

Changes in Bank Leverage: Evidence from US Bank Holding Companies

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March 2015

Abstract

This paper examines how banks respond to shocks to their equity. If banks react to equity shocks by more than proportionately adjusting liabilities, then this will tend to generate a positive correlation between asset growth and leverage growth. However, we show that in the presence of changes in liabilities that are uncorrelated with shocks to equity, a positive correlation of this sort can occur without banks adjusting to equity shocks by more than proportionately adjusting liabilities. The paper uses data from US bank holding companies to estimate an empirical model of bank balance sheet adjustment. We identify shocks to equity as well as orthogonal shocks to bank liabilities and show that both equity and liabilities tend to adjust to move leverage towards target ratios. We also show that banks allow leverage ratios to fall in response to positive equity shocks, though this pattern is weaker for large banks, which are more active in adjusting liabilities after these shocks. We show how this explains why large banks have lower correlations between asset growth and leverage growth.

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1 Introduction

Banks are leveraged institutions and shocks to their equity capital can have a large effect on the size of their balance sheets if banks are concerned about their leverage ratios. As emphasised by Geanakoplos (2010), Adrian and Shin (2010) and others, endogenous management of leverage ratios by financial institutions can potentially act as an important propagation mechanism for business cycles: Positive macroeconomic shocks boosting bank equity can lead to balance sheet expansion that fuels asset price increases and further boosts bank equity, a cycle that can also work in reverse during downturns. That this mechanism played an important role in the 2008-2009 recession was emphasized in a number of papers documenting the links between the financial crisis and the wider economy.¹

How an economic unit's leverage changes after a shock to its equity depends on how it adjusts liabilities to this shock. If positive shocks to equity are accompanied by less than proportional increases in liabilities, then there will be an increase in assets accompanied by a reduction in leverage, while if they are accompanied by a more than proportional increase in liabilities, then there will be an increase in assets accompanied by an increase in leverage. Adrian and Shin (2010) present evidence that the correlation between household asset growth and leverage growth has been negative while the same correlation for investment banks is positive. Adrian and Shin (2011) also report a positive correlation for U.S. bank holding companies. They interpret these positive correlations as evidence that U.S. banks engage in active balance sheet management so that they react to changes in equity by more than proportionately raising liabilities

This paper further explores how banks respond to equity shocks by making two contributions, one methodological and one empirical. Our methodological contribution is to provide a framework for describing the factors that determine the relationship between asset growth and changes in leverage. Our framework includes the possibility that shocks to a bank's equity have a direct effect on its liabilities. Importantly, it also has two other features: Shocks to bank liabilities that are unrelated to shocks to equity and adjustments to liabilities and equity to bring about gradual convergence towards a target leverage ratio.

We use this framework to show that there may be multiple explanations for a particular correlation between asset growth and leverage growth. In particular, we show that while positive correlations between asset growth and leverage growth could occur because banks

¹For example, Greenlaw, Hatzius, Kashyap and Shin (2008) and Hatzius (2008) both emphasize this mechanism.

choose to react to changes in equity by more than proportionately raising liabilities, they can also occur because shocks to liabilities unrelated to equity shocks are an important source of bank balance sheet dynamics.

We show that once liability shocks of this type exist, then there is a U-shaped relationship between the short-term reaction of bank liabilities to equity shocks and the correlation between leverage growth and asset growth: As the contemporaneous response of liabilities to equity shocks increases away from zero, the correlation between asset growth and leverage growth falls and then starts to increase again.

Our empirical contribution applies our framework to a large panel dataset for U.S. bank holding companies. We model bank equity and liabilities jointly using a panel Vector Error Correction Mechanism (VECM) framework which allows for adjustment of both equity and liabilities in response to the deviation of the leverage ratio from target levels. We use a recursive identification scheme to identify equity and liability shocks, i.e. our “liability shock” comes second in the ordering so it is uncorrelated with shocks to equity. We find that the two new elements introduced in our framework are empirically important. Shocks to bank liabilities that are unrelated to shocks to equity play an important role in affecting the dynamics of bank balance sheets. In addition, we find banks gradually adjust both liabilities and equity over time to move towards target leverage ratios.

We show how our approach explains the pattern of correlations between leverage growth and asset growth observed for various types of banks. We find the correlation between asset growth and leverage growth to be positive across a wide range of different types of banks, even though none of these samples exhibit liabilities responding to equity shocks with an elasticity greater than one. We also report some interesting differences between banks in how they manage their balance sheets. We provide evidence that large banks engage in more active balance sheet management in response to shocks. However, our estimates of the reaction of bank liabilities to equity are all in the region of the downward slope of the U-shaped relationship just mentioned, i.e. larger banks that manage balance sheets more actively have a less positive correlation between asset growth and leverage growth.

The paper is organized as follows: Section 2 briefly reviews the evidence on correlations between changes in bank leverage and asset growth. Section 3 presents our methodological framework describing the factors that determine the correlation. Section 4 discusses our data and describes our empirical model. Section 5 reports the empirical results and Section 6 concludes.

2 Evidence on Leverage and Asset Growth

Adrian and Shin (2010) described different possible ways that economic units can adjust their balance sheets over time, presenting aggregate evidence on the correlation between asset growth and leverage growth for different sectors of the US economy.

Figures 1 and 2 use data from the Flow of Funds accounts to replicate Adrian and Shin’s evidence for the household sector and for broker-dealer financial institutions (investment banks). Figure 1 shows a strong negative correlation between leverage growth and asset growth for the household sector, consistent with households reacting to rising housing and financial asset prices without taking on extra liabilities to offset the impact on their net equity position. In contrast, Figure 2 shows that the broker-dealer sector exhibits a strong positive correlation between asset growth and leverage growth. Adrian and Shin interpreted this correlation as implying that broker-dealers respond to increases in equity by taking on proportionately larger increases in liabilities.² This interpretation influenced the calculations of Greenlaw, Hatzius, Kashyap and Shin (2008) on the balance sheet effects of mortgage-related losses at U.S. banks.

Most bank credit in the US is provided by bank holding companies (BHCs). A BHC is any company that controls one or more commercial banks. In this paper, we use quarterly data from the Consolidated Financial Statements for individual BHCs in the United States from 1986:Q3 to 2013:Q4. We describe the dataset in detail later in the paper. Figure 3 shows asset growth and leverage growth for each BHC-quarter observation in our dataset. A positive correlation is clearly evident for the sample as a whole. Adrian and Shin (2011) report a similar result from an exercise that calculates aggregate correlations from similar source data. Damar, Meh and Terajima (2013) also reported positive leverage growth-asset growth correlations for Canadian banks.

²Their paper describes the positive correlation between asset growth and leverage growth as “procyclical leverage” and they describe this situation as follows: “The perverse nature of the reactions to price changes are even stronger when the leverage of the financial intermediary is procyclical. When the securities price goes up, the upward adjustment of leverage entails purchases of securities that are even larger than that for the case of constant leverage.”

