *Total Factor Productivity* (TFP), and this is the term most commonly used in empirical papers that attempt to calculate this series.

Usually, we will be more interested in the determination of output per person in the economy, rather than total output. Output per person is often labelled *productivity* by economists with increases in output per worker called *productivity growth*. Productivity is obtained by dividing both sides of equation (1) by L_t to get

$$\frac{Y_t}{L_t} = A_t K_t^{\alpha} L_t^{\beta-1} \tag{2}$$

which can be re-arranged to give

$$\frac{Y_t}{L_t} = A_t \left(\frac{K_t}{L_t}\right)^{\alpha} L_t^{\alpha+\beta-1} \tag{3}$$

This equation shows that there are three potential ways to increase productivity:

- Technological progress: Improving the efficiency with which an economy uses its inputs, i.e. increases in A_t.
- Capital deepening (i.e. increases in capital per worker)
- Increases in the number of workers: Note that this only adds to growth if $\alpha + \beta > 1$, i.e. if there are increasing returns to scale. Most growth theories assumes constant returns to scale: A doubling of inputs produces a doubling of outputs. If a doubling of inputs manages to more than double outputs, you could argue that the efficiency of production has improved and so perhaps this should be considered an increase in A rather than

something that stems from higher inputs. If, there are constant returns to scale, then $\alpha + \beta - 1 = 0$ and this term disappears and production function can be written as

$$\frac{Y_t}{L_t} = A_t \left(\frac{K_t}{L_t} \right)^{\alpha} \tag{4}$$

The Determinants of Growth

Let's consider what determines growth with a constant returns to scale Cobb-Douglas production function (so $\beta = 1 - \alpha$)

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{5}$$

and let's assume that time is continuous. In other words, the time element t evolves smoothly instead of just taking integer values like $t = 1$ and $t = 2$.

How do we characterise how this economy grows over time? Let's denote the growth rate of Y_t by G_t^Y. This can be defined as

$$G_t^Y = \frac{1}{Y_t} \frac{dY_t}{dt} \tag{6}$$

In other words, the growth rate at any point in time is the change in output (the derivative of output with respect to time, $\frac{dY_t}{dt}$) divided by the level of output. We can characterise the growth rate of Y_t as a function of the growth rates of labour, capital and technology by differentiating the right-hand-side of equation (5) with respect to time. Before we do this, you should recall the product rule for differentiation, i.e. that

$$\frac{dAB}{dx} = B\frac{dA}{dx} + A\frac{dB}{dx} \tag{7}$$

For products of three variables (like we have in this case) this implies

$$\frac{dABC}{dx} = BC\frac{dA}{dx} + AC\frac{dB}{dx} + AB\frac{dC}{dx} \tag{8}$$

In our case, we have

$$\frac{dY_t}{dt} = \frac{dA_t K_t^{\alpha} L_t^{1-\alpha}}{dt} = K_t^{\alpha} L_t^{1-\alpha} \frac{dA_t}{dt} + A_t L_t^{1-\alpha} \frac{dK_t^{\alpha}}{dt} + A_t K_t^{\alpha} \frac{dL_t^{1-\alpha}}{dt} \tag{9}$$

We can use the chain rule to calculate the terms involving the impact of changes in capital and labour inputs:

$$\frac{dK_t^{\alpha}}{dt} = \frac{dK_t^{\alpha}}{dK_t} \frac{dK_t}{dt} = \alpha K_t^{\alpha-1} \frac{dK_t}{dt} \tag{10}$$

$$\frac{dL_t^{1-\alpha}}{dt} = \frac{dL_t^{1-\alpha}}{dL_t} \frac{dL_t}{dt} = (1-\alpha) L_t^{-\alpha} \frac{dL_t}{dt} \tag{11}$$

Plugging these formulae into the right places in equation (9) we get

$$\frac{dY_t}{dt} = K_t^{\alpha} L_t^{1-\alpha} \frac{dA_t}{dt} + \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} \frac{dK_t}{dt} + (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha} \frac{dL_t}{dt} \tag{12}$$

The growth rate of output is calculated by dividing both sides of this by Y_t which is the same as dividing by $A_t K_t^{\alpha} L_t^{1-\alpha}$.

