# New Evidence on Balanced Growth, Stochastic Trends, and Economic Fluctuations

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**Abstract**

The one-sector Solow-Ramsey growth model informs how most modern researchers characterize macroeconomic trends and cycles, and evidence supporting the model's balanced growth predictions is often cited. This paper shows, however, that the inclusion of recent data leads to the balanced growth predictions being rejected. An alternative balanced growth hypothesis—that the ratio of nominal consumption to nominal investment is stationary—is put forward, and new measures of the stochastic trends and cycles in aggregate US data are derived based on this hypothesis. The contrasting behavior of real and nominal ratios is consistent with a two-sector model of economic growth, with separate production technologies for consumption and investment and two stochastic trends underlying the long-run behavior of all macroeconomic series. Empirical estimates of these stochastic trends are presented based on a structural VAR and the role played in the business cycle by shocks to these trends is discussed.

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# 1 Introduction

The one-sector Solow-Ramsey growth model remains the workhorse underlying a vast amount of macroeconomic research. In particular, its "balanced-growth" prediction that the ratios of real consumption, investment, and output should all be stationary (and thus that the logs of the level series share a common stochastic trend) has played a key role in modern macroeconomics. Sample averages of these "great ratios" are regularly used to calibrate the long-run properties of a wide range of theoretical macroeconomic models. In addition, the common trend prediction has played an important role in modern empirical characterizations of macroeconomic trends and cycles. To cite a few well-known examples, King, Plosser, Stock, and Watson (1991) have developed empirical methods to identify a time series for this common trend, while Cochrane (1994) and Rotemberg and Woodford (1996) have relied on the stationarity of the ratio of real consumption to real output to provide econometric estimates of the transitory cyclical components of consumption, investment, and output.

This paper presents new evidence on the traditional balanced growth hypothesis and on the role of stochastic trends in the determination of U.S. macroeconomic fluctuations. The evidence underlying the one-sector growth model is reconsidered and rejected, an alternative balanced growth hypothesis consistent with a two-sector framework is suggested, and the implications of this framework for macroeconomic trends and fluctuations are considered.

The contents of the paper are as follows. First, it is demonstrated that the inclusion of recent data produces test results that point strongly against the balanced growth hypothesis. For example, King, Plosser, Stock and Watson's evidence for the hypothesis is still widely cited. However, the sample in their study ends in 1988; using data through 2004, the traditional balanced growth hypothesis turns out to be firmly rejected. Underlying this rejection is the fact that the ratio of real investment to real consumption has moved substantially upwards over time and now displays no evidence of mean-reversion.

Importantly, these results have been derived using standard National Income and Product Accounts (NIPA) data. This implies a crucial distinction between these the formal statistical tests presented here and the more informal evidence presented in the well-known paper by Greenwood, Hercowitz, and Krusell (1997), who argue in favor of a model in which real equipment investment rises relative to real GDP, but in which the real equipment series is defined using Robert Gordon's (1990) alternative price index. That the balanced growth property is clearly rejected for NIPA data has important implications that go well

beyond those discussed by Greenwood et al because NIPA data form the basis for the vast majority of empirical work on US macroeconomics, and much of this work relies explictly or implicitly on the balanced growth hypothesis.

Second, the paper discusses the implications of the failure of the traditional balanced growth hypothesis for the measurement of stochastic trends and cycles. It is shown that the estimated Vector Error Correction Model (VECM) implied by the traditional one-sector balanced growth hypothesis produces unintuitive trending estimates of the cyclical components of consumption, investment and output. Because these results contradict much of the received wisdom on this issue that was outlined in previous papers such as Cochrane (1994), the paper discusses how the incorporation of new data and revisions to historical data have contributed to overturning patterns that were previously considered to be stylized facts.

Third, an alternative "balanced growth" hypothesis is put forward, which is that the ratio of *nominal* investment to *nominal* consumption is stationary and evidence is presented that this hypothesis is consistent with US macroeconomic data. This nominal-ratio balanced growth hypothesis implies that the logs of real consumption, real investment, and the relative price of consumption to investment can together be characterized according to a VECM with a single cointegrating vector. Using this alternative VECM, new measures of the transitory components of aggregate consumption, investment, and output are developed and the implications of these alternative measures for the interpretation of recent business cycles are explored.

Fourth, a simple structural model is proposed that is compatible with the long-run properties of the data and the model's implications are explored. The differing behavior over time of the real and nominal ratios of consumption and investment are, by definition, due a decline over time in the price of investment relative to the price of consumption, a pattern that cannot occur in the one-sector growth model. However, this pattern is consistent with a *two-sector* model of economic growth in which separate technological trends in the consumption and investment sectors drive the relative price of output in the two sectors. In addition, the existence of a single cointegrating vector among consumption, investment, and their relative price implies a common trends representations in which the long-run behavior of these variables depends on two separate $I(1)$ stochastic trends. Using a simple two-sector model, a set of long-run restrictions are derived that allow for the use of the King, Plosser, Stock, Watson (1991) methodology to identify these stochastic trends

as corresponding to the states of technology in the consumption- and investment-producing sectors. The dynamic responses to the two types of technology shocks are discussed, as are the roles played by these shocks in generating business cycle fluctuations. It is concluded that technology shocks play a limited role in generating business cycle fluctuations in consumption, investment, and output.

## 2 Tests of the Traditional Balanced Growth Hypothesis

The starting point for the standard neoclassical growth model is an aggregate resource constraint of the form $C_t + I_t = Y_t = F(K_t, A_t L_t)$ where the production function displays diminishing marginal returns productivity with respect to capital accumulation. It is also usually assumed that a representative consumer maximizes the present discounted value of utility from real consumption, subject to a law of motion for capital and a process for aggregate technology. If the technology process grows at a constant long-run average rate, then it is well known that the model's solution features all real variables, $C, I, K$, and $Y$, growing at the same average rate in the long-run. Thus, ratios of any of the real aggregates will be stationary stochastic processes.

The intuition for this property of the one-sector growth model is very general and it holds across a wide range of specifications for preferences and technology. Given diminishing marginal returns to capital accumulation, a once-off increase in the savings rate can only allow capital to grow faster than output for a temporary period, and a trend in the savings rate will not be optimal for any standard specification of preferences.[1] Thus, ratios involving any of the variables capital, investment, consumption and output must be stationary.

Beyond theory, the hypothesis of stationary "great ratios" has been held as a crucial stylized fact in macroeconomics at least as far back as the well-known contributions of Kaldor (1957) and Kosobud and Klein (1961). More recently, King, Plosser, Stock, and Watson (1991, henceforth KPSW), using NIPA data through 1988, presented formal econometric evidence for the balanced growth hypothesis using modern time-series methods. KPSW's results are still widely cited as important evidence for the one-sector model's long-run restrictions. However, Figure 1(a) shows that the traditional balanced growth predictions appear empirically invalid once we use updated NIPA data. The figure shows the ratio of real private fixed investment to real consumption expenditures (the same measures of

[1] See, for instance the discussion of capital-output ratio dynamics in the Solow Model in Chapter 4 of DeLong (2002)

investment and consumption used in KPSW's study). That this ratio should be stable is the essence of the balanced growth hypothesis: The model's other long-run restrictions—such as the stationarity of the ratio of real consumption to aggregate real output or of real investment to aggregate real output—will follow directly from stationarity of the ratio of consumption to investment.[2]

Figure 1(a) shows that the ratio of real investment to real consumption has moved upwards substantially over time and shows little apparent tendency for mean-reversion. Importantly, most of the increase in this ratio occurred after 1988, the last year in KPSW's sample. Also notable is the fact that, despite the substantial slump in investment during the recession beginning in 2001, the cyclical low point for this ratio still exceeded the high point for all expansions prior to the 1990s, and the ratio has reversed most of this decline by the final period shown, 2004:Q3. Table 1 shows that formal statistical evidence confirms the absence of mean reversion suggested by the chart. Panel A reports results from three tests of the null hypothesis that the ratio series contains a unit root. The first is the standard Augmented Dickey-Fuller $t$-test; the second is the $DF^{GLS}$ test of Elliot, Rothenberg and Stock (1996) which has superior power to the ADF test; the third is the $\bar{M}Z_{\alpha}^{GLS}$ test of Ng and Perron (2001) which has been shown to have excellent size and power properties. For each test, the lag lengths for the test regressions was chosen using Ng and Perron's Modified AIC procedure. In all three cases, the tests do not come close to rejecting the unit root hypothesis at conventional levels of significance.