Figure 1: Leverage Growth and Asset Growth for US Households

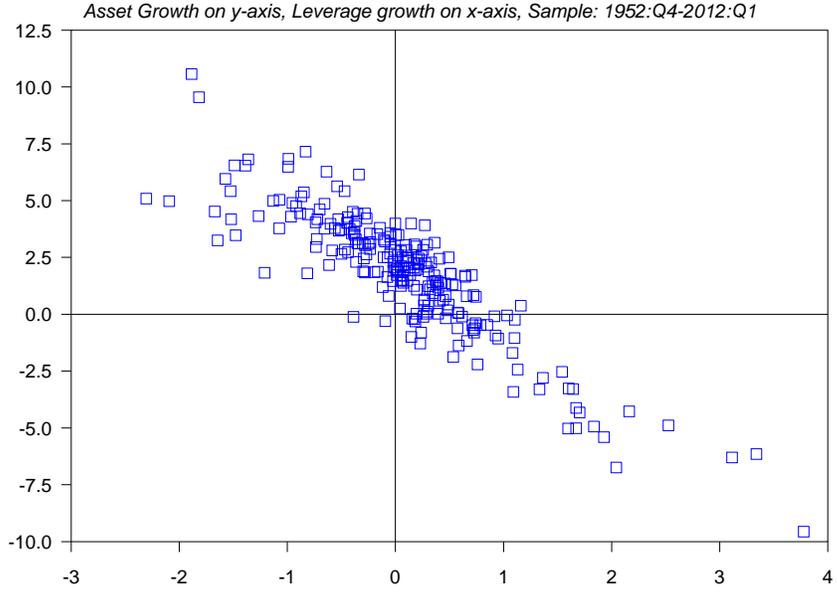


Figure 2: Leverage Growth and Asset Growth for US Broker Dealers

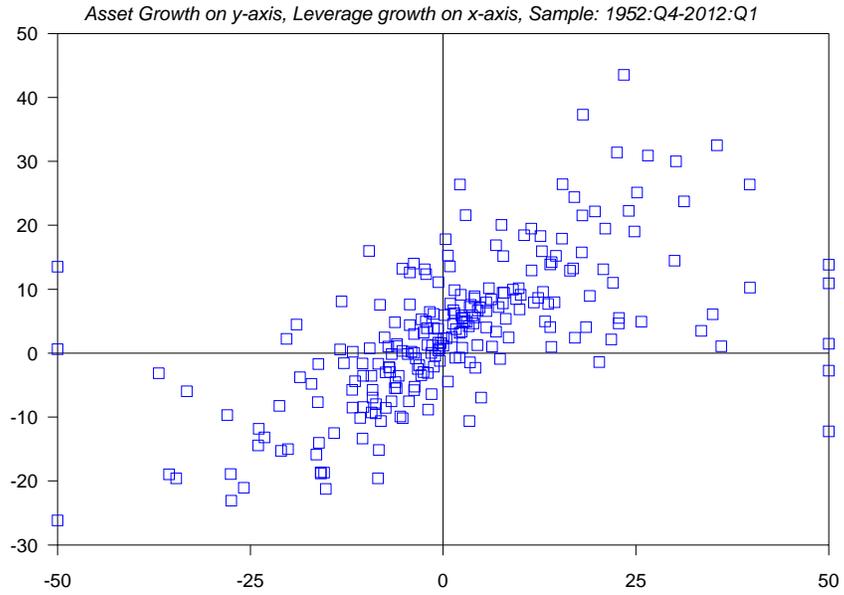
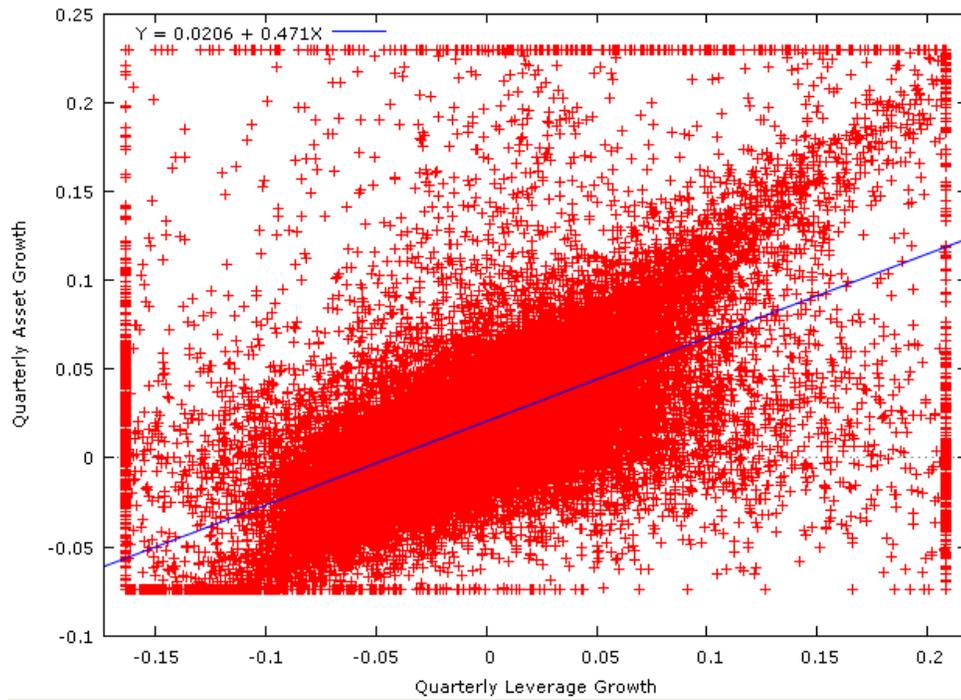


Figure 3: BHC-Level Data on Asset Growth and Leverage Growth



3 The Asset Growth-Leverage Growth Correlation

So what kind of behavior drives the relationship between changes in bank assets and changes in leverage? In this section, we first present a simple result that describes the determinants of this relationship. We then describe a number of different examples of how the correlation between asset growth and leverage growth behaves depending on how banks adjust their liabilities and equity capital over time.

3.1 A Useful Formula

Defining L_t as a bank's liabilities, A_t as its assets and $E_t = A_t - L_t$ as its equity capital, the leverage ratio can be expressed as

$$LEV_t = \frac{E_t + L_t}{E_t} = 1 + \frac{L_t}{E_t} \quad (1)$$

So the leverage ratio is driven by the ratio of liabilities to equity, which we will denote as

$$LEV_t^a = \frac{L_t}{E_t} \quad (2)$$

Here, we will calculate the covariance between the growth rate of this ratio and asset growth, as this is identical to the covariance between leverage growth and asset growth.

To obtain a simple analytical formula describing the covariance of asset growth and leverage growth, we approximate the log of total assets as

$$\log A_t = \theta \log L_t + (1 - \theta) \log E_t \quad (3)$$

where θ is the average ratio of liabilities to assets. Asset growth is thus a weighted average of liability growth and equity growth.

$$\Delta \log A_t = \theta \Delta \log L_t + (1 - \theta) \Delta \log E_t \quad (4)$$

From this, we can calculate the covariance between asset growth and leverage growth as

$$\begin{aligned} \text{Cov}(\Delta \log A_t, \Delta \log LEV_t^a) &= \text{Cov}(\theta \Delta \log L_t + (1 - \theta) \Delta \log E_t, \Delta \log L_t - \Delta \log E_t) \\ &= \theta \text{Var}(\Delta \log L_t) - (1 - \theta) \text{Var}(\Delta \log E_t) \\ &\quad + (1 - 2\theta) \text{Cov}(\Delta \log L_t, \Delta \log E_t) \end{aligned} \quad (5)$$

Given this formula, we can consider a number of different cases depending on how liabilities and equity evolve over time.

3.2 Two Extreme Cases

Here we consider two different cases for how bank liabilities and equity change over time.

Liability Response to Equity: Consider the following simple rule of thumb for bank liabilities:

$$\Delta \log L_t = \mu \Delta \log E_t \quad (6)$$

Inserting this formula into equation (5), the covariance between $\Delta \log A_t$ and $\Delta \log LEV_t^a$ becomes

$$\text{Cov}(\Delta \log A_t, \Delta \log LEV_t^a) = (1 + \theta\mu - \theta)(\mu - 1) \text{Var}(\Delta \log E_t) \quad (7)$$

The first term on the right-hand-side (i.e. $1 + \theta\mu - \theta$) will be positive if μ is non-negative, which is likely to be the case for financial institutions. In this case, the sign of the correlation between asset growth and leverage growth will depend on whether μ is greater than, equal to or less than one.

If $0 < \mu < 1$, so that an increase in equity produces a less-than-proportional increase in liabilities, then leverage growth will be negatively correlated with asset growth. This is the type of behavior that Adrian and Shin (2010) attribute to households. A value of $\mu = 1$ would imply a zero correlation between asset growth and leverage growth because leverage would be constant in this case. Finally, a value of $\mu > 1$, so that liabilities adjusted more than proportionally in response to a change in equity, will produce a positive correlation between asset growth and leverage growth. This is the type of behavior that Adrian and Shin (2010) attribute to broker-dealers.

Equation (7) provides one way to interpret the correlation between asset growth and leverage growth. However, these results rely on the assumption of a simple link between liability growth and equity growth, as described by equation (6). Moving beyond this assumption, one could observe positive, negative or zero correlations without being able to make direct inferences about the contemporaneous response of liabilities to changes in equity.

Liabilities Independent of Equity: Consider the case in which liabilities evolve completely independently from equity so that $\text{Cov}(\Delta \log L_t, \Delta \log E_t) = 0$. In this case, the covariance between asset growth and leverage growth simplifies to

$$\text{Cov}(\Delta \log A_t, \Delta \log LEV_t^a) = \theta \text{Var}(\Delta \log L_t) - (1 - \theta) \text{Var}(\Delta \log E_t) \quad (8)$$

The covariance is determined by the variance of liability growth, the variance of equity growth and the share of each in total assets. So, for example, if the variance of equity growth and liability growth are equal and they have an equal share in funding ($\theta = 0.5$), then the correlation between leverage growth and asset growth will be zero.

