Advanced Macroeconomics
11. Before Growth: The Malthusian Model

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Before Economic Growth

- We have been studying models of economies that grow steadily over time.
- However, prior to around the year 1800, there is very little evidence of steady growth in income levels.
- The chart on the next page is taken from *A Farewell to Alms* by economic historian Greg Clark.
- It summarises world economic history as a long period in which living standards fluctuated over time showing no growth trend before the Industrial Revolution lead to steady growth over time.
- There is some of controversy over Clark's particular interpretation of the evidence but all agree that the average rate of economic growth was very low before 1800.
- In addition, global population growth was extremely slow until 1800 and then increased to much higher rates.

World Economic History (from Greg Clark's book)

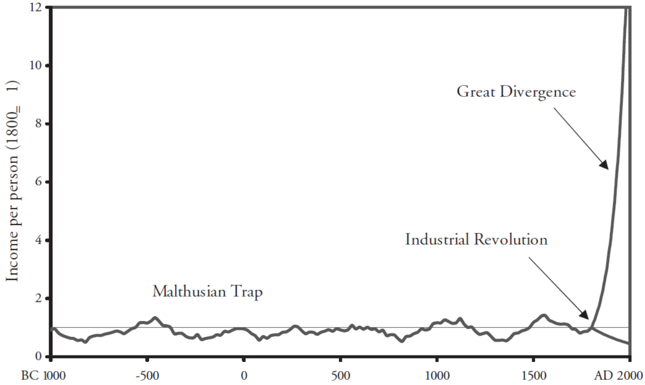
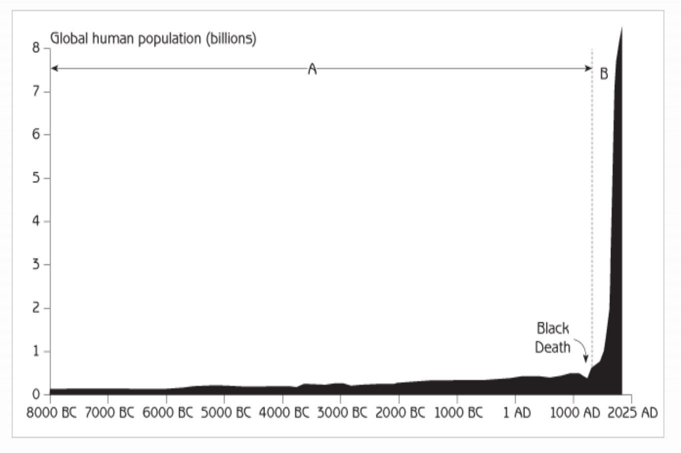


Figure 1.1 World economic history in one picture. Incomes rose sharply in many countries after 1800 but declined in others.

The History of Global Population



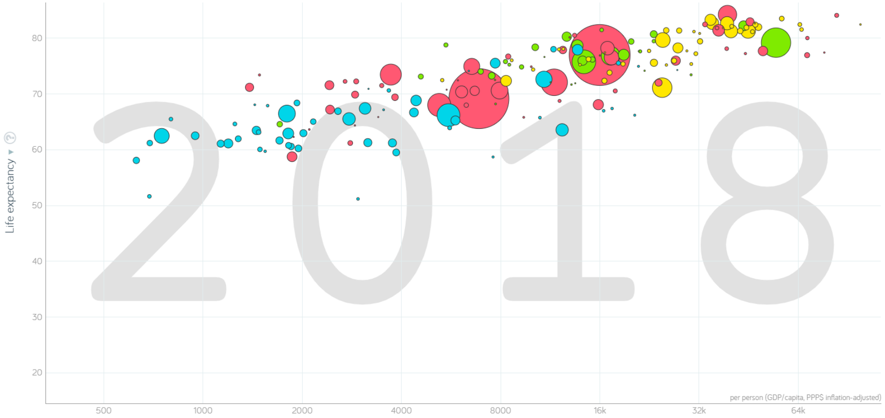
A Model With Slow Technological Progress