The table also reports some more direct tests of the balanced growth hypothesis. In general, the null hypothesis of no cointegration between the log of real investment and the log of real consumption can be rejected. However, normalizing the coefficient on real investment to be one, one can reject the hypothesis that the coefficients on consumption in the estimated long-run relationship is minus one, as implied by the one-sector model's balanced growth hypothesis. Panel B's point estimate of -0.865 is derived from the Stock-Watson dynamic least-squares estimation methodology and the balanced growth hypothesis

[2]This statement implicitly follows the model in defining real output as an aggregate of consumption and fixed investment. In contrast, KPSW tested for stationarity of the ratio of real consumption to real private GDP (real GDP excluding government purchases). This differs from an aggregate of consumption and fixed investment when net exports and inventory investment are non-zero. The preference here for the simpler output measure is due to its consistency with the pure one-sector growth model, but the results in this section are not overturned by instead looking at ratios involving private GDP. See Section 3.4 below.

that this coefficient is minus one is tested using an asymptotically valid $t$-test.[3] Panel C's point estimate of -0.843 is derived from the maximum likelihood systems estimation methodology of Søren Johansen (1995), while its test of the balanced growth hypothesis is based on a Wald test statistic derived from comparison of the log-likelihoods for the constrained and unconstrained systems (this has an asymptotic $\chi_1^2$ distribution.) In both cases, the traditional balanced growth hypothesis is rejected at significantly greater than the one percent level.

The fact that the null hypothesis of cointegration between the logs of consumption and investment still cannot be rejected implies that, technically, one cannot reject the idea that there still exists a single-common-trend representation for these two series. However, it is hard to imagine what the *economic* basis for such a single trend representation would be. For example, in an economy with a single technology process, why would a unit shock to that process result in a one-percent increase in investment but only a 0.86 percent increase in consumption. And why would this long-run relationship always have to take the form $(1, -0.86)$? The one-sector growth model provides a powerful intuition about the convergent forces that make it unsustainable for the levels of consumption and investment to have different long-run trends. This provides a theoretical case for $(1, -1)$ as a cointegrating vector that is completely absent for the vector $(1, -0.86)$ or any other vector $(1, -b)$ where $b$ does not equal one.[4]

Figure 1(b) provides an alternative way to think about balanced growth and long-run relationships. The figure shows the ratio of *nominal* private fixed investment to nominal consumption. Unlike the ratio of the real series, this ratio exhibits no apparent trend over time. Table 2 confirms that this alternative notion of balanced growth—that the ratio of nominal expenditures on the two categories is stable over time—receives strong empirical support from the same statistical tests that rejected the traditional formulation of balanced growth. Panel A reports that the hypothesis that the nominal ratio contains a unit root is firmly rejected by each of our tests, while Panels B and C show that point estimates of the cointegrating vectors for the logs of the nominal series are very close to $(1, -1)$, and statistical tests cannot come close to rejecting this null hypothesis.

[3] The exact procedure followed here is discussed on pages 608-612 of Hamilton (1994).

[4] This is not to say that one cannot find empirical models based on this alternative notion of balanced growth. For example, Kim and Piger (2002) use the one-sector growth model to motivate a single common trend for consumption, investment, and output, and then implement this idea by equating log-consumption with the trend and allowing for non-unit weights on the trend for log-investment and log-output.

A first (somewhat obvious) observation about this alternative formulation of balanced growth is that the difference between the series in Figures 1(a) and 1(b) is in itself evidence against the one-sector growth model. The model's assumption that consumption and investment goods are produced using the same technology implies that any decentralized market equilibrium must feature them having the same price, and so real ratios and nominal ratios should be the same thing. The differences between the two charts reflect different price developments for the types of goods: Specifically, it reflects a substantial decline over time in the ratio of investment prices to consumption prices. This relative price movement likely reflects the existence of different production technologies for consumption and investment.

Once one allows for the idea that consumption and investment can be produced using different technologies, and thus that their prices do not always move together, then the stability of the nominal ratio of investment to consumption also provides an intuitive version of the same "sustainability" idea that underlies the one-sector model's balanced growth prediction. In the one-sector context, the ratio of real investment to real consumption cannot trend upwards over time because households will not wish to allocate ever higher-fractions of their incomes towards saving. However, in a two-sector model with faster technological progress in the production of investment goods, then real investment can grow faster than real consumption without requiring ever-increasing sacrifices on the part of households. Moreover, the original intuition behind the one-sector balanced growth hypothesis still holds as an explanation for why the nominal ratio of investment to consumption should be stationary: Increases in the household savings rate (the fraction of nominal income that is saved) will only provide a temporary boost to the growth rates of capital and output.

## 3 Implications for Measurement of Trends and Cycles

The empirical properties just documented suggest that the appropriate theoretical framework for thinking about long-run restrictions on US macro data is one with two types of production technology (thus allowing investment and consumption prices to be different) and with balanced growth formulated in terms of stability of the nominal ratio. We will outline such a model, and how to identify the two technology shocks and their dynamic effects, in Sections 4 and 5.

First, however, we will present some implications of the competing balanced growth hypotheses that hold independent of any specific theoretical framework used to derive them. Specifically, the long-run cointegrating restrictions implied by these hypotheses provide a

natural way to decompose macroeconomic time series into a stochastic trend component on the one hand, and a transitory cyclical component on the other. We show here that the traditional one-sector balanced growth hypothesis produces unsatisfactory estimates of the cyclical components in consumption, investment, and output, and these estimates are compared with those based on the alternative nominal ratio hypothesis.

### 3.1 Traditional Balanced Growth Method

According to the Granger representation theorem, the traditional formulation of the balanced growth hypothesis—that log real investment $i_t$ and log real consumption, $c_t$ are cointegrated with a (1,-1) cointegrating vector—implies the existence of a VECM representation of the following type:

$$\begin{pmatrix} \Delta c_t \\ \Delta i_t \end{pmatrix} = \begin{pmatrix} \alpha_c \\ \alpha_i \end{pmatrix} + B(L) \begin{pmatrix} \Delta c_{t-1} \\ \Delta i_{t-1} \end{pmatrix} + \begin{pmatrix} \gamma_i \\ \gamma_c \end{pmatrix} (i_{t-1} - c_{t-1}) + \epsilon_t. \tag{1}$$

The results from the estimation of a two-lag version of this VECM are reported in Table 3.[5] The coefficients on the error-correction term (the investment-consumption ratio) are both negative, with the coefficient in the investment regression being larger. This implies that a high value of the real investment to real consumption ratio is a negative signal for future growth in both consumption and investment, and that the ratio should tend to decline by a process of investment falling by more than consumption. However, while these coefficient estimates technically imply a stable VAR, it should be kept in mind that both cointegration tests and unit root tests on the investment-consumption ratio reject the hypothesis that this is a satisfactory model of investment and consumption dynamics.