In reality, of course, bank liabilities are typically multiple times bigger than equity and, as we will discuss below, the variance of their growth rates are relatively similar. For this reason, we would expect to observe $\theta \text{Var}(\Delta \log L_t) > (1 - \theta) \text{Var}(\Delta \log E_t)$ implying a positive correlation in this case. Put more simply, a bank that tends to expand or contract mainly by adding or subtracting liabilities will display a positive correlation between asset growth and leverage growth. So, a positive correlation also doesn't necessarily imply a conscious pattern of reacting to equity shocks by raising leverage. And, indeed, a zero correlation isn't necessarily a sign that liabilities are moving proportionately with equity.

3.3 A More General Model

The two examples we just considered are both extreme cases. The first example views liabilities moving mechanically in response to changes in equity with no other sources of variation. This is unlikely to be a good model of how bank liabilities change over time as we are likely to see movements in bank liabilities that are not simply a response to changes in equity: For example, banks may choose add or repay liabilities independently from equity-related developments or liabilities may move up or down depending on the amount of customer money being deposited. However, the second example, in which liabilities evolve over time without any reference to the bank's equity is also a highly unrealistic case.

Indeed, a serious problem with both of these examples is that, with the exception of the knife-edge case of $\mu = 1$ in the first example (when the leverage ratio is constant), there is nothing in either example to prevent bank leverage ratios wandering off towards arbitrarily high or low levels. Even in the absence of capital adequacy rules, such outcomes are extremely unlikely. Moreover, the existing literature on bank capital has provided some evidence for the idea that banks adjust leverage ratios over time towards target levels. For example, Hancock and Wilcox (1993, 1994) presented evidence of partial adjustment towards target capital ratios and presented evidence of the effect on lending of a gap between actual and target capital, a result also reported more recently by Berrospide and Edge (2010). Berger et al (2008) also provide evidence that banks make adjustments to move themselves towards target capital ratios. These adjustments can be made by adding or

subtracting liabilities but they can also be made by adjusting bank equity. While changes in bank equity may be mainly driven by asset returns, bad loan provisions and other factors that are mainly outside a bank’s control, equity can be consciously adjusted via dividend payments, share repurchases or new equity issuance.

Taken together, these considerations suggest we should consider a model in which liabilities can react to changes in equity but where there are also other sources of variation in liabilities and both equity and liabilities tend to adjust over time to move the bank towards a target leverage ratio. The simplest model with each of these features is an error-correction model of the following form:

$$\Delta \log E_t = g + \lambda_E (\log L_{t-1} - \log E_{t-1} - \theta) + \epsilon_t^E \quad (9)$$

$$\Delta \log L_t = g + \mu \Delta \log E_t - \lambda_L (\log L_{t-1} - \log E_{t-1} - \theta) + \epsilon_t^L \quad (10)$$

where g is a common trend growth rate of both equity and liabilities. When the error correction parameters, λ_L or λ_E and the parameter μ are set to zero, log-equity and log-liabilities follow random walks with drifts with the same trend growth rate. When the error correction parameters, λ_L or λ_E are positive, the model tends to adjust towards a ratio of liabilities to equity of $\exp(\theta)$, implying a target leverage ratio of $\exp(\theta) + 1$.

In an appendix, we derive analytical results for the true population regression coefficient generated by this model from a regression of leverage growth on asset growth, again using the log-linear approximation of equation (3).³ The derivations assume that ϵ_t^E and ϵ_t^L are uncorrelated iid shock terms with $\text{Var}(\epsilon_t^E) = \sigma_E^2$ and $\text{Var}(\epsilon_t^L) = \sigma_L^2$. Because the model has quite a few “moving parts” (different shock variances, error-correction speeds and the coefficient for how liabilities react to equity shocks) the formula is long and complicated and we don’t repeat it here. Instead, we provide some charts to illustrate how this regression coefficient changes as we vary the parameters of the model and discuss the intuition for these results.

We start by considering the role of the parameter μ , which describes the contemporaneous response of liabilities to changes in equity. Figure 4 shows the true regression coefficient for various values of μ for a case in which $\theta = 0.9$ (liabilities provide ninety percent of funding), the variance of the equity and liability shocks are equal (the coefficient only depends on the ratio of the variances, not their levels) and the error-correction coefficients are $\lambda_E = \lambda_L = 0.04$. For this configuration of parameters, the true coefficient from a

³Model simulations confirm that the calculations based on a log-linear approximation are highly accurate.

regression of leverage growth on asset growth is positive for all values of μ . The coefficient starts off at a high value at $\mu = 0$, then reaches a minimum just below $\mu = 1$ and increases after this point.

These results can be explained as follows. When $\mu = 0$, liability shocks dominate because (in this example) liabilities account for 90 percent of funding. This generates a strong positive regression coefficient because increases in assets usually stem from increases in liabilities that generate higher leverage. When μ increases above zero, then positive equity shocks become more correlated with asset growth because they lead to the bank also adding more liabilities. As long as $\mu < 1$ then asset growth driven by equity shocks (and consequent addition of liabilities) coincides with lower leverage (because liabilities have grown by less than equity) so the correlation between asset growth and leverage becomes less positive. However, as μ increases, the reduction in leverage associated with equity shocks gets smaller and smaller, so at some point, higher values of μ become associated with a more positive correlation between asset growth and leverage growth.

Figure 4 is based on the assumption that shocks to equity and liabilities have identical variance ($\sigma_E^2 = \sigma_L^2$) an assumption that fits well with the empirical evidence we present later. However, as would be expected from our previous discussion, the relationship between asset growth and leverage growth is very sensitive to the relative size of these shocks.

Figure 5 shows the relationship between the regression coefficient and the value of μ for a number of different values of the ratio of the variance of equity shocks to the variance of liability shocks. For all values of this ratio, there is a U-shaped relationship between the regression coefficient and the μ parameter. And for each value of μ , the higher the variance of equity shocks, the lower the value of the coefficient in a regression of leverage growth on asset growth. However, we find that the variance of equity shocks needs to be at least three times the variance of liability shocks before negative values of the regression coefficient can be seen for any value of μ . Similar results apply for other realistic values of the share of liabilities in funding. These calculations show that we should generally expect the correlation between leverage growth and asset growth to be positive and this positive correlation need not imply a conscious pattern in which banks react to positive equity shocks by raising leverage (i.e. that $\mu > 1$).

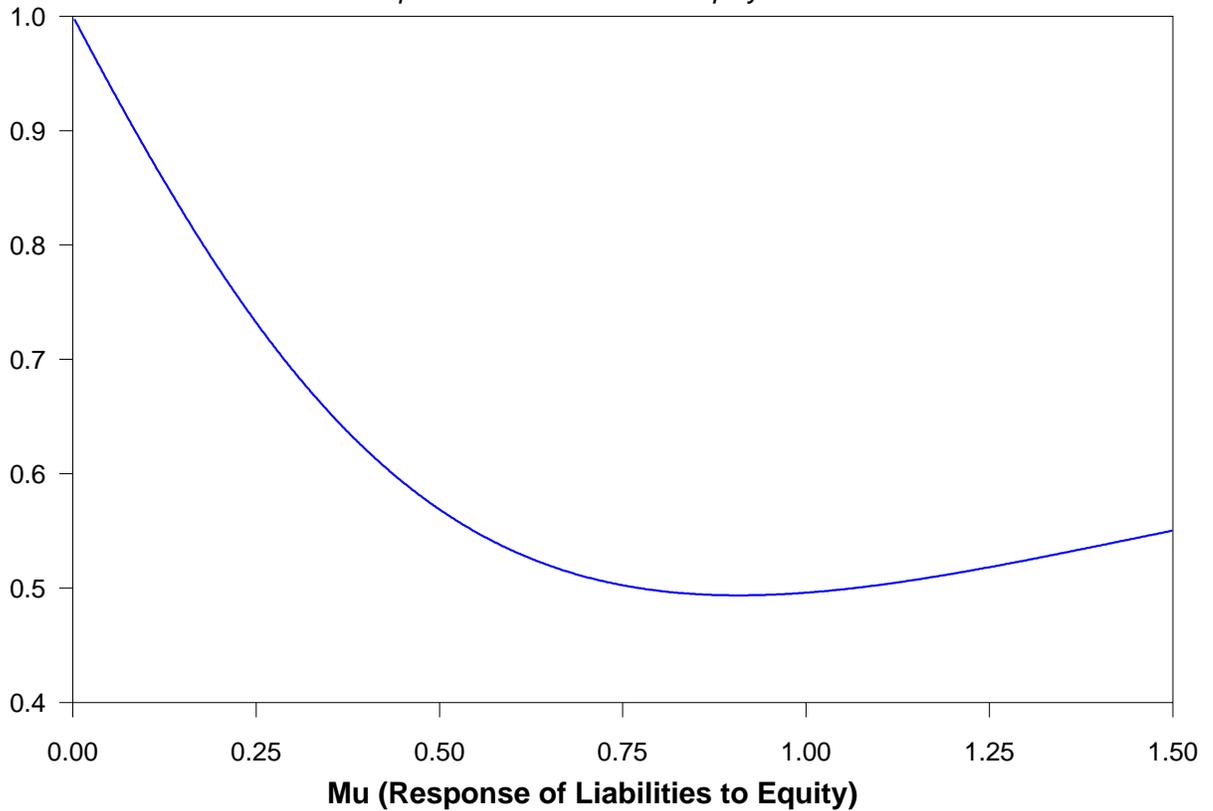
The results reported up to now have been based on error-correction values of $\lambda_E = \lambda_L = 0.04$, which implies a pace of adjustment of the liabilities-to-equity ratio similar to the pace estimated in our empirical analysis reported later in the paper. Clearly, the introduction