$$\frac{1}{Y_t} \frac{dY_t}{dt} = \left(\frac{K_t^{\alpha} L_t^{1-\alpha}}{A_t K_t^{\alpha} L_t^{1-\alpha}} \right) \frac{dA_t}{dt} + \alpha \left(\frac{A_t K_t^{\alpha-1} L_t^{1-\alpha}}{A_t K_t^{\alpha} L_t^{1-\alpha}} \right) \frac{dK_t}{dt} + (1-\alpha) \left(\frac{A_t K_t^{\alpha} L_t^{-\alpha}}{A_t K_t^{\alpha} L_t^{1-\alpha}} \right) \frac{dL_t}{dt} \tag{13}$$

Cancelling the various terms that appear multiple times in the terms inside the brackets and we get

$$\frac{1}{Y_t} \frac{dY_t}{dt} = \frac{1}{A_t} \frac{dA_t}{dt} + \alpha \frac{1}{K_t} \frac{dK_t}{dt} + (1-\alpha) \frac{1}{L_t} \frac{dL_t}{dt} \tag{14}$$

This can written in more intuitive form as

$$G_t^Y = G_t^A + \alpha G_t^K + (1-\alpha) G_t^L \tag{15}$$

The growth rate of output equals the growth rate of the technology term plus a weighted average of capital growth and labour growth, where the weight is determined by the parameter α. This is the key equation in growth accounting studies. These studies provide estimates of how much GDP growth over a certain period comes from growth in the number of workers, how much comes from growth in the stock of capital and how much comes from improvements in Total Factor Productivity.

One can also show that the growth rate of output per worker is the growth rate of output minus the growth in the number of workers, so this is determined by

$$G_t^Y - G_t^L = G_t^A + \alpha \left(G_t^K - G_t^L \right) \tag{16}$$

This is a re-statement in growth rate terms of our earlier decomposition of output growth into technological progress and capital deepening when the production function has constant returns to scale.

Ideally, I'd like you to be able to understand how equation (15) was derived but certainly you should know this formula and understand its meaning.

- For example, remember the production function is $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$. The reason an increase of x percent in A_t translates into an increase of x percent in output is because A_t multiplies the other terms.

- In contrast, K_t is taken to the power of α. An increase in K_t, say by replacing it with $(1+x) K_t$ is equivalent to multiplying the existing level of output by $(1+x)^{\alpha}$. Because α is assumed to be less than one, this is a smaller increase than comes from increasing A_t by a factor of $(1+x)$.

- To understand why a 1% increase in both K_t and L_t leads to a 1% increase in output, note that if we multiplied both the inputs in $K_t^{\alpha} L_t^{1-\alpha}$ by $(1+x)$, we would get

$$A_t \left((1+x) K_t \right)^{\alpha} \left((1+x) L_t \right)^{1-\alpha} = (1+x)^{\alpha} (1+x)^{1-\alpha} A_t K_t^{\alpha} L_t^{1-\alpha} = (1+x) A_t K_t^{\alpha} L_t^{1-\alpha}$$

How to Calculate the Sources of Growth: Solow (1957)

For most economies, we can calculate GDP, as well as the number of workers and also get some estimate of the stock of capital (this last is a bit trickier and usually relies on assumptions about how investment cumulates over time to add to the stock of capital.) We don't directly observe the value of the Total Factor Productivity term, A_t. However, if we knew the value of the parameter α, we could figure out the growth rate of TFP from the following equation based on re-arranging (15)

$$G_t^A = G_t^Y - \alpha G_t^K - (1-\alpha) G_t^L \tag{17}$$

But where would we get a value of α from? In a famous 1957 paper, MIT economist Robert Solow pointed out that we could arrive at an estimate of α by looking at the shares of GDP paid to workers and to capital.[1]

To see how this method works, consider the case of a perfectly competitive firm that is seeking to maximise profits. Suppose the firm sells its product for a price P_t (which it has no control over), pays wages to its workers of W_t and rents its capital for a rental rate of R_t (this last assumption—that the firm rents its capital—isn't important for the points that follow but it makes the calculations simpler.) This firm's profits are given by

$$\Pi_t = P_t Y_t - R_t K_t - W_t L_t \tag{18}$$

$$= P_t A_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - W_t L_t \tag{19}$$

Now consider how the firm chooses how much capital and labour to use. It will maximise profits by differentiating the profit function with respect to capital and labour and setting the