This VECM can be used to derive a Beveridge-Nelson-style decomposition of consumption and investment into their stochastic trend and transitory cycle components. Letting $\omega$ and $\psi$ represent the unconditional expectations for $\Delta c_t$ and $i_t - c_t$, the terms in the VECM can be re-arranged to arrive at a VAR representation in $\Delta c_t - \omega$ and $i_t - c_t - \psi$:

$$(I - C(L)) \begin{pmatrix} \Delta c_t - \omega \\ i_t - c_t - \psi \end{pmatrix} = \epsilon_t. \tag{2}$$

This can be written in companion matrix form as

$$Z_t = AZ_{t-1} + \epsilon_t. \tag{3}$$

[5] This two-lag version is consistent with results from lag-length selection tests which favor three lags in the levels specification.

Our definition of the transitory cyclical component of consumption is

$$c_t^{cyc} = \lim_{k\to\infty} E_t \left(c_t - c_{t+k} + k\omega\right). \tag{4}$$

In other words, the transitory component is the expected cumulative shortfall in future consumption growth relative to its trend growth rate, so that when this component is positive, one should expect the average future growth rate of consumption to be below its trend rate. This can be estimated from the VAR as

$$c_t^{cyc} = -e_1' \left(A + A^2 + A^3 + ....\right) Z_t = -e_1' A \left(I - A\right)^{-1} Z_t \tag{5}$$

where $e_1$ is a $(1, 0)$ vector. The transitory cyclical component of investment can also be estimated from this VAR as

$$\begin{aligned} i_t^{cyc} &= \lim_{k\to\infty} E_t \left(i_t - i_{t+k} + k\omega\right) \\ &= \lim_{k\to\infty} E_t \left(i_t - c_t + c_t - c_{t+k} + c_{t+k} - i_{t+k} + k\omega\right) \\ &= c_t^{cyc} + \left(i_t - c_t - \psi\right) \end{aligned} \tag{6}$$

Figure 2(a) shows the transitory components of investment and consumption generated by the VECM reported in Table 3. Both series display the unsatisfactory property of appearing to trend upwards over time, with the values early in the sample tending to be negative and the values later in the sample tending to be positive.[6] This feature is more evident for investment than for consumption: The non-stationarity is due to both series placing some weight on the real investment-consumption ratio, and the VECM coefficients suggest that this is a more important signal for future investment growth than for future consumption growth.

### 3.2 Nominal Ratio Method

The same technique can be used to derive the transitory components implied by the nominal-ratio balanced growth hypothesis. In this case, the VECM is formulated as

$$\begin{pmatrix} \Delta c_t \\ \Delta i_t \\ \Delta p_t \end{pmatrix} = \begin{pmatrix} \alpha_c \\ \alpha_i \\ \alpha_p \end{pmatrix} + B(L) \begin{pmatrix} \Delta c_{t-1} \\ \Delta i_{t-1} \\ \Delta p_{t-1} \end{pmatrix} + \begin{pmatrix} \gamma_i \\ \gamma_c \\ \gamma_p \end{pmatrix} \left(i_{t-1} - p_{t-1} - c_{t-1}\right) + \epsilon_t. \tag{7}$$

[6] It should be noted that the general pattern reported here turns out to be robust to the addition of a wide range of other cyclical variables to the forecasting VAR.

where $p_t$ is the log of the price of consumption relative to the price of investment. The results from the estimation of a two-lag version of this VECM are reported in Table 4. As with the real-ratio system, the coefficients on the error-correction term (in this case the nominal investment-consumption ratio) are both negative, with the coefficient in the investment regression being larger. However, the sizes of the adjustment coefficients are larger and more statistically significant. In addition, the unit root and cointegration tests suggest that this system provides an acceptable model of the joint dynamics of real investment, real consumption and their relative price.

Letting $\omega^c$, $\omega^i$, and $\mu$ represent the unconditional expectations of consumption growth, investment growth and the nominal ratio of investment to consumption, the nominal-ratio system implies a VAR representation of the form:

$$(I - C(L)) \begin{pmatrix} \Delta c_t - \omega^c \\ \Delta i_t - \omega^i \\ i_t - p_t - c_t - \mu \end{pmatrix} = \epsilon_t. \tag{8}$$

Again, writing this system in the same companion matrix format as equation (3), the cyclical components of consumption and investment can be measured as

$$c_t^{cyc} = -e_1' A \left(I - A\right)^{-1} Z_t \tag{9}$$

$$i_t^{cyc} = -e_2' A \left(I - A\right)^{-1} Z_t \tag{10}$$

where $e_i$ is a vector with one as its $i$th component and zeros elsewhere.

Figure 2(b) shows these new measures of the transitory components of consumption and investment. As with the measures based on the real-ratio VECM, the nominal-ratio approach implies that the transitory component of investment is generally larger and more volatile than the transitory component of consumption. However, unlike the real ratio approach, these measures do not trend over upwards over time.

### 3.3 Measures of the Aggregate Cycle

Figure 3 presents measures of the transitory component of aggregate output generated by the two alternative VECMs. These measures have been defined as

$$y^{cyc} = \theta c^{cyc} + (1 - \theta) i^{cyc} \tag{11}$$

where $\theta$ is the sample average of the ratio of nominal consumption to the sum of nominal consumption and nominal investment. This measure is consistent with the concept of

aggregate real output as a Divisia index of consumption and investment, which is essentially identical to the Fisher chain-index methodology that has been used to construct real aggregates in the US NIPAs since 1996.[7]

Beyond the fact that the transitory series implied by the real ratio method trends upwards, while the nominal-ratio-based measure does not, the most striking aspect of Figure 3 is the substantially different stories these series tell about the behavior of output relative to trend since the early 1990s. The real ratio model sees the long expansion of the 1990s as having brought the economy way above its stochastic trend, with output a massive 12 percent above trend at its peak in 2000:Q2. And this model views the recession beginning in 2001 as only having partially reversed this pattern, with output still being 5 percent above its trend level at the cyclical lowpoint of 2002:Q4.

In contrast, the nominal ratio model sees the recession of the early 1990s as having being particularly severe in the sense of bringing output farthest below its stochastic trend. This model thus sees the long, largely investment-led, expansion of the 1990s as mainly an unwinding of this development. The series generated by the nominal ratio model is also in keeping with the common perception of the post-2000 period as one of cyclical weakness, with the economy seen as 5 percent below its stochastic trend in 2003:Q2. By the end of the sample, at 2004:Q3, the nominal ratio model views output as 1 percent below its stochastic trend, compared with 9.5 percent above trend according to the real ratio model.