- What explains these patterns?
- Our previous models would suggest the pace of technological progress must have been slower before the Industrial Revolution and this is true.
- But cumulatively, there was a lot of technological progress from ancient times to 1800. Based on our previous models, you might have expected this to translate into growth in average living standards over time but the evidence suggests such progress was limited.
- We will now discuss the Malthusian model, which explains why the world works very differently when rates of technological progress are slow and why there was limited growth in living standards before 1800.
- The Malthusian model has two key elements:
  1. A positive relationship between income levels and population growth.
  2. A negative relationship between income levels and the size of population
- Let's start with the first relationship.

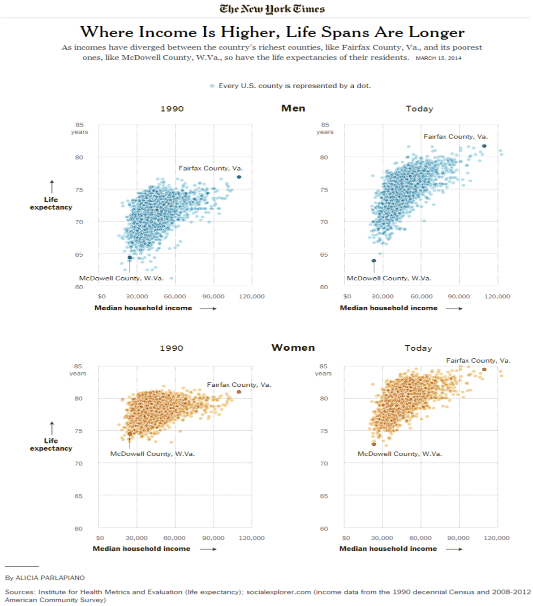
Death Rates and Income Levels

- By definition, population growth increases with birth rates and falls with death rates.
- Death rates, in turn, are what determines life expectancy.
- Throughout history, there has been a strong relationship between a country's average level of income per capita and its average life expectancy.
- This relationship still holds strongly today. See the figure on the next page.
- This pattern is mainly due to variations in rates of child mortality. See the figure two pages on.
- This relationship between income levels and the rate of death among the population will be a key element of the version of the Malthusian model that we will cover.

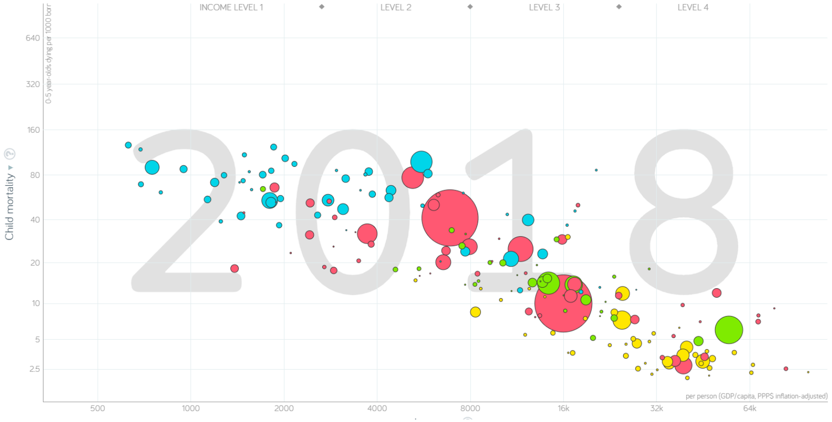
Life Expectancy and GDP Per Capita Around the World



Life Expectancy and Income Levels: U.S. Counties



Child Mortality and GDP Per Capita Around the World



Population and Income Levels

- Consider an economy with aggregate Cobb-Douglas production function

$$Y_t = AK^{\alpha} L_t^{1-\alpha}$$

- Assume capital and technology are fixed.
- Firms maximise

$$\pi = pAK^{\alpha} L_t^{1-\alpha} - wL - rK$$

- The first-order condition for labour is

$$pAK^{\alpha} L^{-\alpha} - w = 0 \Rightarrow \frac{w}{p} = A\left(\frac{K}{L}\right)^{\alpha}$$

- Assume a constant fraction θ of the population is working $L = \theta N$, we get

$$\frac{w}{p} = A\left(\frac{K}{\theta N}\right)^{\alpha}$$

- The higher the population, the lower will be the real wage. This is because of diminishing marginal returns to labour and the fact that workers are being paid their marginal wage product.