[1] "Technical Change and the Aggregate Production Function", *Review of Economics and Statistics.*

resulting derivatives equal to zero. This gives two conditions

$$\frac{\partial \Pi_t}{\partial K_t} = \alpha P_t A_t K_t^{\alpha-1} L_t^{1-\alpha} - R_t = 0 \quad (20)$$

$$\frac{\partial \Pi_t}{\partial L_t} = (1-\alpha) P_t A_t K_t^{\alpha} L_t^{-\alpha} - W_t = 0 \quad (21)$$

These can be simplified to read

$$\frac{\partial \Pi_t}{\partial K_t} = \alpha \frac{P_t Y_t}{K_t} - R_t = 0 \quad (22)$$

$$\frac{\partial \Pi_t}{\partial L_t} = (1-\alpha) \frac{P_t Y_t}{L_t} - W_t = 0 \quad (23)$$

Solving these we get

$$\alpha = \frac{R_t K_t}{P_t Y_t} \quad (24)$$

$$1-\alpha = \frac{W_t L_t}{P_t Y_t} \quad (25)$$

Take a close look these equations.

- $P_t Y_t$ is total nominal GDP (the price level times real output)
- $W_t L_t$ is the total amount of income paid out as wages (the wage rate times number of workers).
- $R_t K_t$ is the total amount of income paid to capital (the rental rate times the amount of capital).

These equations tell us that we can calculate $1-\alpha$ as the fraction of income paid to workers rather than to compensate capital. (In real-world economies, non-labour income mainly takes the form of interest, dividends, and retained corporate earnings). National income accounts come with various decompositions. One of them describes how different types of incomes add up to GDP. In most countries, these statistics show that wage income accounts for most of GDP, meaning $\alpha < 0.5$. A standard value that gets used in many studies, based on US estimates, is $\alpha = \frac{1}{3}$. I would note, however, that some studies do this calculation assuming firms are imperfectly competitive – if this is the case (as it is in the real world) then the shares of income earned by labour and capital depend on the degree of monopoly power. So one needs to be cautious about growth accounting calculations as they rely on theoretical assumptions that could potentially be misleading.

Solow's 1957 paper concluded that capital deepening had not been that important for U.S. growth for the period that he examined (1909-1949). In fact, he calculated that TFP growth accounted for 87.5% of growth in output per worker over that period. The calculation became very famous – it was one his papers that was cited by the Nobel committee when awarding Solow the prize for economics in 1987. TFP is sometimes called "the Solow residual" because it is a "backed out" calculation that makes things add up: You calculate it as the part of output growth not due to input growth in the same way as regression residuals in econometrics are the part of the dependent variable not explained by the explanatory variables included in the regression.

Example: The BLS Multifactor Productivity Figures

Most growth accounting calculations are done as part of academic studies. However, in some countries the official statistical agencies produce growth accounting calculations. In the U.S. the Bureau of Labor Statistics (BLS) produces them under the name "multifactor productivity" calculations, (i.e. they use the term MFP instead of the term TFP but conceptually they are the same thing.) Many of the studies add some "bells and whistles" to the basic calculations just described. For example, the BLS try to account for improvements in the "quality" of the labour force by accounting for improvements in the level of educational qualifications and work experience of employees. In other words, they view the production function as being of the form

$$Y_t = A_t K_t^{\alpha} \left(q_t L_t\right)^{1-\alpha} \tag{26}$$

where q_t is a measure of the "quality" of the labor input.

Figure 1 shows a summary of the BLS's calculations of the sources of growth in the US from 1987 to 2010. They conclude that average growth of 2.2 percent in the U.S. private nonfarm economy can be explained as follows: 0.9 percent comes from capital deepening, 0.3 percent comes from changes in labour composition and 0.9 percent comes from changes in what they call multifactor productivity. Looking at different samples, however, we can see large changes between different periods in the contribution of MFP.

- From 1987-1995, productivity growth averaged only 1.5 percent and MFP growth was weak, contributing only 0.5 percent per year to growth. During this period, there was a lot of discussion about the slowdown in growth relative to previous eras, with much of the focus on the poor performance of TFP growth. Paul Krugman's first popular economics book was called *The Age of Diminished Expectations* because people seemed to have accepted that the US economy was doomed to low productivity growth.