Of course, it is also clear that the transitory component of output generated by the nominal ratio method does not always look like the standard interpretation of the business cycle. For instance, it sees the long decline in output 1980-82 period as merely restoring output back to its trend after having been well above it in the late 1970s. However, such results point to an advantage of this method, inasmuch as it can illustrate how the declines in output associated with standard recessions can be due to either cyclical weakness and/or declines in the stochastic trend underlying output. And, as we will discuss in the next section, the stochastic trend generated by this method can be seen as a combination of two technology shocks, so this method allows for the illustration of the role played in business

[7] In light of the evidence of a declining relative price of investment, this measure is clearly superior to the Laspeyeres or fixed-weight approach, which amounts to using the "real shares" implied by a particular base year to weight the growth rates. The level of such a real investment share will be higher the farther back in time is the base year, and for any fixed base year, this share will trend upwards over time, implying that real output growth would tend to asymptote over time to simply equal the growth rate of real investment. See Whelan (2002) for a discussion of these issues.

cycles by technology shocks.

### 3.4 Comparison with Cochrane (1994)

The results just presented may be somewhat surprising because they seem to contradict the conclusions of a number of previous researchers that the restriction of stationary real ratios can be used to generate apparently satisfactory measures of the cyclical component of output. For example, they appear to contradict the results in a well-known paper by John Cochrane (1994). This paper examined the ratio of real consumption of nondurables and services to real GDP and presented evidence in favor of its stationarity. Cochrane concluded that one could construct a useful measure of the transitory component of output from the VECM implied by this cointegrating relationship.

How can Cochrane's conclusions be reconciled with our earlier results, which showed the ratio of real consumption to real fixed investment declining over time? The explanation has two parts.

The first relates to Cochrane's use of a ratio involving *total* real GDP. Figure 4(a) shows that the ratio studied by Cochrane has been relatively stable over time, and has fluctuated within a narrow band. However, when one uses real *private* GDP—consistent with our approach of only looking at private consumption and investment—a downward trend becomes evident, as implied by our earlier results. This interpretation is backed up by formal hypothesis tests which indicate stationarity of the ratio featuring total GDP and non-stationarity of the ratio featuring private GDP, both for the full sample and for the sample ending in 1989:Q3, as in Cochrane's paper.

Figure 4(b) shows the differences between the Beveridge-Nelson measures of the transitory component of output that emerge from these two alternative potential "cointegrating" VARs, i.e. one featuring the ratio involving total real GDP and the other featuring private real GDP. Consistent with our earlier results, the cyclical measure based on the private GDP ratio is unsatisfactory because it trends upwards over time. In contrast, the transitory measure based on total real GDP does appear to be stationary.

How does one interpret the stationarity of the ratio involving total real GDP? Clearly, in light of the evidence already presented, one cannot draw on the logic of the one-sector model. Instead, the stationarity of this series appears to rest on a somewhat fortuitous offset: Real government expenditures have declined relative to real GDP in a fashion that has helped to mask the decline in real consumption relative to the rest of private GDP.

The second part of the explanation is that there have been important revisions to historical data. Cochrane's paper acknowledged that it may be preferable on theoretical grounds to employ a VAR that uses the private GDP ratio, and he reports that for his data, this ratio appears stationary. Because our results for this same sample period point towards this ratio being nonstationary, it is clear that revisions to historical data have contributed to the different conclusions reached here. Such revisions have included the introduction of hedonic price indexes for various categories of investment goods, as well as other steps aimed at addressing some of the issues raised about capital goods price indexes by researchers such as Robert Gordon (1990). These revisions have had the effect of boosting the historical growth rates of real investment relative to real consumption.

# 4 A Simple Two-Sector Model

We have described the econometric implications of our alternative nominal ratio formulation of balanced growth. Here, we provide a simple two-sector model that provides a theoretical foundation for this approach, and document some additional implications of the model for the long-run behavior of consumption, investment, and their relative price.

## 4.1 The Nominal Ratio Balanced Growth Hypothesis

The production functions for consumption and investment in the model take the form

$$C = A_C K_C^{\beta_c} L_C^{1-\beta_c}, \tag{12}$$

$$I = A_I K_I^{\beta_i} L_I^{1-\beta_i}, \tag{13}$$

where $K_I, K_C, L_I, L_C$ are the capital and labor inputs of the consumption and investment-producing sectors. Models similar to this one, featuring two production technologies, have been discussed by Greenwood, Hercowitz and Krusell (1997) and Whelan (2003).

We assume that firms in both sectors are price-takers and maximize the profit functions:

$$\pi_C = P_C A_C K_C^{\beta_c} L_C^{1-\beta_c} - wL_C - P_I \left( i + \delta - \frac{\dot{P}_I}{P_I} \right) K_C, \tag{14}$$

$$\pi_I = P_I A_I K_I^{\beta_i} L_I^{1-\beta_i} - wL_I - P_I \left( i + \delta - \frac{\dot{P}_I}{P_I} \right) K_I. \tag{15}$$

Here we have assumed that the rental rate for capital is determined by the Jorgensonian user cost of capital formula, where $i$ is the nominal required rate of return on investments

in capital and $\delta$ is the depreciation rate for capital. Solving the first-order conditions for profit-maximization, we get the following factor demand equations:

$$K_C = \frac{P_C}{P_I}\frac{\beta_c C}{i+\delta-\frac{\dot{P}_I}{P_I}} \tag{16}$$

$$K_I = \frac{\beta_i I}{i+\delta-\frac{\dot{P}_I}{P_I}} \tag{17}$$

$$L_C = \frac{(1-\beta_c)\,P_C C}{w} \tag{18}$$

$$L_I = \frac{(1-\beta_i)\,P_I I}{w} \tag{19}$$

An implication of these conditions is that for both factors, the ratios of the quantities of the factor used in the two sectors are strictly proportional to the ratio of nominal outputs:

$$\frac{K_I}{K_C} = \frac{\beta_i}{\beta_c}\frac{P_I I}{P_C C}, \tag{20}$$

$$\frac{L_I}{L_C} = \frac{1-\beta_i}{1-\beta_c}\frac{P_I I}{P_C C}. \tag{21}$$

Now consider the properties of this economy along a steady-state growth path, that is a growth path in which all real variables in the economy grow at constant rates. Note first that if capital in the $j$ sector is growing at a constant rate $G_j$, then investment in that type of capital must be given by

$$I_j = (G_j+\delta)\,K_j \tag{22}$$

Thus, the composition of output in the investment sector along a steady-state growth path can be written as

$$I = \frac{P_C}{P_I}\frac{\beta_c\,(G_C+\delta)\,C}{i+\delta-\frac{\dot{P}_I}{P_I}} + \frac{\beta_i\,(G_I+\delta)\,I}{i+\delta-\frac{\dot{P}_I}{P_I}} \tag{23}$$

This re-arranges to give

$$\frac{P_I I}{P_C C} = \frac{\beta_c\,(G_C+\delta)}{i+\delta-\frac{\dot{P}_I}{P_I}-\beta_i\,(G_I+\delta)} \tag{24}$$

Because the quantity on the right can be assumed to be constant along a steady-state growth path, the ratio of nominal outputs is also constant.

If we move away from deterministic steady-states, and instead assume that the technology shocks for consumption and technology sectors evolve in a stochastic fashion with trend growth rates $\mu_I$ and $\mu_C$, i.e. that

$$\Delta a_{C,t} = \mu_C + \eta_{C,t} \tag{25}$$

$$\Delta a_{I,t} = \mu_I + \eta_{I,t} \tag{26}$$

where $\eta_{C,t}$ and $\eta_{I,t}$ are $I(0)$ series, then for a wide range of standard specifications for preferences, the economy's dynamic stochastic general equilibrium will feature the growth rates of consumption, investment, and relative prices all equalling their nonstochastic steady-state values plus a set of stationary deviations. Thus, these results provide a simple theoretical basis for our alternative formulation of long-run balanced growth.