of error-correction into the model has a dramatic effect on the behavior of the variables as it forces mean-reversion in the leverage ratio rather than allowing them to wander off towards arbitrary values. However, perhaps surprisingly, it doesn't have much effect on the true population coefficient for the regression of leverage growth on asset growth. Figure 6 again shows the relationship between this coefficient and μ with each of the different lines corresponding to different amounts of error correction, ranging from no error correction to $\lambda_E = \lambda_L = 0.04$.⁴ The chart shows that the error-correction speeds have little impact on the regression coefficient of interest.

⁴The figures in this chart are again based on the assumption of equal variances for equity and liability shocks.

Figure 4: Effect on Regression Coefficient of Changing Mu

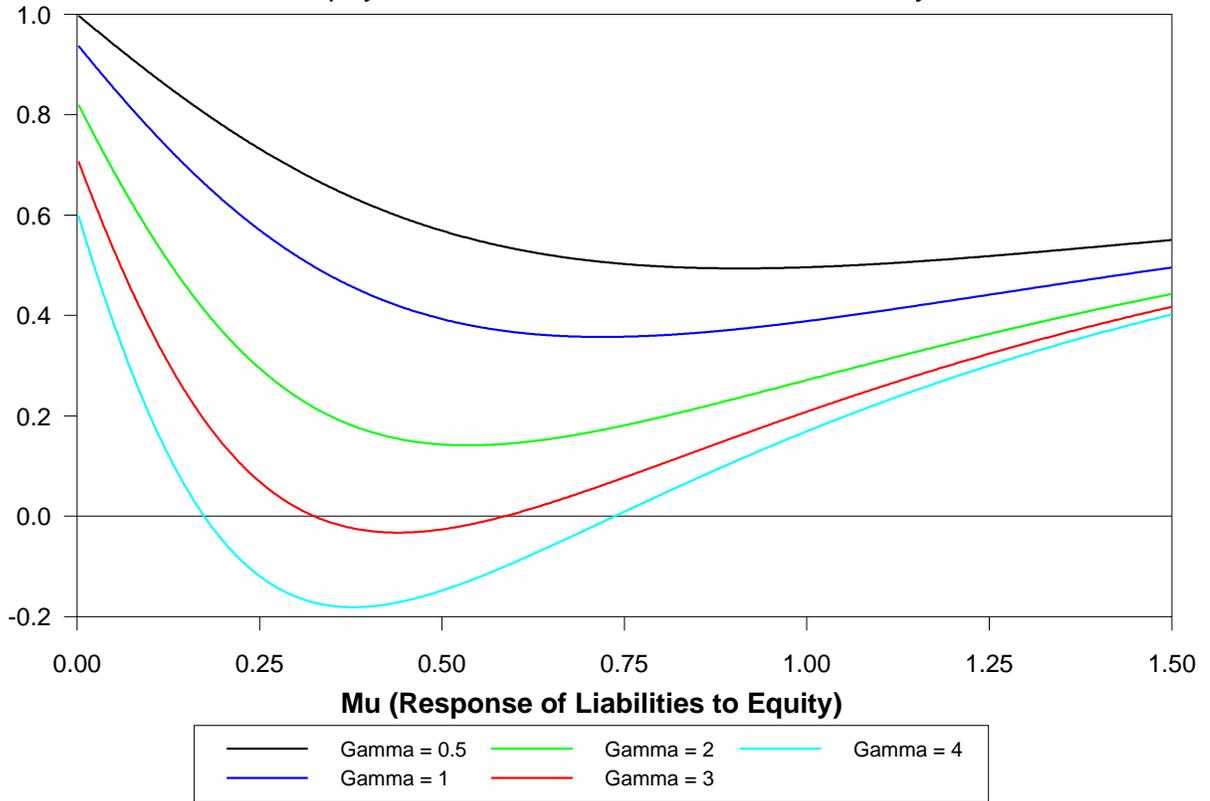
Assumes Equal Error Variances for Equity and Liabilities



This graph shows the true population coefficient from a regression of leverage growth on asset growth for various values of the parameter μ in the model described by equations (9) and (10). The variance of equity and liability shocks are set equal and we set $\lambda_E = \lambda_L = 0.04$ and $\theta = 0.9$

Figure 5: Effect of Relative Error Variances

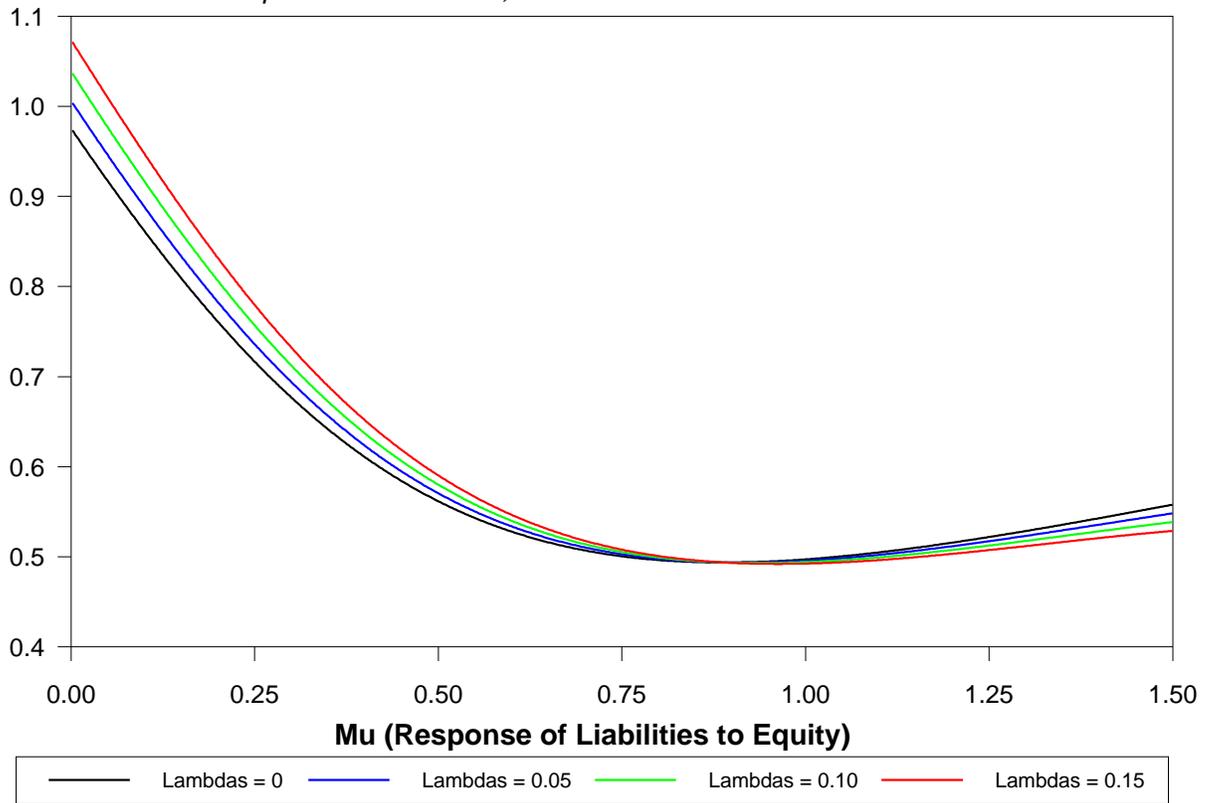
Variance of Equity Shocks = Gamma Times Variance of Liability Shocks



This graph shows the true population coefficient from a regression of leverage growth on asset growth for various values of the parameter μ in the model described by equations (9) and (10). We set $\lambda_E = \lambda_L = 0.04$ and $\theta = 0.9$. The different colored lines reflect different values for the ratio of the variance of equity shocks to the variance of liability shocks.