- From 1995-2007, productivity growth averaged a very respectable 2.65 percent, with MFP growth contributing 1.35 percent. During this period, there was a lot ofdiscussion of the impact of new Internet-related technologies that improved efficiency. While the peak of this enthusiasm was around the dot-com bubble of the 2000s when there was lots of talk of a "New Economy", post-tech-bubble productivity performance was also pretty good.

- From 2007-2013, productivity growth has been weaker than in the previous decade, averaging only 1.3 percent. MFP growth has been particularly weak, averaging only 0.6 percent over this period. "New Economy" optimism has receded, a topic that we will return to later.

In addition to the poor performance of U.S. productivity growth, another factor that is weighing on the potential for output growth is a slow growth rate of the labour force. After years of increasing numbers of people available for work due to normal population growth, immigration and increased female labour participation, the US labour force has flattened out (see Figure 2). This is being driven by long-run demographic trends as the large "'baby boom" generation starts to retire. This trend is set to continue over the next few decades. Figure 3 shows that the dependency ratio (the ratio of non-working to working people) is projected to increase significantly as the populations grows older on average. See my blog post "Is the U.S, Set for an Era of Slow Growth?"

Figure 1: Growth Accounting Calculations for the U.S.

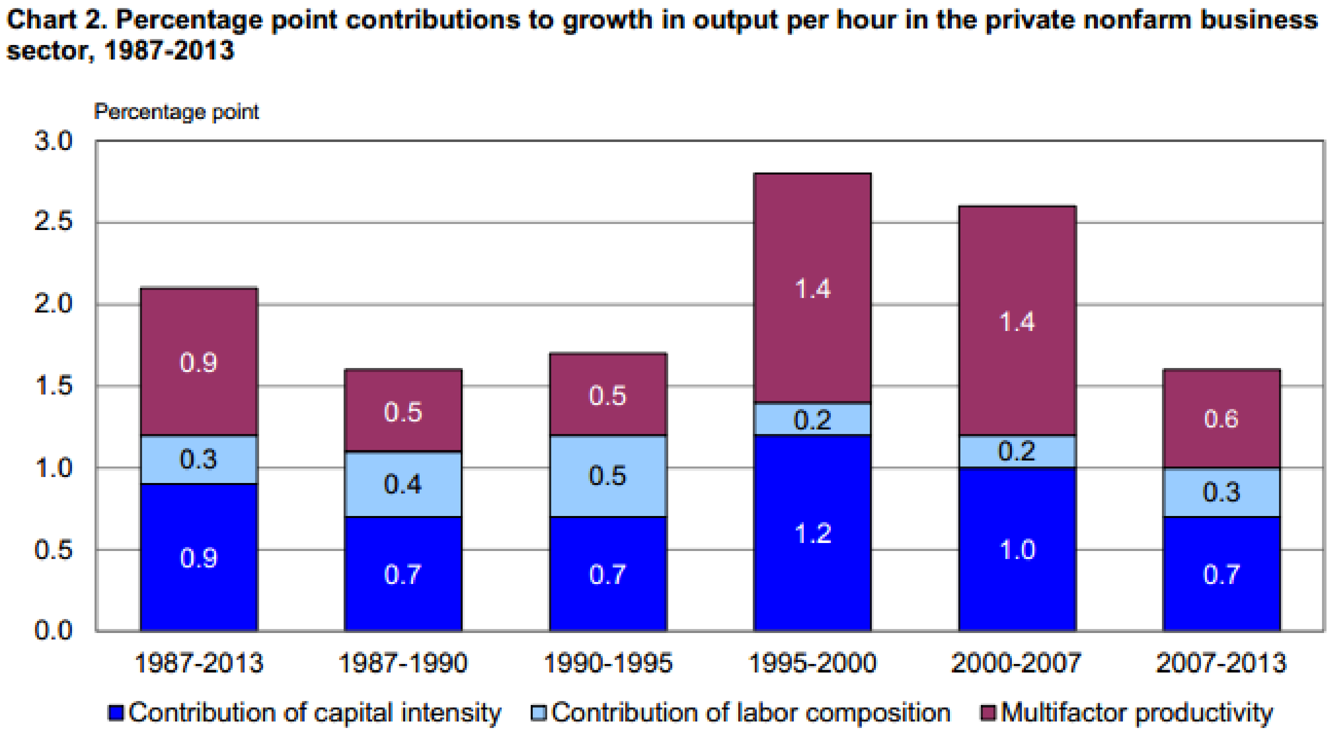


Figure 2: The U.S. Labour Force

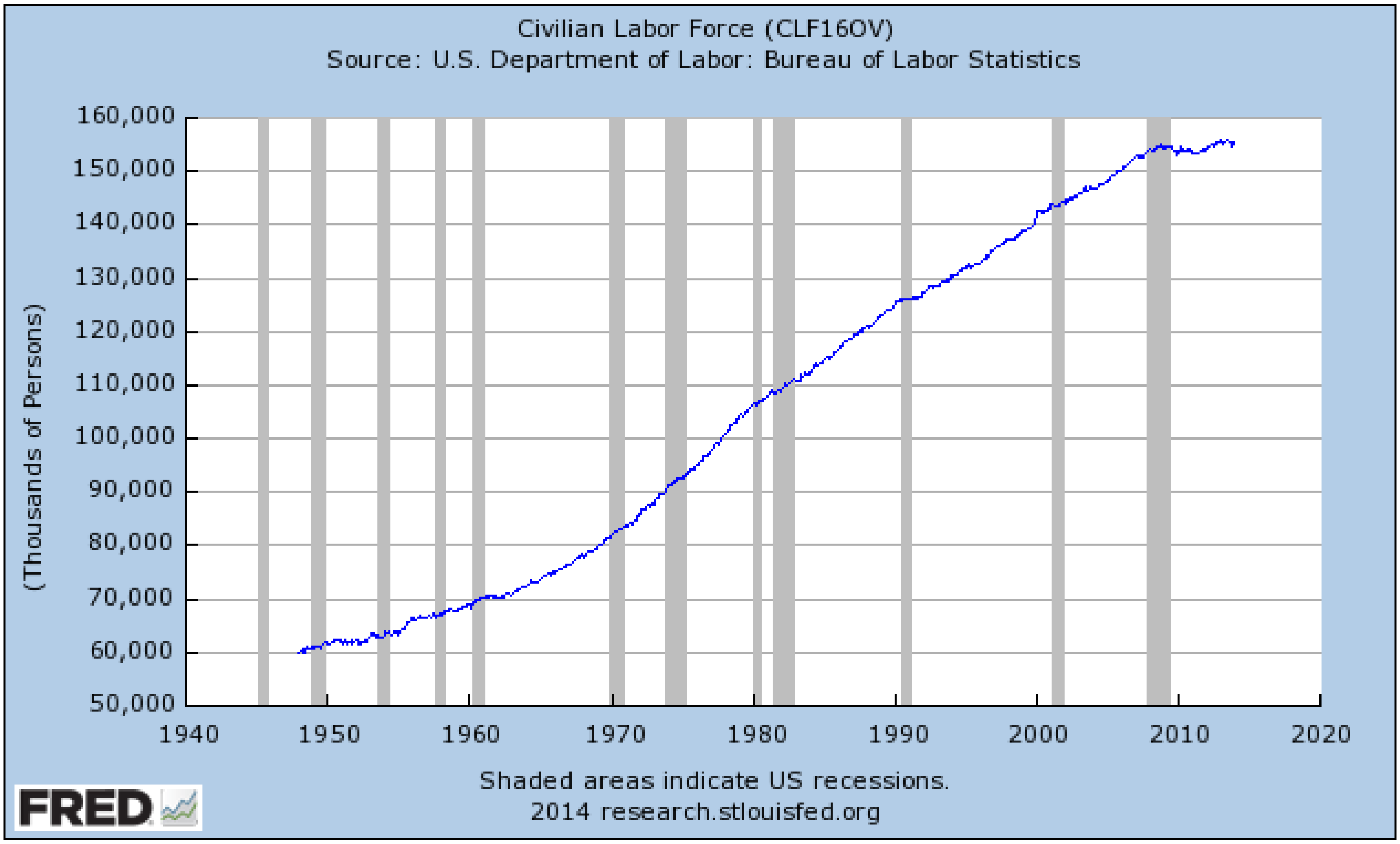
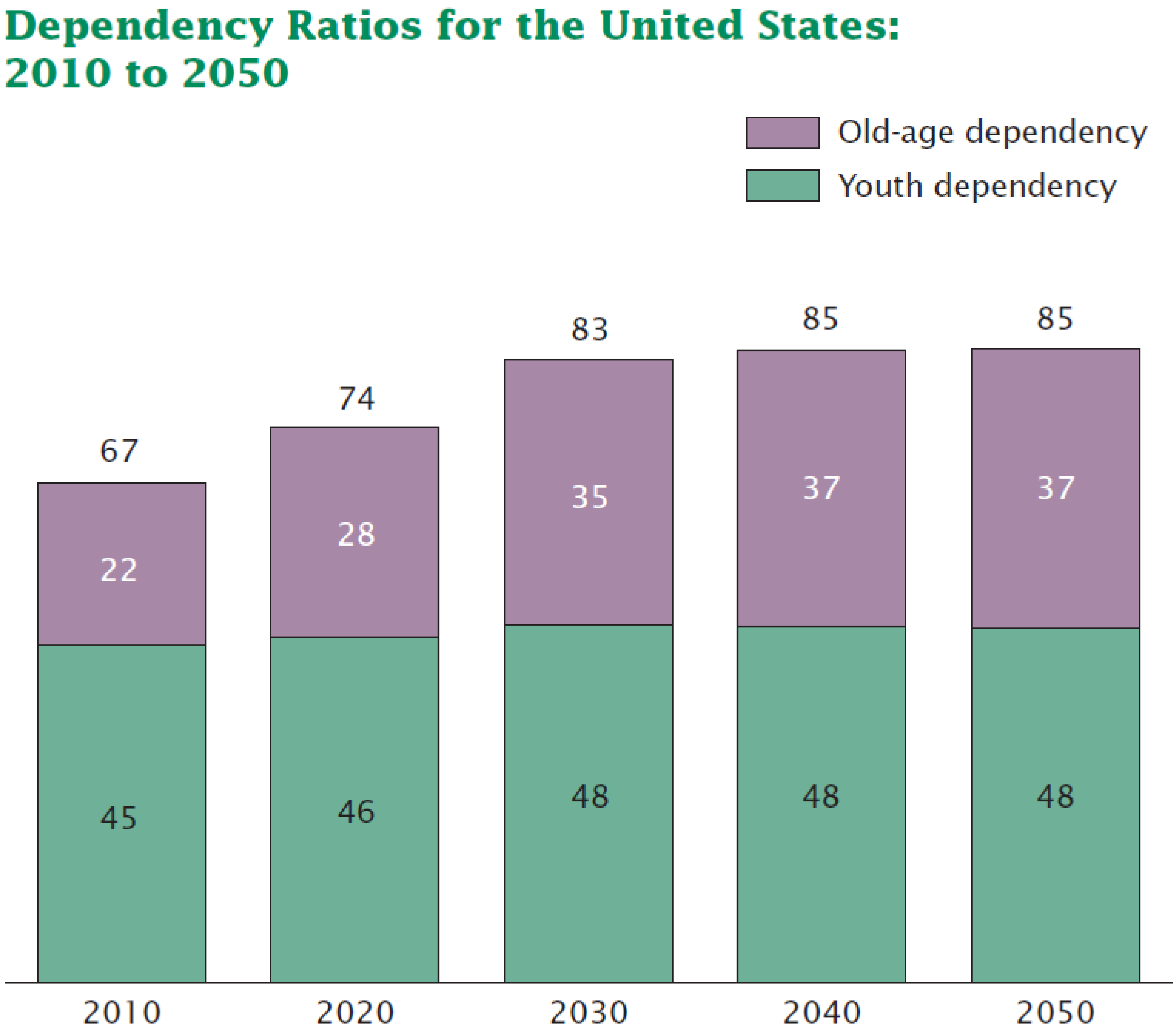