Before moving on, it can be noted that the assumption of Cobb-Douglas technology—and most specifically its implication of unit-elastic factor demands—is essential to the derivation of the result of a stable long-run nominal ratio of consumption to investment. While some may feel uncomfortable with this restrictive specification of the production technology, the model has been designed to fit the evidence on the stability of this nominal ratio and unit-elastic factor demands (and hence Cobb-Douglas technology) appear necessary to fit this pattern. In addition, the recent work of Jones (2005) demonstrates that Cobb-Douglas-shaped aggregate production functions can be derived from ideas-based models under very general assumptions.

### 4.2 Other Long-Run Restrictions

In addition to providing an economic basis for the nominal ratio balanced growth hypothesis, the two-sector model implies a clear set of restrictions on the joint long-run behavior of consumption, investment, and their relative price. To see this, note that equations (20) and (22) together imply that, along the steady-state growth path, $I_C$ and $I_I$—and thus $K_C$ and $K_I$—must expand at the same growth rate as aggregate real investment.

Normalizing the steady-state growth rate of aggregate labor input to zero (implying constant labor input in both sectors) and denoting the steady-state growth rates of the consumption- and investment-sector technologies by $\mu_C$ and $\mu_I$, these considerations imply that the steady-state growth rates of consumption and investment are determined by:

$$g_C = \mu_C + \beta_c g_I \tag{27}$$

$$g_I = \mu_I + \beta_i g_I \tag{28}$$

These equations solve to give

$$g_C = \mu_C + \frac{\beta_c \mu_I}{1 - \beta_i} \tag{29}$$

$$g_I = \frac{\mu_I}{1 - \beta_i} \tag{30}$$

The real growth rates of consumption and investment will generally differ along the steady-state growth path, with investment growing faster as long as $\mu_I > \mu_C$. So, the fact that the ratio of nominal outputs of the two sectors is constant along the steady-state growth path implies that the growth rate of the price of consumption relative to the price of investment will be the negative of the relative growth rates of the quantities, i.e. that

$$g_P = g_I - g_C = \left(\frac{1-\beta_c}{1-\beta_i}\right)\mu_I - \mu_C \tag{31}$$

# 5 Identifying Two Technology Shocks

We will now discuss how the long-run restrictions implied by the two-sector model allow us to identify the stochastic processes for consumption- and investment-sector technology as well as the dynamic effects of shocks to these technologies.

## 5.1 The VMA Representation

We have described how our preferred nominal-ratio balanced growth hypothesis implied two different representations, the VECM system of equation (7), and the VAR system of equation (8). In addition, however, it is well known that any cointegrated VAR with $n$ variables and $r$ cointegrating relations can also be represented using a Vector Moving Average (VMA) representation of the form

$$\Delta X_t = \theta + D(L)\,\epsilon_t, \tag{32}$$

where $D(1)$ has a reduced rank of $n - r$ reflecting the restrictions on long-run behavior imposed by the cointegrating relations. In the case of the nominal ratio model, this means the existence of a representation of the form

$$\Delta X_t = \begin{pmatrix} \Delta c_t \\ \Delta i_t \\ \Delta p_t \end{pmatrix} = \theta + D(L)\,\epsilon_t. \tag{33}$$

where $D(1)$ has a rank of two.

A reduced-form VMA representation can be derived directly from inverting the VECM representation and this can be used to derive impulse responses to the shock terms $\epsilon_t$. However, the shock terms and impulse responses obtained from this reduced-form representation

are not unique. For any nonsingular matrix $G$, there exists an observably equivalent representation of the form

$$\Delta X_t = \theta + \left(D(L)\,G^{-1}\right) G\epsilon_t, \tag{34}$$

with shock terms $G\epsilon_t$ and impulse responses given by $D(L)\,G^{-1}$. So, to obtain shocks and impulse responses that have a useful economic interpretation, it is necessary to impose theory-based restrictions on the $G$ matrix.

In the rest of this section, we show how the long-run restrictions implied by our two-sector model can be combined with the method of King, Plosser, Stock and Watson (1991) to identify structural shocks and impulse responses that are consistent with this model. In other words, we identify a structural representation

$$\Delta X_t = \theta + \Gamma(L)\,\eta_t \tag{35}$$

in which the shocks $\eta_t$ have an economic interpretation consistent with our model.

## 5.2 Identification Methodology

The identification of the structural representations proceeds in a number of steps. The first step is to note that any system of $n$ different $I(1)$ variables with $r$ cointegrating relationships can be expressed in terms of the Stock and Watson (1988) common trends representation with $n-r$ common trends. This means that the non-stationary $I(1)$ components of our three variables can be expressed as functions of two $I(1)$ stochastic trends. Algebraically, these so-called "permanent" components can be written as

$$X_t^p = X_0 + A\tau_t \tag{36}$$

$$\tau_t = \mu t + \sum_{s=1}^{t} \eta_s^p \tag{37}$$

where $A$ is a $3 \times 2$ matrix, $\eta_t^p$ is a vector containing the two shocks to the permanent component, and $A\mu = \theta$.

The second step notes that our two-sector model provides the restrictions to allow us to identify the two common trends as corresponding to the states of technology in the consumption- and investment-producing sectors. This allows one to write the matrix of long-run effects in the structural VMA represention as

$$\Gamma(1) = \begin{pmatrix} A & 0 \end{pmatrix} \tag{38}$$

with the vector of shocks being written as

$$\eta_t = \begin{pmatrix} \eta_t^p \\ \eta_t^t \end{pmatrix} \tag{39}$$

where $\eta_t^t$ represents the model's sole transitory shock.

Third, the model pins down the $A$ matrix. Together, equations (29), (30), and (31) imply that the long-run multipliers can be written as

$$\Gamma(1) = \begin{pmatrix} 1 & \frac{\beta_c}{1-\beta_i} & 0 \\ 0 & \frac{1}{1-\beta_i} & 0 \\ -1 & \frac{1-\beta_c}{1-\beta_i} & 0 \end{pmatrix} \tag{40}$$

where the first two columns describe the long-run effects of the consumption technology shock and the investment technology shock respectively.

This information is sufficient to identify the two technology shocks from the reduced-form VMA. This is done as follows. Starting from the definition of the structural shocks, we have

$$\Gamma(1)\,\eta_t = D(1)\,\epsilon_t. \tag{41}$$

Multiplying both sides by $\Gamma'(1)$, this becomes

$$\Gamma'(1)\,\Gamma(1)\,\eta_t = \Gamma'(1)\,D(1)\,\epsilon_t. \tag{42}$$

This can be re-written as

$$\begin{bmatrix} A'A & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \eta_t^p \\ \eta_t^t \end{pmatrix} = \begin{bmatrix} A'D(1) \\ 0 \end{bmatrix} \epsilon_t. \tag{43}$$

So, the technology shocks are identified as

$$\eta_t^p = (A'A)^{-1} A'D(1)\,\epsilon_t. \tag{44}$$

The transitory shock can be identified from the assumption that it is uncorrelated with either of the technology shocks. This is because there is a unique (up to a scalar multiple) $1 \times 3$ vector of coefficients that describes the linear combination of the reduced-form shocks that is orthogonal to both of the structural shocks. Specifically, letting $\Omega$ be the covariance matrix of the reduced-form errors, $\epsilon_t$, and defining $\left[(A'A)^{-1} A'D(1)\right]_\perp$ to be a $3 \times 1$ vector such that

$$(A'A)^{-1} A'D(1) \left[(A'A)^{-1} A'D(1)\right]_\perp = 0 \tag{45}$$

then the transitory shock is defined as

$$\eta_t^t = \left[ (A'A)^{-1} A'D(1) \right]_{\perp}' \Omega^{-1} \epsilon_t. \tag{46}$$

In terms of the notation used in equation (34), the long-run restriction methodology achieves identification by setting

$$G = \begin{pmatrix} (A'A)^{-1} A'D(1) \\ \left[ (A'A)^{-1} A'D(1) \right]_{\perp}' \Omega^{-1} \end{pmatrix} \tag{47}$$

Finally, note that this identification of the structural shocks allows for a direct economic interpretation of the Beveridge-Nelson decompositions presented earlier. This is because, by definition, the expected values of deviations in the distant future from the variables' permanent components—defined in equations (36) and (37)—are all zero, so these are identical to the permanent components identified by our Beveridge-Nelson decompositions in Section 3. And the economic interpretation of the transitory components produced by these decompositions is that they represent the components of consumption, investment, and output that are unrelated to the states of technology in either the consumption- or investment-producing sectors.