Figure 6: Effect of Error-Correction Speeds

Equal Error Variances, Various Values of ECM Coefficients



This graph shows the true population coefficient from a regression of leverage growth on asset growth for various values of the parameter μ in the model described by equations (9) and (10). The variance of equity and liability shocks are set equal and the different colored lines reflect different values for the parameters λ_E and λ_L and $\theta = 0.9$.

4 An Empirical Model of Bank Balance Sheet Adjustment

In the rest of the paper, we estimate a Vector Error-Correction Model (VECM) for bank liabilities and equity using the panel of data on bank holding companies discussed in Section 2. Here we discuss the data used in more detail and describe our empirical specification.

4.1 Data

Our data come from the quarterly Consolidated Financial Statements for Bank Holding Companies in the United States which are available from the Federal Reserve Bank of Chicago.⁵ BHCs are subject to regulation by the Federal Reserve Board of Governors under the Bank Holding Company Act of 1956 and Regulation Y.

Our data cover the entire activities of the BHC and subsidiary commercial banks on a consolidated basis, removing the impact of intra-group balances on the aggregate size of the balance sheet. The various commercial banks in any given BHC are subject to regulation by the Comptroller of the Currency or the Federal Deposit Insurance Corporation (FDIC). However, the relationship between commercial banks within a BHC is in part defined by the broader regulatory environment. Regulators can force both parent BHCs and affiliated commercial banks to support failing subsidiaries and affiliates under the FDIC cross-guarantee rule or the Fed’s “source-of-strength” doctrine. Consequently, the behaviour and performance of individual commercial banks is potentially not independent of other banks in the BHC and examining issues such as those addressed in this paper, is better achieved using consolidated data at the BHC level.⁶

Data files for each quarter from 1986:Q3 to 2013:Q4 were downloaded, with each file containing approximately 2,200 balance sheet, income statement and related variables for each BHC.⁷ From March 2006 onwards, the dataset covers all BHCs with total assets of \$500 million or above. Prior to this period, BHCs with total assets of \$150 million or

⁵See www.chicagofed.org/webpages/banking/financial_institution_reports/bhc_data.cfm.

⁶Aschcraft (2008) finds that commercial banks that are part of a multi-bank holding company are less likely to experience financial distress than stand-alone banks, and even in the cases where they do experience financial distress, they are more likely than single banks to survive because they receive capital injections from their parent BHCs or affiliated banks

⁷The reporting forms have changed a number of times over the sample period causing changes to some variables available in the raw data over time. Where reporting changes have impacted on variables of interest in this paper, we have created consistent time series by methodically tracing these changes through the reporting form vintages and merging data as appropriate.

above were required to report. The total number of unique BHCs over the entire sample period is 7,712, with an average of 1,493 BHCs reporting per quarter up to 2005:Q4 and 867 per quarter from 2006:Q1 to 2013:Q4. Despite the smaller number of BHCs reporting in recent years, the data offer practically full coverage of the assets held by the U.S. chartered banking population.

We restrict our sample to those BHCs with at least 30 contiguous observations over the period in order to ensure we have sufficient time series variation in our data to allow for good estimates of the dynamic elements of our empirical model. After cleaning and dealing with other anomalies in the raw data files, our analysis below includes 986 BHCs covering 59,530 BHC-quarter observations, meaning we have an average of 60 observations per BHC in our dataset.⁸

4.2 Empirical Model

Our empirical approach to modelling bank balance sheet adjustments is to use the following Vector Error-Correction Model (VECM) for bank's i 's equity, E_{it} , and liabilities, L_{it} .

$$\begin{aligned} \Delta \log E_{it} &= \alpha_t^E + \alpha_i^E + \beta^{EE}(L) \Delta \log E_{it} + \beta^{EL}(L) \Delta \log L_{it} \\ &\quad + \gamma^E (\log L_{i,t-1} - \log E_{i,t-1}) + \epsilon_{it}^E \end{aligned} \tag{11}$$

$$\begin{aligned} \Delta \log L_{it} &= \alpha_t^L + \alpha_i^L + (\mu + \beta^{LE}(L)) \Delta \log E_{it} + \beta^{LL}(L) \Delta \log L_{it} \\ &\quad + \gamma^L (\log L_{i,t-1} - \log E_{i,t-1}) + \epsilon_{it}^L \end{aligned} \tag{12}$$

where $\beta^{EE}(L)$, $\beta^{EL}(L)$, $\beta^{LE}(L)$, and $\beta^{LL}(L)$ are lag operators.

The model also has a number of features worth noting. First, as with the stylised VECM discussed above (i.e. equations (9) and (10)) the model allows for the estimation of error-correction terms so that equity and liabilities can adjust to move towards a target leverage ratio. Despite its simplicity, we believe ours is the first paper to estimate a VECM of this sort for bank balance sheets. While a number of other papers have provided evidence that banks adjust their balance sheets in response to deviations from target levels of capital, they do not focus on the separate adjustments to equity and liabilities that drive these adjustments. For example, Hancock-Wilcox (1993, 1994) estimate the effect of estimated capital shortfalls on changes in total bank assets and sub-components of these assets where

⁸Observations with missing values for total assets, equity capital and those with implausible rates of change from quarter to quarter (i.e. less than -100 percent) were removed. To remove the impact of extreme outliers, the remaining variables in the dataset were winsorized at the 1st and 99th percentile.

the measures of capital shortfalls are constructed separately from the estimated regression. (In our analysis, the target leverage ratios are functions of time and bank-specific dummy variables). Berger et al (2008) and Berrospide and Edge (2010) estimate partial adjustment models for various definitions of capital ratios. Partial adjustment models of this sort are a subset of the VECM model estimated in this paper but cannot allow for differential responses of the numerator and denominator in the ratios. Worth noting, however, is that Berger et al (2008) provide significant evidence that banks use equity issuance and share repurchases to manage their capital ratios.

Second, it is not possible to identify contemporaneous responses of both equity to liability shocks and liabilities to equity shocks within this VECM framework, as this would result in two collinear regressions. Thus, as with the stylised framework above, the model features liabilities responding to contemporaneous changes in equity but does not have a contemporaneous response of equity to liabilities. In other words, the shocks are estimated using a recursive identification. One can justify this assumption on the grounds that the various sources of changes to equity (profits, dividend payments, equity raising etc.) are unlikely to be very sensitive to within-quarter changes in liabilities. Perhaps more importantly for our paper is that this identification produces a model that understates the points made in this paper about the role of liability shocks. As discussed earlier, the inclusion of liability shocks that are uncorrelated with equity shocks can change the interpretation of the relationship between changes in leverage and changes in assets with positive relationships between these changes more likely as the variance of liability shocks increases. This identification maximizes the estimated variance of shocks to equity in the model and minimizes the estimated variance of orthogonal shocks to liabilities.

Third, beyond the contemporaneous identification assumption, we allow for a general pattern of dynamic relationships between equity growth and liability growth. In our empirical specification, we include four quarterly lags of each as explanatory variables in both regressions. Thus, our analysis allows for the possibility of positively autocorrelated liability growth as well as other relationships between equity and liabilities that are separate from those associated with longer-run targeting of a particular leverage ratio.

Fourth, we include both bank-level and time fixed effects. In relation to the two bank-level effects, assuming a stationary leverage ratio, these parameters can be mapped directly into the long-run common growth rate of equity and liabilities (and thus assets) as well as the long-run equilibrium leverage ratio. Specifically, assuming a long-run average for the

time effects of zero, the long-run equilibrium growth rate for both equity and liabilities for bank i will be

$$g_i = \frac{\gamma^E \alpha_i^L - \gamma^L \alpha_i^E}{\gamma^E (1 - \mu - \beta^{LE}(1) - \beta^{LL}(1)) - \gamma^L (1 - \beta^{EE}(1) - \beta^{EL}(1))} \quad (13)$$

while the equilibrium ratio for of liabilities to equity for bank i will be

$$\theta_i = \frac{(1 - \beta^{EE}(1) - \beta^{EL}(1)) g_i}{\gamma^E} - \frac{\alpha_i^E}{\gamma^E} \quad (14)$$

So the inclusion of bank-specific fixed effects in our two equations means we are allowing banks to differ in their growth trajectories and their target leverage ratios. The presence of time effects in both equations means that macroeconomic factors can influence the growth rates of equity and liabilities as well as the average leverage ratios that banks are targeting.