Malthus (1798)

- Malthus didn't write about technology or diminishing returns. He was more concerned about the pressure on food supplies of higher population: "*An increase of population without a proportional increase of food will evidently have the same effect in lowering the value of each man's patent. The food must necessarily be distributed in smaller quantities, and consequently a day's labour will purchase a smaller quantity of provisions.*
- He also wrote about how higher living standards would raise population. He discussed two mechanisms: The effect on death rates that we have already identified ("*the actual distresses of some of the lower classes, by which they are disabled from giving the proper food and attention to their children, act as a positive check to the natural increase of population.*" and an additional effect on birth rates (which Malthus called "the preventative check").
- In practice, as discussed in Greg Clark's book on the Malthusian model, the evidence for a link between living standards and birth rates prior to the Industrial Revolution is fairly weak and I will assume a constant birth rate in the model.

The Model

- Population equals last period's population plus last period's level of births minus deaths.

$$N_t = N_{t-1} + B_{t-1} - D_{t-1}$$

- Births are a constant fraction of the population

$$\frac{B_t}{N_t} = b$$

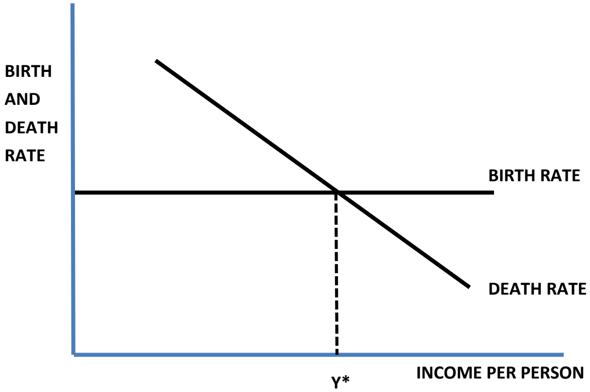
- While deaths are a decreasing function of real income per person

$$\frac{D_t}{N_t} = d_0 - d_1 Y_t$$

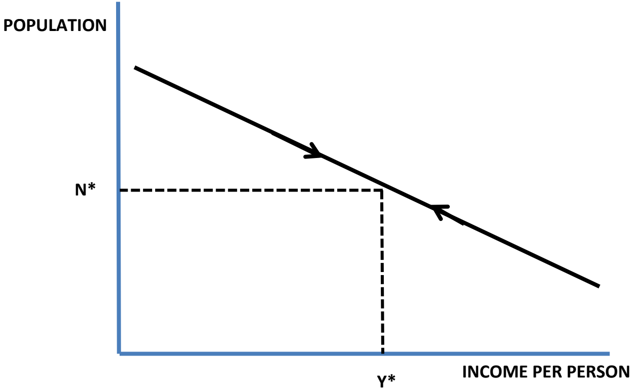
- Finally, real income per person is a negative function of the population size:

$$Y_t = a_0 - a_1 N_t$$

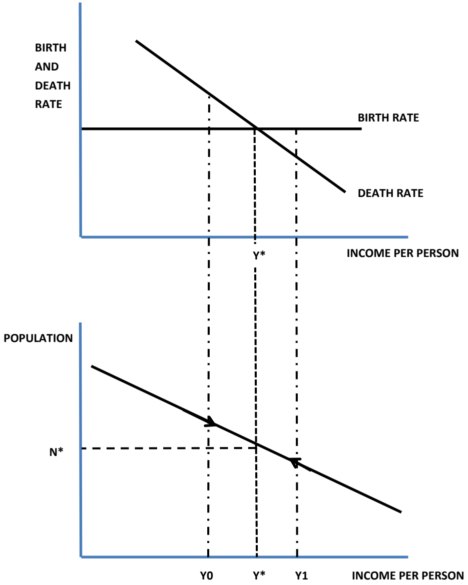
Malthus Model: Birth and Death Rate Schedules