Figure 3: The Ratio of Non-Working to Working People in U.S.



Example: The Euro Area

Longer-term growth prospects in Europe appear to be worse than in the United States. My paper with Kieran McQuinn ("Europe's Long-Term Growth Prospects: With and Without Structural Reforms") reports a growth accounting analysis for the euro area and constructs longer-term growth projections. The following discussion is based on this work.

Table 1 shows that growth in output per worker in the countries that make up the euro area has gradually declined over time. In particular, TFP growth has collapsed. From 2.7 percent per year over 1970-76, TFP growth has fallen to an average of 0.2 percent per year over the period 2000-2013. Table 2 shows that weak performances for TFP growth can be seen widely across different European countries.

Europe is also on the cusp of a significant demographic change that will reduce the potential for GDP growth: See Figure 4. Population growth is slowing and total population is set to peak in before the middle of this century. The population is also ageing significantly. Indeed the total amount of people aged between 15 and 64 (i.e. the usual definition of work-age population) has peaked and is set to decline substantially over the next half century.

While Europe has many short-term macroeconomics problems due to weak aggregate demand and high levels of public and private debt, it is also the case that it faces severe challenges in relation to long-term growth. Maintaining growth rates at close to those experienced historically will likely involve reforms to raise the size of the labour force (such as raising retirement ages and immigration) and boost productivity.

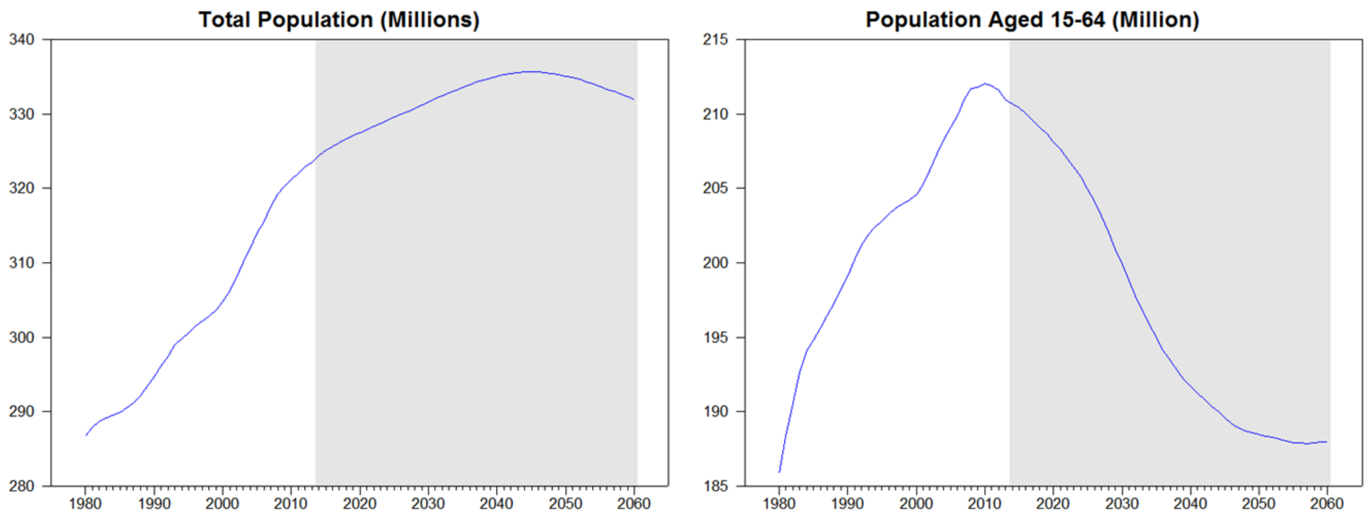
Table 1: The Euro Area's Growth Performance

	Euro Area			
Period	$\triangle y$	$\triangle a$	$\triangle k$	$\triangle l$
1970-1976	3.6	2.7	1.5	-0.5
1977-1986	2.1	1.6	0.8	-0.4
1987-1996	2.3	1.5	0.8	0.0
1997-2006	2.2	0.7	0.8	0.7
2007-2013	-0.3	-0.2	0.5	-0.6
2000-2013	0.9	0.2	0.7	0.0
2010-2013	0.1	0.3	0.3	-0.5