We estimate equations (11) and (12) for our entire sample and also separately across the distribution of banks by size (total assets) and funding profile (relative use of wholesale funding). Finally, before presenting results from our VECM analysis, it is important to clarify that this is an appropriate specification to run with our data. The VECM formulation is only appropriate if the ratio of liabilities to equity (i.e. the leverage ratio minus one) is stationary and that liabilities and equity series are cointegrated. Table 1 presents results of a range of panel unit root tests. The logs of all the series used here are identified as $I(1)$ in levels with the exception of the leverage ratio, for which the null hypothesis of a unit root is rejected at the one percent level. Using panel cointegration tests developed by Pedroni (1999, 2004), we also find that liabilities and equity are cointegrated. As a result of these time series characteristics of the variables concerned the specification we employ in this paper appears to be appropriate.

Table 1: Panel Unit Root Tests

	Test	Levels	First Differences	Order of Integration
Total Liabilities				
	<i>IPS W-stat</i>	7.61	-150.00***	I(1)
	<i>ADF</i>	15.18	-98.18***	I(1)
	<i>PP</i>	1.93	-201.97***	I(1)
Equity				
	<i>IPS W-stat</i>	13.77	-140.00***	I(1)
	<i>ADF</i>	15.38	-80.41***	I(1)
	<i>PP</i>	5.17	-182.96***	I(0)
Leverage Ratio				
	<i>IPS W-stat</i>	-2.26**		I(0)
	<i>ADF</i>	9.16		I(0)
	<i>PP</i>	-2.09**		I(0)
Cointegration:				
Liabilities and Equity				
	<i>Panel v</i>	4.02***		
	<i>Panel rho</i>	-5.18***		
	<i>Panel PP</i>	-0.66		
	<i>Panel ADF</i>	-1.95**		
	<i>Group rho</i>	-7.83***		
	<i>Group PP</i>	-9.98***		
	<i>Group ADF</i>	-8.85***		

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Unit root test statistics are W statistics proposed by Im, Pesaran and Shin (2003) and Z statistics from Fisher-type Augmented Dickey Fuller and Phillips-Perron tests proposed by Maddala and Wu (1999) and Choi (2001). H_0 : All panels contain unit roots; H_a : At least one panel is stationary. Cointegration test statistics are those proposed by Pedroni (1999) and Pedroni (2004), with H_0 : series are not cointegrated; H_a : series are cointegrated. Significance of the test statistics at conventional levels implies rejection of H_0 . Series are cross-sectionally demeaned in the unit root tests and a constant is included in all test regressions. Optimum lags are included based on the lowest SIC score. The sample covers BHCs with a minimum of 30 contiguous observations over the sample period (1986:Q3-2013:Q4). All variables are expressed in natural logs. The leverage ratio is a linear combination of two I(1) variables.

5 Estimation Results

In this section, we present our baseline estimation results and then discuss results from estimating our model across various sub-samples of the data.

5.1 Full Sample Estimation

Table 2 presents the results from the estimation via OLS of our VECM model described by equations (11) and (12). The specification also contains time effects, seasonal effects and bank-specific fixed effects.

Recall that the specification allows for a within-period impact of changes in equity on changes in liabilities, but changes in liabilities are assumed to not have a contemporaneous impact on changes in equity⁹. Looking at the results for the liabilities regression, it can be seen that a within-period change in equity of 1 percent results in a 0.385 percent increase in liabilities, i.e. we estimate a value of $\mu = 0.385$. Perhaps surprisingly, autoregressive terms have little impact on liability growth, as might have been expected if “leverage cycles” were playing an important role. In contrast, there is evidence of some weak autoregressive effects for bank equity, so that quarters in which banks have high rates of equity growth tend to be followed by other strong quarters for equity.

Importantly, both error-correction terms enter significantly and with the expected sign. The size of the error-correction coefficient for liability adjustment, at minus 0.042, is larger in absolute terms than the coefficient for equity, which is 0.031. Still, it is clear that both liabilities and equity play a role in moving leverage ratios back towards target levels. Taken together, our estimates suggest that leverage ratios tend to be adjusted by 7.3 per cent per quarter towards their target levels, with 60 percent of this adjustment taking the form of liability adjustments and 40 percent taking the form of equity adjustments. This relatively slow speed of adjustment suggests that shocks to equity and liability will tend to take a long time to play out.

Table 2 also reports the estimated coefficient that we obtain from regressing leverage growth on asset growth, also controlling for BHC-specific fixed effects, seasonal and time effects. The table labels this parameter as “the Adrian-Shin regression coefficient”. As would be expected from the data already illustrated in Figure 3, the coefficient of 0.53

⁹We did re-order the specification to allow for contemporaneous affects of changes in liabilities on equity. This did not lead to any implications for our current estimates.

is significantly positive. Note, however, that this positive coefficient does not stem from banks reacting to equity shocks by choosing to raise leverage. The coefficient of $\mu = 0.385$ means that leverage declines temporarily in response to positive equity shocks. Rather, the positive correlation stems from the important role played by liability shocks that are uncorrelated with equity shocks. While the standard deviation of equity shocks of 0.077 is higher than the standard deviation of liability shocks of 0.063, the ratio of these variances is well below what would be required to generate a negative correlation.

One way to check whether the magnitude of the Adrian-Shin coefficient is consistent with our estimated VECM model is to run simulations of the model and check whether the observed coefficient is consistent with the range generated by these simulations. 5000 Monte Carlo simulations of the estimated dynamic model using normally-distributed draws for equity and liability shocks with variances that match the data generated a median regression coefficient from regressing asset growth on leverage growth that is 0.46. This is slightly lower than the Adrian-Shin coefficient estimated from the data but the estimate of 0.53 lies within the 95-th percentile band of the Monte Carlo distribution.

Table 2: Liabilities and Equity Error Correction Mechanism

	Liabilities	Equity
	Log-Difference	Log-Difference
Equity: Log-Difference	0.385*** (0.029)	
Equity: Log-Difference (Sum of Lagged 4 Quarters)	0.013 (0.019)	0.153*** (0.039)
Liabilities: Log-Difference (Sum of Lagged 4 Quarters)	-0.002 (0.020)	-0.031 (0.021)
Leverage Ratio: Lagged 1 Quarter	-0.042*** (0.003)	0.031*** (0.005)
R^2	0.16	0.06
N	46,909	46,909
Variance of Residuals	0.004	0.006
Adrian-Shin (A-S) Leverage-Assets Coefficients	0.532*** (0.027)	
R^2	0.26	
N	58,554	
<i>A-S Coeff. Monte Carlo Simulations</i>		
5th percentile	0.381	
Median	0.456	
95th percentile	0.531	

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Intercept, seasonal and time dummies included. Cluster robust standard errors in parentheses. All variables are expressed in natural logs. The sample covers BHCs with a minimum of 30 contiguous observations over the sample period (1986:Q3-2013:Q4).

5.2 Differences Across Banks: Size and Funding Profiles

The regressions just reported allowed for banks to differ in their target capital ratios and in their long-run average growth rates. The behavioural coefficients, however, were restricted to be the same across banks. Here, we loosen this constraint by separately estimating our VECM specification for banks in different size categories and with different liability funding profiles. Specifically, we present four different liability and equity regressions, one each for banks in the 25th, 50th, 75th and 100th percentile on the distribution of total assets and the distribution of the share in debt securities in total liabilities (a proxy for wholesale funding). Note that the quartiles have been defined for each time period, so an individual BHC could be in different quartiles at different points in time depending on its position relative to the population of BHCs in a given quarter.

Table 3 presents the results of the liability regressions and Table 4 presents the results of the equity regressions across the distribution of total assets. We find that the contemporaneous response of liabilities to changes in equity increases with the size of the BHC, the coefficient rising from 0.10 to 0.56 from the first to the fourth quartile. Generally speaking, the error correction terms on the lagged leverage ratio for liability and equity changes also increase in magnitude across the distribution, so that larger BHCs move towards their target leverage ratio at a faster rate than smaller BHCs. Despite this, the response could still be argued to be gradual even for large BHCs, with any disequilibrium from target leverage ratios for the largest BHCs being corrected by 11 percent each quarter (6.3 percent from liability adjustment and 4.3 percent from equity adjustment).

These results show that large banks are much more active in adjusting their balance sheet in response to shocks. They adjust liabilities by more in response to shocks to equity and are quicker to move towards their target leverage ratios. As we discussed in Section 3, the reported pattern of higher μ coefficients for large banks could imply these banks have either higher or lower Adrian-Shin coefficients, depending on whether the values of μ are on the downward- or upward-sloping parts of the curves described in Figures 3 to 5. However, given the observed variances of equity and liability shocks (which are relatively similar in size) and the fact that the largest μ coefficient in Table 3 equals 0.562, we would expect these values to be on the downward-sloping part of the curve, as illustrated in Figure 3.