Malthus Model: Income-Population Schedule



The Full Model



Calculating the Long-Run Equilbrium

- We can figure N^* and Y^* out algebraically as follows. Combining the birth and death schedules with the equation for population change gives

$$\frac{N_t - N_{t-1}}{N_{t-1}} = b - d_0 + d_1 Y_{t-1}$$

- Inserting the dependence of income levels on wages, we get

$$\frac{N_t - N_{t-1}}{N_{t-1}} = b - d_0 + d_1 a_0 - d_1 a_1 N_{t-1}$$

- Equilibrium population level determined by

$$b - d_0 + d_1 a_0 - d_1 a_1 N^* = 0 \Rightarrow N^* = \frac{b - d_0 + d_1 a_0}{d_1 a_1}$$

- The long-run equilibrium level of real income per person can be derived as the income level that gives a growth rate of population of zero

$$\frac{N_t - N_{t-1}}{N_{t-1}} = b - d_0 + d_1 Y^* = 0 \Rightarrow Y^* = \frac{d_0 - b}{d_1}$$

What Matters in Long-Run Equilibrium?

- Note what matters for the long-run equilibrium level of real income per person and also what doesn't.

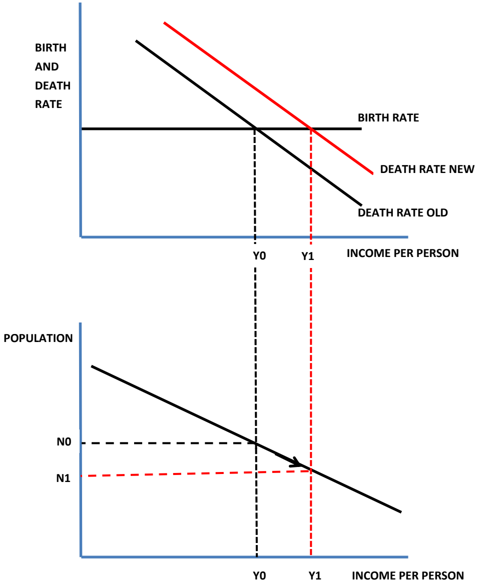
$$Y^* = \frac{d_0 - b}{d_1}$$

- Income per persons depends positively on the two parameters that raise the death rate (d_0 and d_1) and negatively on the birth rate b. It does not depend at all on the parameters of the real wage equation a_0 and a_1.
- Note that, in this model, an increase in technological efficiency means an increase in a_0 because it raises the amount that workers can earn at any given level of population.
- Note now that all the elements of the model influence long-run population:

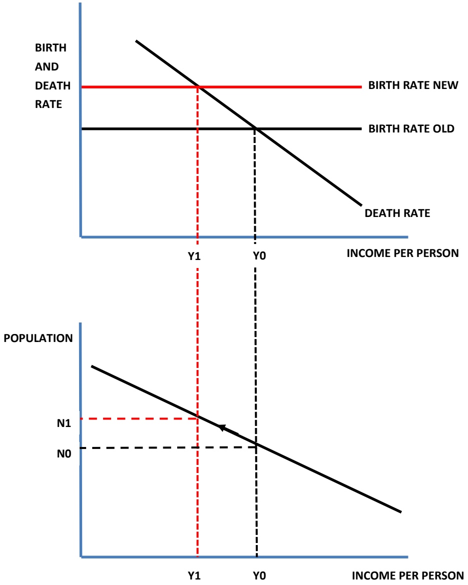
$$N^* = \frac{b - d_0 + d_1 a_0}{d_1 a_1}$$

- Higher birth rates and lower death rates raise population. An increases in technological efficiency acts via (a_0) to raise the population. An increase in the sensitivity of wages to population (a_1) reduces population.