### 5.3 Discussion of Identification Methodology

The generality of this identification methodology is worth noting: It relies only on the long-run restrictions implied by the two-sector growth model and on the assumption that the technology shocks are uncorrelated with the transitory shock. Importantly, no assumption whatsoever has been made about the covariance structure of the technology shocks themselves. This is important because many methodologies for identifying structural shocks in VAR models proceed by assuming each of the shocks are uncorrelated. In this case, such an assumption would clearly be unwarranted: There is no reason to expect that technological innovations in the investment sector should be uncorrelated with similar innovations in the consumption sector. Moreover, as we discuss in Section 7, our identification allows one to assess the validity of other methodologies which have relied on alternative assumptions about the covariance structure of technology shocks across sectors.

# 6 Results

## 6.1 Implementation Details

In the analysis that follows, the standard value of the capital elasticity of one-third was applied to both sectors, i.e $\beta_i = \beta_c = \frac{1}{3}$ is used. This is based on evidence relating to the composition of value-added by industry which suggest that sectors oriented towards the production of capital goods appear to have similar capital shares to the rest of the sectors.[8] With these assumptions, our long-run identifying restrictions can be written as

$$\Gamma(1) = \begin{pmatrix} 1 & 0.5 & 0 \\ 0 & 1.5 & 0 \\ -1 & 1 & 0 \end{pmatrix}. \tag{48}$$

It should be noted, however, that the pattern of the results reported here remain quite similar when a range of different plausible values are used for $\beta_c$ and $\beta_i$.

The empirical implementation of the two-sector identification was carried out using a three-variable cointegrated VAR featuring the log of per capita real consumption, the log of per capita real fixed investment, and the log of the relative price of consumption to investment.[9] As before, the sample was 1949:1 to 2004:3 and the system was estimated with three lags.[10]

## 6.2 Technology Shocks and Their Effects

We first consider the technology shocks that emerge from the imposition of the model's long-run restrictions. Figure 5(a) displays the estimates of the two technology series, $a_{C,t}$ and $a_{I,t}$. Not surprisingly, in light of the facts documented earlier, the series for investment technology has grown faster over the whole sample than the series for consumption technology. However, what may be somewhat more surprising is that this pattern only becomes evident after 1980. The reasons for this can be seen in Figure 5(b). The gap between the two technology series can be equated with the long-run component of the price of consumption relative to investment. The figure shows that the transitory component for the relative price series is quite small, and that the series appeared to be relatively stationary up until about 1980, after which it increased steadily.

[8] Greenwood, Hercowitz and Krusell (1997) also argue that this is a reasonable assumption.

[9] The population measure used is the civilian population aged sixteen and over.

[10] This is the lag length consistent with the optimal value for the Akaike Information Criterion; however, the substance of the results discussed here were not found to be sensitive to the lag length chosen.

Our empirical estimates of the average growth rates of the stochastic trends ($\mu_C$ and $\mu_I$) allow us to calculate—via equation (29)—the average contributions of the two types of technological progress to the steady-state growth in consumption per capita. The average growth rate of consumption technology, $\mu_C$, is estimated to be 0.32 percent per quarter, while $\mu_I$ is estimated at 0.45 percent per quarter. These parameters are estimated fairly tightly: Standard errors based on 5000 bootstrap replications of the reduced-form VAR process are 0.03 percent for $\mu_C$ and 0.06 percent for $\mu_I$. Using these figures along with our assumption of $\beta_c = \beta_i = \frac{1}{3}$, equation (29) implies that 42 percent of long-run growth in consumption per capita is due to technological improvements in the investment sector, with the rest being due to technological progress in the consumption sector. And using a long-run ratio of nominal consumption to nominal investment of 4:1, these figures imply that technological progress in the investment sector accounts for 53 percent of long-run growth in a Divisia index of output per capita.

Turning from long-run trends to the behavior of the observed series relative to their estimated permanent components, an important pattern that emerges from Figures 2(b) and 5(b) is that consumption and relative prices tend to stay much closer to their permanent components than does investment. This result, of course, echoes the conclusions of Fama (1992), Cochrane (1994), and others that transitory shocks play a much smaller role in determining consumption than they do in determining investment. However, whereas these and subsequent researchers have viewed this result as implying that consumption is a useful proxy for a unique single common trend that also underlies investment and output, in our case, this pattern simply means that consumption stays close to a weighted average of two different stochastic trends, one due to consumption technology and one due to investment technology.

Figure 6 illustrates the impulse responses of consumption, investment, and their relative price to shocks to the consumption and investment technologies as well as to the model's single transitory shock. It should be noted that, as expected, our two technology shock series have a positive correlation of 0.36. However, the series in Figure 6 are the structural impulse responses given by $D(L)G^{-1}$ and have not been calculated based on any Choleski-type re-ordering of the shocks. In other words, these series provide a direct answer to the question "what happens when there is a positive unit shock to the technology of sector $i$ without any shock to sector $j$'s technology?" rather than "what tends to be observed on average when there is a positive shock to technology in sector $i$?"

The impulse responses help to flesh out why it is that investment deviates more substantially than the other variables from its permanent component (see Figure 2(b)). First, they show that investment is substantially more responsive to the transitory shock than the other variables, both in the magnitude of its maximum response and the time it takes for the shock to wear off. Second, in the short-run, investment displays large reactions to technology shocks that differ significantly from its long-run responses. For instance, although consumption technology has no effect on investment in the long run, the short-run response of investment to a consumption technology shock is almost as positive as the response of consumption. More intriguingly, the initial response of investment to shocks to investment-sector technology is *negative*, with the response only moving towards its long-run positive value with a substantial lag.

### 6.3 Sources of Business Cycle Fluctuations

Having identified the technology shocks for the two sectors and their dynamic effects, we can also be more precise about the role that these shocks play in generating the business cycle. Here, we report some estimates of the role played by technology shocks in business cycle fluctuations in consumption, investment, and output using a method previously employed in a one-sector context by Christiano, Eichenbaum, and Vigfusson (2003).

This method is implemented as follows. First, the estimated structural VAR model is re-simulated over history with all of the historical estimated non-technology shocks set to zero. Second, the simulated technology-driven series for consumption, investment and output are cyclically adjusted using a Hodrick-Prescott filter. Then, the cyclically-adjusted simulated technology-driven series are compared with the cyclically-adjusted version of the historical series. Figure 7 charts the actual and technology-driven cyclically-adjusted series for output, consumption, and investment.