The results confirm this pattern, with progressively smaller Adrian-Shin coefficients as bank size increases. The overall magnitudes of the declines are a bit larger than predicted by our Monte Carlo simulations of the estimated VECM models but the estimated models

do a good job of explaining why larger banks have lower Adrian-Shin coefficients than big ones. This results show that the active balance sheet management by these larger banks acts to reduce the correlation between asset growth and leverage growth. This is perhaps a bit counter-intuitive relative to would be expected in a world where there are only shocks to bank equity but the results fit well with the more general model that we have presented.

Tables 5 and 6 repeats the analysis across the distribution of funding profiles. The contemporaneous response of liabilities to changes in equity (μ) rises, from 0.28 to 0.41, as banks recourse to wholesale funding through debt markets increases. The lagged leverage ratio coefficient also increases in magnitude across the quartiles, indicating that banks that depend more on wholesale funding adjust to their target leverage ratio at a faster pace than banks which have a lower share of debt securities in their total liabilities. Again, the A-S coefficients follow a pattern broadly consistent with what that predicted by the VECM with the coefficients declining up to the third quartile and then increasing in the fourth quartile (this latter patten apparently due to the higher variance of liability shocks in the highest quartile).

5.3 Econometric Issues

It is well-known that OLS estimation of dynamic panel regressions with fixed effects can lead to significant biases.¹⁰ Specifically, least squares dummy variable estimation is equivalent to estimating a de-meanded model, i.e. a specification in which the individual-level average of each variable has been subtracted off and the error-term has had its average value subtracted off. Because the lagged dependent variable is correlated with one of the terms in the transformed error term, this results in finite-sample biases. This is a non-trivial issue because most of the alternative methods also suffer from a range of potential problems. For example, the commonly-used Arellano-Bond estimator uses lagged first differences as instruments but these instruments work poorly when you have persistent series, as we have here.

One step that we have taken to minimize biases is to restrict our sample to BHCs with at least 30 contiguous observations. In fact, our panel has an average number of observations per BHC of about 58, which is high enough to suggest that econometric biases are likely to be less severe than in the shorter panels used in most empirical work. We carried out a Monte Carlo exercise in which we simulated our estimated model replicating the standard

¹⁰See Judson and Owen (1999) and Bond (2002) for reviews.

deviations of residuals and fixed effects. The results indicated that there should be very little bias for the key parameter, μ i.e. the contemporaneous effect of equity changes on liability changes.

The Monte Carlo exercise did suggest that the error-correction coefficients may be somewhat over-stated. However, estimation of the model via the Arellano-Bond technique indicated the opposite, producing estimates of adjustment speeds that were larger than those from our OLS estimation. These results may not be reliable, though, because the instruments failed the over-identifying restrictions tests. On balance, we don't believe our conclusions are the result of econometric biases.

Table 3: Liabilities VECM and Adrian-Shin Coefficients Across the Distribution of Total Assets

	1st Quartile	2nd Quartile	3rd Quartile	4th Quartile
Equity: Log-Difference	0.104*** (0.034)	0.193*** (0.040)	0.407*** (0.042)	0.562*** (0.050)
Equity: Log-Difference (Sum of Lagged 4 Quarters)	0.042 (0.032)	-0.010 (0.031)	0.042 (0.039)	-0.017 (0.047)
Liabilities: Log-Difference (Sum of Lagged 4 Quarters)	-0.028 (0.028)	-0.075* (0.042)	-0.104** (0.048)	-0.072* (0.043)
Leverage Ratio: Lagged 1 Quarter	-0.046*** (0.006)	-0.051*** (0.007)	-0.073*** (0.015)	-0.063*** (0.009)
R^2	0.09	0.12	0.21	0.33
N	11,877	11,753	11,760	11,619
Variance of Residuals	0.004	0.004	0.005	0.005
Adrian-Shin (A-S) Leverage-Assets Coefficients	0.850*** (0.027)	0.728*** (0.029)	0.525*** (0.043)	0.328*** (0.060)
R^2	0.45	0.36	0.28	0.17
N	14,508	14,619	14,672	14,745
<i>A-S Coeff. Monte Carlo Simulations</i>				
5th percentile	0.660	0.559	0.417	0.369
Median	0.754	0.649	0.489	0.429
95th percentile	0.846	0.736	0.561	0.492

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Intercept, seasonal and time dummies included. Cluster robust standard errors in parentheses. All variables are expressed in natural logs. The sample covers BHCs with a minimum of 30 contiguous observations over the sample period (1986:Q3-2013:Q4). The Adrian-Shin coefficients are from regressions of leverage growth on asset growth.

Table 4: Equity Error Correction Mechanism Across the Distribution of Total Assets

	1st Quartile	2nd Quartile	3rd Quartile	4th Quartile
Equity: Log-Difference (Sum of Lagged 4 Quarters)	0.060** (0.029)	0.177** (0.071)	0.124** (0.057)	0.035 (0.078)
Liabilities: Log-Difference (Sum of Lagged 4 Quarters)	-0.024 (0.029)	0.036 (0.035)	-0.083** (0.034)	-0.055 (0.055)
Leverage Ratio: Lagged 1 Quarter	0.036*** (0.005)	0.038*** (0.008)	0.048*** (0.009)	0.043*** (0.013)
R^2	0.10	0.10	0.07	0.05
N	11,877	11,753	11,660	11,619
Variance of Residuals	0.006	0.006	0.006	0.006

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Intercept, seasonal and time dummies included. Cluster robust standard errors in parentheses. All variables are expressed in natural logs. The sample covers BHCs with a minimum of 30 contiguous observations over the sample period (1986:Q3-2013:Q4).

Table 5: Liabilities VECM and Adrian-Shin Coefficients Across the Distribution of Securities Issued Share of Liabilities

	1st	2nd	3rd	4th
	Quartile	Quartile	Quartile	Quartile
Equity: Log-Difference	0.278*** (0.057)	0.377*** (0.051)	0.438*** (0.061)	0.408*** (0.047)
Equity: Log-Difference (Sum of Lagged 4 Quarters)	0.077** (0.035)	0.020 (0.032)	-0.023 (0.038)	0.103 (0.065)
Liabilities: Log-Difference (Sum of Lagged 4 Quarters)	-0.187*** (0.049)	-0.114*** (0.037)	-0.093** (0.037)	-0.281** (0.142)
Leverage Ratio: Lagged 1 Quarter	-0.049*** (0.008)	-0.048*** (0.007)	-0.064*** (0.007)	-0.093*** (0.014)
R^2	0.15	0.21	0.29	0.26
N	11,796	11,843	11,584	11,686
Variance of Residuals	0.004	0.004	0.005	0.005
Adrian-Shin (A-S) Leverage-Assets Coefficients	0.658*** (0.048)	0.499*** (0.041)	0.433*** (0.058)	0.548*** (0.047)
R^2	0.37	0.26	0.22	0.27
N	14,569	14,675	14,632	14,668
<i>A-S Coeff. Monte Carlo Simulations</i>				
5th percentile	0.484	0.401	0.397	0.489
Median	0.570	0.477	0.467	0.557
95th percentile	0.649	0.552	0.534	0.626

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Intercept, seasonal and time dummies included. Cluster robust standard errors in parentheses. All variables are expressed in natural logs. The sample covers BHCs with a minimum of 30 contiguous observations over the sample period (1986:Q3-2013:Q4). The Adrian-Shin coefficients are from regressions of leverage growth on asset growth.

Table 6: Equity Error Correction Mechanism Across the Distribution of Securities Issued Share of Liabilities

	1st Quartile	2nd Quartile	3rd Quartile	4th Quartile
Equity: Log-Difference (Sum of Lagged 4 Quarters)	0.048 (0.039)	0.086 (0.053)	0.137* (0.079)	0.025 (0.083)
Liabilities: Log-Difference (Sum of Lagged 4 Quarters)	0.030 (0.035)	-0.025 (0.047)	-0.128** (0.052)	-0.036 (0.041)
Leverage Ratio: Lagged 1 Quarter	0.031*** (0.007)	0.042*** (0.010)	0.035** (0.012)	0.046*** (0.013)
R^2	0.08	0.07	0.06	0.06
N	11,796	11,843	11,584	11,686
Variance of Residuals	0.006	0.006	0.006	0.006

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$

Intercept, seasonal and time dummies included. Cluster robust standard errors in parentheses. All variables are expressed in natural logs. The sample covers BHCs with a minimum of 30 contiguous observations over the sample period (1986:Q3-2013:Q4).

