Sample Questions for Part 3 of Advanced Macro Final Exam

Final Version

1. **The IS-MP-PC Model**

This question refers to the IS-MP-PC model, described by the IS curve

$$y_t = y_t^* - \alpha \left(i_t - \pi_t - r^* \right) + \epsilon_t^y$$

the monetary policy rule

$$i_t = r^* + \pi^* + \beta_\pi \left(\pi_t - \pi^* \right)$$

and the Phillips curve:

$$\pi_t = \pi_t^e + \gamma \left(y_t - y_t^* \right) + \epsilon_t^\pi$$

where y_t is output, π_t is inflation, i_t is the interest rate, π^* is the central bank's inflation target and α and γ are assumed to be positive.

(a) Show how to derive the IS-MP curve and explain what determines its position and whether it slopes upwards or downwards.

(b) Show how to derive a solution for inflation as function of expected inflation and the central bank's inflation target. Discuss the factors determining the coefficient on expected inflation.

(c) How do the dynamics of the model change when β_π moves from being greater than one to slightly less than one? What is the explanation for the change in the behaviour of the model?

2. **Asset Prices**

This question relates to the dividend-discount model.

(a) Starting from the definition of the rate of return on stocks, show that stock prices obey the following first-order difference equation

$$P_t = \frac{D_t}{(1 + r_{t+1})} + \frac{P_{t+1}}{(1 + r_{t+1})}$$

where D_t represents dividends and r_t is the rate of return on stocks.

(b) Now assume that agents have rational expectations and that expected future stock returns are constant and equal to r. Show that this difference equation implies the following representation for stock prices

$$P_t = \sum_{k=0}^{\infty} \left(\frac{1}{1 + r} \right)^{k+1} E_t D_{t+k}$$

Discuss any assumptions that you make in deriving this equation.

(c) Suppose dividend payments are expected to grow at rate g forever. What does the dividend-discount model (i.e. the model derived in part (b) above) imply for the current level of stock prices?

(d) Now suppose that instead of growing at rate g each period, dividends can be expressed as the sum of two components—a trend component that grows at rate g each period and a cyclical component u_t that follows an AR(1) process $u_t = \rho u_{t-1} + \epsilon_t$ where $0 < \rho < 1$ and ϵ_t is a zero-mean noise process. What does the dividend-discount model imply for stock prices in this case?

3. **Rational Expectations and Consumption**

(a) Starting from the household budget constraint

$$A_{t+1} = (1+r)(A_t + Y_t - C_t)$$

where A_t is the value of household assets, Y_t is after-tax labour income, and C_t is consumption expenditures, derive the intertemporal budget constraint

$$\sum_{k=0}^{\infty} \frac{E_t C_{t+k}}{(1+r)^k} = A_t + \sum_{k=0}^{\infty} \frac{E_t Y_{t+k}}{(1+r)^k}$$

(b) State the assumptions required to derive the random walk hypothesis that expected changes in consumption should be unpredictable, i.e. that $C_t = E_t C_{t+1}$ and sketch out how this result is derived.

(c) Show that the random walk hypothesis implies that current consumption depends in a specific fashion on current assets and on current and expected future labour income.

(d) Now suppose that consumers expect after-tax labour income to grow at rate g forever (where $g < r$). Maintaining the random walk assumption, what does this imply for the relationship between consumption, labour income and assets?

(e) Could econometric estimates of the relationship derived in (d) be used to assess the effects of a temporary tax cut? Explain your answer.

4. **Uncovered Interest Parity**

European bonds have an interest rate of i_t^E and US bonds have an interest rate of i_t^{US}. Let e_t represent the amount of dollars that can be obtained with one Euro.

(a) Under what conditions would a risk-neutral investor be willing to invest in either or both US and European bonds? Why might we expect this so-called Uncovered Interest Parity (UIP) relationship to hold?

(b) What are the implications of the UIP relationship for a country's choice of monetary policy and exchange rate regimes?

(c) Show how to re-write the UIP condition as a first-order stochastic difference equation.

(d) Assuming investors have rational expectations, derive an expression relating the value of e_t to expected future values of i_t^E and i_t^{US}. Show how this expression can be used to explain the factors that influence changes in exchange rates.

(e) How does the rational expectations version of the UIP model fit with the empirical evidence on exchange rate fluctuations?

5. **Growth Accounting and the Solow Model**

(a) Suppose aggregate output is produced using a Cobb-Douglas production function

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Derive an expression linking the growth rate of output with the growth rate of capital and labour inputs as well as total factor productivity.

(b) In measuring TFP growth, we need to have an estimate of α. Show how you can use information on the composition of national income to estimate α under the assumption that firms operate under perfect competition.

(c) Suppose output grows at a constant rate G_Y. Derive an expression for the growth rate of the capital-output ratio in a Solow model economy with a savings rate of s and a depreciation rate of δ.

(d) Using the result obtained in (c), derive a formula for the growth rate of output in the Solow model along a steady growth path.

6. **The Malthusian Model**

This question relates to the Malthusian model in which population changes over time according to

$$N_t = N_{t-1} + B_{t-1} - D_{t-1}$$

where births are determined by

$$B_t = bN_t$$

where deaths are determined by

$$D_t = (d_0 - d_1 Y_t)N_t$$

and where Y_t is real income per person as determined by

$$Y_t = a_0 - a_1 N_t$$

(a) Show how to illustrate the model's properties by deriving the birth rate and death rate schedules and illustrating them on a graph with income per person and population on the axes.

(b) Illustrate how an increase in the death rate affects population and income per person in the model.

(c) Derive the long-run equilibrium levels of population and income per person in the model.

(d) Show how to derive the speed at which this economy tend to converge towards its long-run equilibrium.

7. **Population and Resources**

This question relates to the "Easter Island" model of Brander and Taylor. In this model, population changes according to

$$\frac{dN_t}{dt} = -dN_t + \theta H_t$$

where d is a parameter determining the death rate (not to be confused with the "d" in the derivatives in the model), H_t is the harvest, which is determined by

$$\frac{H_t}{N_t} = \gamma S_t$$

and S_t is a stock of resources that changes according to

$$\frac{dS_t}{dt} = rS_t (1 - S_t) - H_t$$

(a) Briefly explain each of the model's equations.

(b) Show how to derive a graph that shows the change in population as a function of the current level of population and resources.

(c) Show how to derive a graph that shows the change in resources as a function of the current level of population and resources.

(d) What is the equilibrium level of population and resources in this economy? (By equilibrium, we mean the levels of population and resources that, if obtained, see both variables being unchanged.)

(e) Assuming this model economy starts out with high resources and a low population, what are the subsequent dynamics of population and resources?