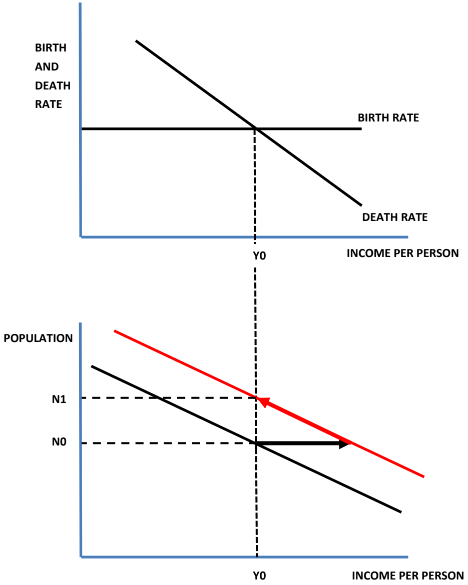
A Shift in the Death Rate Schedule



A Shift in the Birth Rate Schedule



An Increase in Technological Efficiency



Convergence Speed

- Remembering the formula for the growth rate of population

$$\frac{N_t - N_{t-1}}{N_{t-1}} = b - d_0 + d_1 a_0 - d_1 a_1 N_{t-1}$$

- And that equilibrium population is

$$N^* = \frac{b - d_0 + d_1 a_0}{d_1 a_1}$$

- We can re-write the dynamics as

$$\frac{N_t - N_{t-1}}{N_{t-1}} = (d_1 a_1)(N^* - N_{t-1})$$

- The growth rate of population is determined by how far population is from its equilibrium level, with the speed of adjustment to this equilibrium, $d_1 a_1$, determined by the sensitivity of income levels to population and the sensitivity of the death rate to income levels.

Malthus versus Solow

- There is an interesting contrast here between what happens when there is technological progress in the Solow model and when technology improves in the Malthusian model.
- The difference relates to the assumption in the Solow model that there is a consistent and non-trivial pace of technology increase.
- In the Malthusian model, the instantaneous effect of an increase in efficiency is an improvement of living standards. But this is offset over time by population increases if there aren't any further increases in technology.
- In the Solow model, technology keeps increasing and keeps pushing up incomes every period, so the population can steadily increase without pushing income levels down.
- Greg Clark argues that while, cumulatively, there was a large increase in technology from ancient times to 1800, the pace of this increase was never fast enough to prevent population growth eroding its effects on living standards, so that prior to the Industrial Revolution, improvements in productive efficiency only translated into higher population.

Malthus on the Poor Laws

- The Malthusian model is one in which our usual understanding of what is good and what is bad is turned on its head.
- Things that we think are good, such as people living longer, turn out to be bad for average living standards, and things that we think are bad, like plagues and diseases, have a positive effect on those who survive.
- This non-intuitive worldview translated into Malthus's own policy recommendations. For example, he argued strongly against "poor laws" that provided assistance to the poor.
- Over the years, Malthus has often been criticised for being overly-pessimistic about the fate of mankind and for opposing socially-progressive policies.
- However, until the time that he wrote his essay (1798) his version of how the world worked actually described the economy remarkably well. It was only after his book was written that technological progress became fast enough to render his analysis less relevant.

Things to Understand From This Topic

1. Facts about income levels and population before and after 1800.
2. Facts about life expectancy and child mortality around the world.
3. The elements that make up the Malthusian model.
4. The properties of the long-run equilibrium of the Malthusian model.
5. How the Malthusian economy responded to shocks.
6. Why the Solow and Malthusian models deliver such different outcomes.
7. Why Malthus opposed helping the poor.