6 Conclusions

This paper has presented a general approach to modelling how banks adjust their balance sheets. In addition to presenting a new framework describing the determinants of the relationship between changes in leverage and changes in assets, we have estimated our model using micro data on US Bank Holding Companies and documented a number of new empirical results.

Our results show that banks adjust their balance sheets to move towards target leverage ratios, with both liabilities and equity being adjusted. Banks react to positive shocks to equity by raising their liabilities but their leverage ratios still fall temporarily. So while we observe a positive correlation between changes in assets and changes in leverage, this relationship is not driven by the reaction of banks to equity shocks. Rather, this correlation reflects the importance of shocks to bank liabilities that are unrelated to equity shocks.

Finally, we show that larger banks tend to engage in more active balance sheet management, with liabilities responding more to contemporaneous changes in equity and by faster adjustment towards target leverage ratios. We have shown how this active balance sheet management produces a smaller correlation between changes in assets and changes in leverage for large banks than for smaller banks.

The model presented here can be extended in various ways. For example, one set of questions that we have not yet addressed are the sources of the equity adjustment that we estimate. One possibility is that equity tends to increase when leverage is high because high leverage generally produces higher profits and thus higher retained earnings. Alternatively (or additionally) equity may increase when leverage ratios are high because of conscious actions to reduce leverage such as selling new shares or reducing dividends. Another question is the role played in balance sheet adjustment of regulatory capital ratios, which feature risk-weighted assets rather than the total unweighted assets series examined here. We plan to investigate these questions in future research.

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A Calculation of Asset-Leverage Regression Coefficients

Using lower-case letters to denote logged variables, we start with a log-linear approximation of assets as a function of liabilities and equity.

$$a_t = \theta l_t + (1 - \theta) e_t \quad (15)$$

Because the intercepts in the model don't affect the relevant long-run correlations, we will derive these results for a simplified version that we will write as follows. Our model of bank equity and liabilities can be written as

$$\Delta e_t = -\lambda_e (e_{t-1} - l_{t-1}) + \epsilon_t^e \quad (16)$$

$$\Delta l_t = \mu \Delta e_t + \lambda_l (e_{t-1} - l_{t-1}) + \epsilon_t^l \quad (17)$$

where ϵ_t^e and ϵ_t^l are uncorrelated iid shock terms. The liabilities equation can be re-written as

$$\Delta l_t = (\lambda_l - \mu \lambda_e) (e_{t-1} - l_{t-1}) + \mu \epsilon_t^e + \epsilon_t^l \quad (18)$$

We can then calculate the covariance of asset growth and leverage growth as

$$\text{Cov}(\theta \Delta l + (1 - \theta) \Delta e, \Delta l - \Delta e) = \theta \text{Var}(\Delta l) - (1 - \theta) \text{Var}(\Delta e) + (1 - 2\theta) \text{Cov}(\Delta l, \Delta e) \quad (19)$$

The relevant long-run variances and co-variances can be calculated as follows:

$$\text{Var}(\Delta e) = \lambda_e^2 \text{Var}(e - l) + \sigma_E^2 \quad (20)$$

$$\text{Var}(\Delta l) = (\lambda_l - \mu \lambda_e)^2 \text{Var}(e - l) + \mu^2 \sigma_E^2 + \sigma_L^2 \quad (21)$$

$$\text{Cov}(\Delta l, \Delta e) = -\lambda_E (\lambda_l - \mu \lambda_e) \text{Var}(e - l) + \mu \sigma_E^2 \quad (22)$$

To derive the long-run variance $\text{Var}(e - l)$, we need to derive the underlying process for this variable. We start by re-expressing the equity and liabilities equations in terms of levels rather than differences:

$$e_t = (1 - \lambda_e) e_{t-1} + \lambda_e l_{t-1} + \epsilon_t^e \quad (23)$$

$$l_t = (1 - \lambda_l + \mu \lambda_e) l_{t-1} + (\lambda_l - \mu \lambda_e) e_{t-1} + \mu \epsilon_t^e + \epsilon_t^l \quad (24)$$

This means the combined process for the log of equity to liabilities is

$$e_t - l_t = (1 - \lambda_e - \lambda_l + \mu \lambda_e) (e_{t-1} - l_{t-1}) + (1 - \mu) \epsilon_t^e - \epsilon_t^l \quad (25)$$

The long-run variance of this process can then be calculated as

$$\text{Var}(e - l) = \frac{(1 - \mu)^2 \sigma_e^2 + \sigma_l^2}{1 - (1 - \lambda_e - \lambda_l + \mu\lambda_e)^2} \quad (26)$$

Putting all the pieces together

$$\begin{aligned} \text{Cov}(\theta\Delta l + (1 - \theta)\Delta e, \Delta l - \Delta e) &= \theta \left[\frac{(\lambda_l - \mu\lambda_e)^2 \left[(1 - \mu)^2 \sigma_e^2 + \sigma_l^2 \right]}{1 - (1 - \lambda_e - \lambda_l + \mu\lambda_e)^2} + \mu^2 \sigma_E^2 + \sigma_L^2 \right] \\ &\quad - (1 - \theta) \left[\frac{\lambda_e^2 \left[(1 - \mu)^2 \sigma_e^2 + \sigma_l^2 \right]}{1 - (1 - \lambda_e - \lambda_l + \mu\lambda_e)^2} + \sigma_E^2 \right] \\ &\quad - (1 - 2\theta) \left[\frac{\lambda_e (\lambda_l - \mu\lambda_e) \left[(1 - \mu)^2 \sigma_e^2 + \sigma_l^2 \right]}{1 - (1 - \lambda_e - \lambda_l + \mu\lambda_e)^2} + \mu\sigma_E^2 \right] \end{aligned} \quad (27)$$

The expression on the right hand side can be simplified slightly to

$$\begin{aligned} \left[\frac{(1 - \mu)^2 \sigma_e^2 + \sigma_l^2}{1 - (1 - \lambda_e - \lambda_l + \mu\lambda_e)^2} \right] &\left[\theta (\lambda_l - \mu\lambda_e)^2 - (1 - \theta) \lambda_e^2 - (1 - 2\theta) \lambda_e (\lambda_l - \mu\lambda_e) \right] \\ &\quad + (1 + \theta\mu - \theta) (\mu - 1) \sigma_E^2 + \theta\sigma_L^2 \end{aligned} \quad (28)$$

The coefficient from a regression of leverage growth on asset growth is derived by dividing this covariance by the variance of asset growth which is calculated as

$$\text{Var}(\Delta a) = \theta^2 \text{Var}(\Delta l) + (1 - \theta)^2 \text{Var}(\Delta e) + 2\theta(1 - \theta) \text{Cov}(\Delta l, \Delta e) \quad (29)$$

This can be calculated as

$$\begin{aligned} \text{Var}(\Delta a) &= \theta^2 \left[\frac{(\lambda_l - \mu\lambda_e)^2 \left[(1 - \mu)^2 \sigma_e^2 + \sigma_l^2 \right]}{1 - (1 - \lambda_e - \lambda_l + \mu\lambda_e)^2} + \mu^2 \sigma_E^2 + \sigma_L^2 \right] \\ &\quad + (1 - \theta)^2 \left[\frac{\lambda_e^2 \left[(1 - \mu)^2 \sigma_e^2 + \sigma_l^2 \right]}{1 - (1 - \lambda_e - \lambda_l + \mu\lambda_e)^2} + \sigma_E^2 \right] \\ &\quad + 2\theta(1 - \theta) \left[\frac{\lambda_e (\lambda_l - \mu\lambda_e) \left[(1 - \mu)^2 \sigma_e^2 + \sigma_l^2 \right]}{1 - (1 - \lambda_e - \lambda_l + \mu\lambda_e)^2} + \mu\sigma_E^2 \right] \end{aligned} \quad (30)$$

The right-hand side here can be re-written as

$$\left[\frac{(1-\mu)^2 \sigma_e^2 + \sigma_l^2}{1 - (1 - \lambda_e - \lambda_l + \mu \lambda_e)^2} \right] \left[\theta^2 (\lambda_l - \mu \lambda_e)^2 + (1 - \theta)^2 \lambda_e^2 + 2\theta (1 - \theta) \lambda_e (\lambda_l - \mu \lambda_e) \right] + (1 + \theta \mu - \theta)^2 \sigma_E^2 + \theta^2 \sigma_L^2 \quad (31)$$