Table 2: Country-by-Country Growth Performance 2000-2013

	2007-2013				2000-2013			
Country	$\triangle y$	$\triangle a$	$\triangle k$	$\triangle l$	$\triangle y$	$\triangle a$	$\triangle k$	$\triangle l$
Belgium	0.4	-0.5	0.7	0.2	1.2	0.1	0.8	0.4
Germany	0.7	-0.1	0.4	0.3	1.1	0.6	0.5	0.0
France	0.1	-0.5	0.7	-0.1	1.0	0.1	0.8	0.1
Greece	-4.5	-2.6	0.6	-2.4	0.1	0.0	0.9	-0.8
Ireland	-1.3	-0.1	0.6	-1.8	2.0	0.4	1.4	0.2
Italy	-1.6	-1.0	0.4	-0.8	0.0	-0.6	0.6	0.0
Spain	-1.0	0.3	0.8	-2.1	1.3	-0.1	1.2	0.2
United Kingdom	-0.2	-1.1	0.7	0.2	1.5	0.2	0.9	0.2
Finland	-0.8	-1.0	0.7	-0.5	1.4	0.6	0.7	0.0
Lux	0.1	-2.4	1.4	1.1	2.2	-0.2	1.6	0.9
Portugal	-1.2	0.2	0.3	-1.7	0.1	0.0	0.7	-0.7
Austria	0.6	0.3	0.5	-0.2	1.5	0.8	0.6	0.1
Netherlands	-0.3	-0.8	0.6	-0.1	0.9	0.0	0.8	0.2

Figure 4: Demographic Projections for the Euro Area from Eurostat



Example: A Tale of Two Cities

Alwyn Young's 1992 paper "A Tale of Two Cities: Factor Accumulation and Technical Change in Hong Kong and Singapore" is an interesting example of a growth accounting study. He compares the growth experiences of these two small Asian economies from the early 1970s to 1990. Young explained his motivation for picking these two economies in terms of their similarities and their differences:

> In the prewar era, both economies were British colonies that served as entrepot trading ports, with little domestic manufacturing activity ... In the postwar era, however, both economies developed large export-dependent domestic manufacturing sectors. Both economies have passed through a similar set of industries, moving from textiles, to clothing, to plastics, to electronics, and then, in the 1980s, gradually moving from manufacturing into banking and financial services ... The postwar population of both was composed primarily of immigrant Chinese from Southern China ... While the Hong Kong government has emphasized a policy of laissez faire, the Singaporean government has, since the early 1960s, pursued the accumulation of physical capital via forced national saving."

Both economies were successful: Hong Kong had total growth of 147% between the early 1970s and 1990 and Singapore had growth of 154%. But Young was interested in exploring the extent to which TFP contributed to growth in these two economies. The results of his growth accounting calculations are shown on the next page. He found that Singapore's approach did not produce any TFP growth while Hong Kong's more free market approach lead to strong TFP growth with this element accounting for almost half of the growth in output per worker. One can argue this was a better outcome because Hong Kong achieved the growth without having to divert a huge part of national income towards investment rather than consumption. As we will see in the next lecture, however, TFP-based growth has an advantage over growth based on capital accumulation because it is more sustainable.

Table from Alwyn Young's 1992 Paper

Table 5 CRUDE ESTIMATE OF TOTAL FACTOR PRODUCTIVITY GROWTH

Time period	Growth of			Average capital share	Percentage contribution of		
	Output	Labor	Capital		Labor	Capital	TFP Δ
Hong Kong							
71–76	0.406	0.165	0.447	0.330	0.27	0.36	0.36
76–81	0.512	0.253	0.527	0.386	0.30	0.40	0.30
81–86	0.294	0.095	0.388	0.421	0.19	0.55	0.26
86–90	0.260	0.036	0.237	0.414	0.08	0.38	0.54
71–90	1.472	0.549	1.599	0.384	0.23	0.42	0.35
Singapore							
70–75	0.454	0.247	1.005	0.553	0.24	1.22	−0.47
75–80	0.408	0.256	0.503	0.548	0.28	0.68	0.04
80–85	0.300	0.069	0.620	0.491	0.12	1.01	−0.13
85–90	0.383	0.252	0.273	0.468	0.35	0.33	0.31
70–90	1.545	0.825	2.402	0.533	0.25	0.83	−0.08

Things to Understand from these Notes

Here's a brief summary of the things that you need to understand from these notes.

1. The sources of growth in output per worker.
2. How to derive the growth rate of output under constant returns as a function of the growth rates of capital, labour and TFP.
3. Solow's method for calculating TFP growth.
4. Evidence from the BLS on US productivity growth.
5. Evidence on growth in Europe.
6. Young's Tale of Two Cities.