The chart shows that there is a relationship between actual business cycles and the cycles generated from a technology-shocks-only simulation of history. There is a positive correlation of 0.42 between cyclically-adjusted output and the technology-driven simulated series (again defining output as a Divisia-index aggregate of consumption and investment). The correlations for consumption and investment are 0.62 and 0.30 respectively. However, as reported in Table 5, the technology-driven series systematically fail to explain the *magnitude* of economic fluctuations, most notably for investment. Technology shocks account for only 21 percent of the variance of business-cycle fluctuations in output, with the corresponding

figures for consumption and investment being 44 percent and 9 percent.

This methodology also allows one to isolate the separate roles played in the business cycle by consumption-sector and investment-sector technology shocks. Simulating the model with only consumption-sector technology shocks, one can explain 17 percent of the business-cycle variance in output. Simulating the model with only investment-sector technology shocks, a mere 3.5 percent of the variance can be explained. This low number is due in part to the very low fraction (6.9 percent) of the variance in investment fluctations that can be explained by investment-sector technology shocks. This, in turn, can be related back to the substantial responses of investment to transitory shocks documented in Figure 6.

Overall, these results confirm and extend the conclusions of King, Plosser, Stock, and Watson (1991) that technology shocks appear to play a limited role in determining business cycle fluctuations, with the effects of these shocks being most notable for investment.

# 7 On "Investment-Specific" Technology Shocks

Following Greenwood, Hercowitz, and Krusell (1997), some researchers have described the technology terms in two-sector models such as ours using a slightly different formulation. This formulation would see our model re-written as

$$C = zK_C^{\beta_c} L_C^{1-\beta_c}, \tag{49}$$

$$I = zqK_I^{\beta_i} L_I^{1-\beta_i}, \tag{50}$$

with $z$ termed "neutral" technology and $q$ termed "investment-specific" technology As a theoretical matter, this formulation provides an isomorphic description of the economy to ours. In terms of identifying the technology series, the $z$ series is identical to our consumption technology series and the $q$ series is the ratio of investment technology to consumption technology.[11] Similarly, our analysis of the effects of technology shocks can be re-phrased in terms of this alternative formulation: The impulse response to a neutral shock that produces a unit increase in the technology series for both sectors can be calculated simply by adding together the impulse responses for the two technology shocks in our analysis. And

[11] For this reason, the "neutral" terminology is not very satisfactory. For instance, we could re-write the model with $z$ multiplying the investment sector production function and $zq$ multiplying the consumption sector production function. In this case, "neutral" technology would actually be investment sector technology, and $q$ would be "consumption-specific" technology.

the effects of a so-called investment-specific shock—a shock that improves investment technology but not consumption technology—are identical to the effects of the investment-sector shock that we have described.

However, beyond its usage as an alternative way of describing the model, this formulation does have *empirical* content if it is assumed that the shocks to the $z$ and $q$ series are uncorrelated, as is done for instance in Greenwood, Hercowitz, and Krusell (2000). This assumption implies that, ceteris paribus, one should expect that any positive shock to consumption technology should also show up one-for-one in the technology of the investment sector. Our analysis allowed for identification of consumption- and investment-sector technology shocks without any assumptions about their covariance, and the results based upon this more general identification call into doubt the idea of uncorrelated neutral and investment-specific shocks.

Specifically, in our analysis, the log of investment-specific technology shocks can be calculated as the difference between the logs of investment-sector and consumption-sector technology. Thus calculated, there is a statistically significant negative correlation between investment-specific shocks and consumption (i.e. neutral) shocks of -0.30. One way of explaining this result is to note that the correlation between technology shocks in the two sectors, while positive, is weak enough that a positive shock to consumption technology tends to imply that consumption technology increases relative to investment technology, (implying a decline in measured investment-specific technology). These calculations place in doubt the accuracy of identifications that rely on the assumption of uncorrelated $z$ and $q$ shocks.

# 8 Conclusions

This paper has had a number of goals.

First, it has been shown that the inclusion of recent data from the US national accounts overturns earlier widely-cited results that supported the one-sector model's balanced growth predictions. This is important because the idea of stable "great ratios" of real consumption to real investment or of real investment to real GDP, has generally been considered a central stylized fact in macroeconomics. The fact that real investment appears to have a different long-run trend growth rate from real consumption in US data should have important implications for macroeconomic analysis, given that many empirical and theoretical studies

take the one-sector growth model as a baseline for characterizing the long-run behavior of the economy.

Second, an alternative formulation of the idea of balanced growth—that the ratio of nominal consumption to nominal investment should be stationary—was suggested and found to provide a good description of the US data. It was shown that this formulation produces estimates of the transitory or cyclical components of consumption and investment that have more satisfactory features than those based on the traditional "real ratio" formulation of balanced growth.

Third, a simple two-sector framework was proposed that is consistent with the long-run properties of the data. The model acknowledges that capital goods appear to be produced with a different technology than consumption goods, with the pace of technological change in the production of capital goods being faster on average. A structural VAR analysis was implemented to explore the implications of this approach for the macroeconomic effects of technology shocks. The results suggest that technology shocks are not a dominant force driving the business cycle. In light of these results, an obvious next direction for research is the development of theoretical models that are consistent with the long-run facts presented here and are also consistent with the evidence concerning the role played in business cycles by technology shocks.

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Table 1: Tests of the Traditional Balanced Growth Hypothesis

*A. Unit Root Tests for $\frac{i}{c}$*

| Test | Statistic | 5%/10% Values |
|---|---|---|
| ADF $t$-statistic | -1.81 | -2.88/-2.57 |
| Elliot-Rothenberg-Stock $DF^{GLS}$ | -1.27 | -1.95/-1.62 |
| Ng-Perron $\bar{M}Z_{\alpha}^{GLS}$ | -1.72 | -8.10/-5.70 |

*B. Estimated Cointegrating Vector: Stock-Watson DOLS*

| Variable | Null Hypothesis | Estimates |
|---|---|---|
| $\log(i)$ | 1 | 1 |
| $\log(c)$ | -1 | -0.865 |

$t$-test of Balanced Growth Restriction: $t = -4.94$ ($p$=0.0000007)

*C. Estimated Cointegrating Vector: Maximum Likelihood*

| Variable | Null Hypothesis | Estimates |
|---|---|---|
| $\log(i)$ | 1 | 1 |
| $\log(c)$ | -1 | -0.843 |

Wald Test of Balanced Growth Restriction: $\chi_1^2 = 17.19$ ($p$=0.00003)

*Notes*: $i \equiv$ Real private fixed investment at 2000 dollars, $c \equiv$ Real consumption expenditures at 2000 dollars. Sample is 1949:1 to 2004:3. DOLS estimation was based on four first-difference leads and four lags; $t$-test calculated as in Hamilton (1994) pages 608-612. Lag length for ML VAR was three (chosen by AIC). See text for additional details.

Table 2: Tests of the Nominal-Ratio Balanced Growth Hypothesis

*A. Unit Root Tests for* $\frac{i^*}{c^*}$

| Test | Statistic | 5%/10% Values |
|---|---|---|
| ADF $t$-statistic | -3.16 | -2.88/-2.57 |
| Elliot-Rothenberg-Stock $DF^{GLS}$ | -2.91 | -1.95/-1.62 |
| Ng-Perron $\bar{M}Z_{\alpha}^{GLS}$ | -9.14 | -8.10/-5.70 |

*B. Estimated Cointegrating Vector: Stock-Watson DOLS*

| Variable | Null Hypothesis | Estimates |
|---|---|---|
| $\log(i^*)$ | 1 | 1 |
| $\log(c^*)$ | -1 | -1.00003 |

$t$-test of Balanced Growth Restriction: $t = -0.0014$ ($p$=0.99)

*C. Estimated Cointegrating Vector: Maximum Likelihood*

| Variable | Null Hypothesis | Estimates |
|---|---|---|
| $\log(i^*)$ | 1 | 1 |
| $\log(c^*)$ | -1 | -1.007 |

Wald Test of Balanced Growth Restriction: $\chi_1^2 = 0.47$ ($p$=0.49)

*Notes*: $i^* \equiv$ Nominal private fixed investment, $c^* \equiv$ Nominal consumption expenditures. Sample is 1949:1 to 2004:3. DOLS estimation was based on four first-difference leads and four lags; $t$-test calculated as in Hamilton (1994) pages 608-612. Lag length for ML VAR was five (chosen by AIC). See text for additional details.

Table 3: VECM Implied by Traditional Balanced Growth Hypothesis

| | $\Delta c_t$ | $\Delta i_t$ |
|---|---|---|
| Constant | -0.008 | -0.051 |
| | (-0.089) | (-2.212) |
| $\Delta c_{t-1}$ | -0.103 | 0.279 |
| | (-1.268) | (1.240) |
| $\Delta c_{t-2}$ | 0.263 | 0.362 |
| | (3.243) | (1.617) |
| $\Delta i_{t-1}$ | 0.065 | 0.424 |
| | (2.186) | (5.144) |
| $\Delta i_{t-2}$ | -0.016 | -0.015 |
| | (-0.557) | (-0.196) |
| $i_{t-1} - c_{t-1}$ | -0.007 | -0.031 |
| | (-1.382) | (2.140) |

*Notes*: $i \equiv$ Real private fixed investment, $c \equiv$ Real consumption expenditures. Sample is 1949:1 to 2004:3. $t-$statistics are given in parentheses below coefficient estimates

Table 4: VECM Implied by Nominal Ratio Balanced Growth Hypothesis

| | $\Delta c_t$ | $\Delta i_t$ | $\Delta p_t$ |
|---|---|---|---|
| Constant | -0.018 | -0.085 | -0.003 |
| | (-1.835) | (-3.194) | (-0.488) |
| $\Delta c_{t-1}$ | -0.106 | 0.305 | 0.104 |
| | (-1.304) | (1.372) | (1.984) |
| $\Delta c_{t-2}$ | 0.279 | 0.393 | 0.088 |
| | (3.457) | (1.774) | (1.682) |
| $\Delta i_{t-1}$ | 0.065 | 0.398 | -0.032 |
| | (2.184) | (4.884) | (-1.663) |
| $\Delta i_{t-2}$ | -0.021 | -0.018 | -0.024 |
| | (-0.074) | (-0.023) | (-1.300) |
| $\Delta p_{t-1}$ | -0.215 | -0.130 | 0.270 |
| | (-2.098) | (-0.462) | (4.072) |
| $\Delta p_{t-2}$ | 0.204 | 0.606 | 0.244 |
| | ( 1.994) | (2.162) | (3.696) |
| $i_{t-1} - p_{t-1} - c_{t-1}$ | -0.015 | -0.059 | -0.002 |
| | (2.255) | (3.164) | (0.484) |

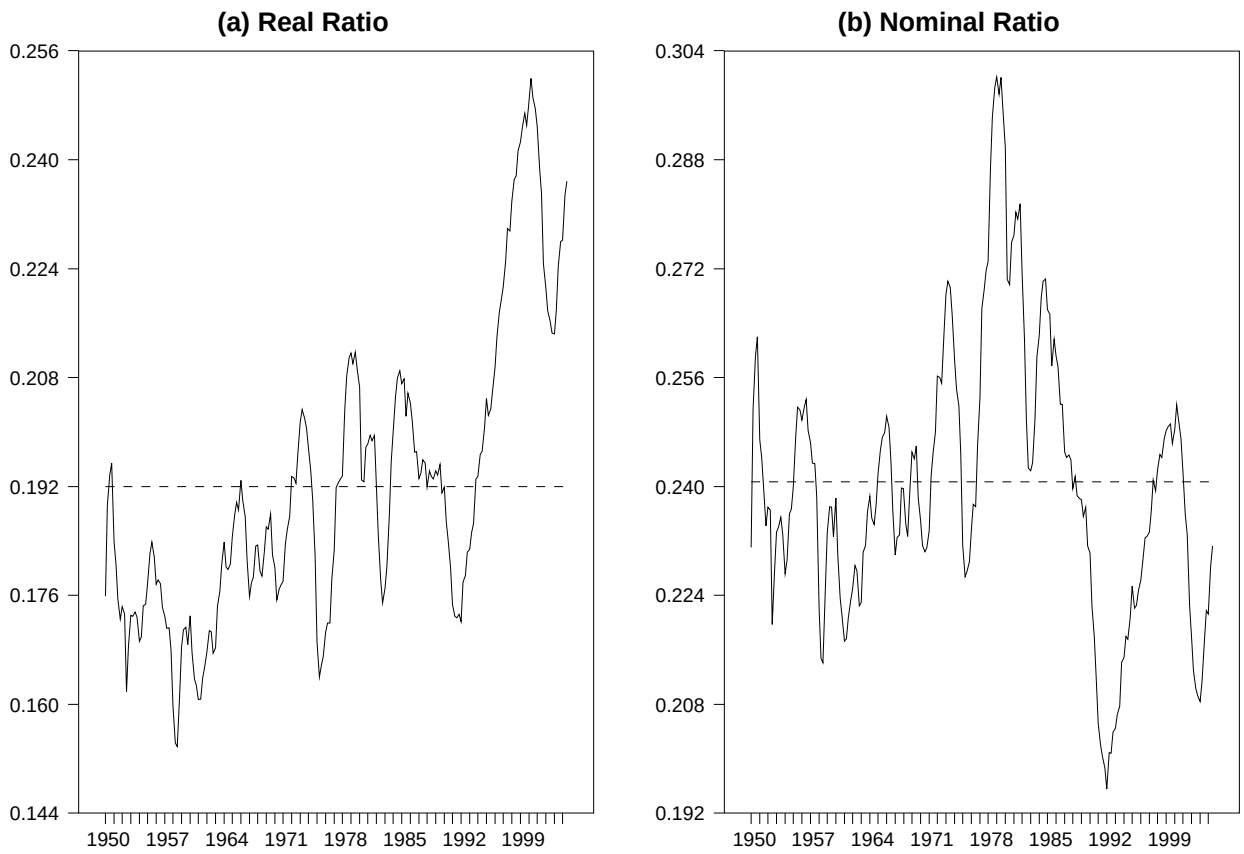
*Notes*: $i \equiv$ Real private fixed investment, $c \equiv$ Real consumption expenditures, $p \equiv$ Price of consumption relative to the price of investment. Sample is 1949:1 to 2004:3. $t-$statistics are given in parentheses below coefficient estimates

Table 5: Contribution of Technology Shocks to Cyclical Variance

| | Consumption | Investment | Output |
|---|---|---|---|
| Both Technology Shocks | 0.444 | 0.092 | 0.209 |
| Consumption Shocks Only | 0.380 | 0.037 | 0.174 |
| Investment Shocks Only | 0.022 | 0.069 | 0.035 |

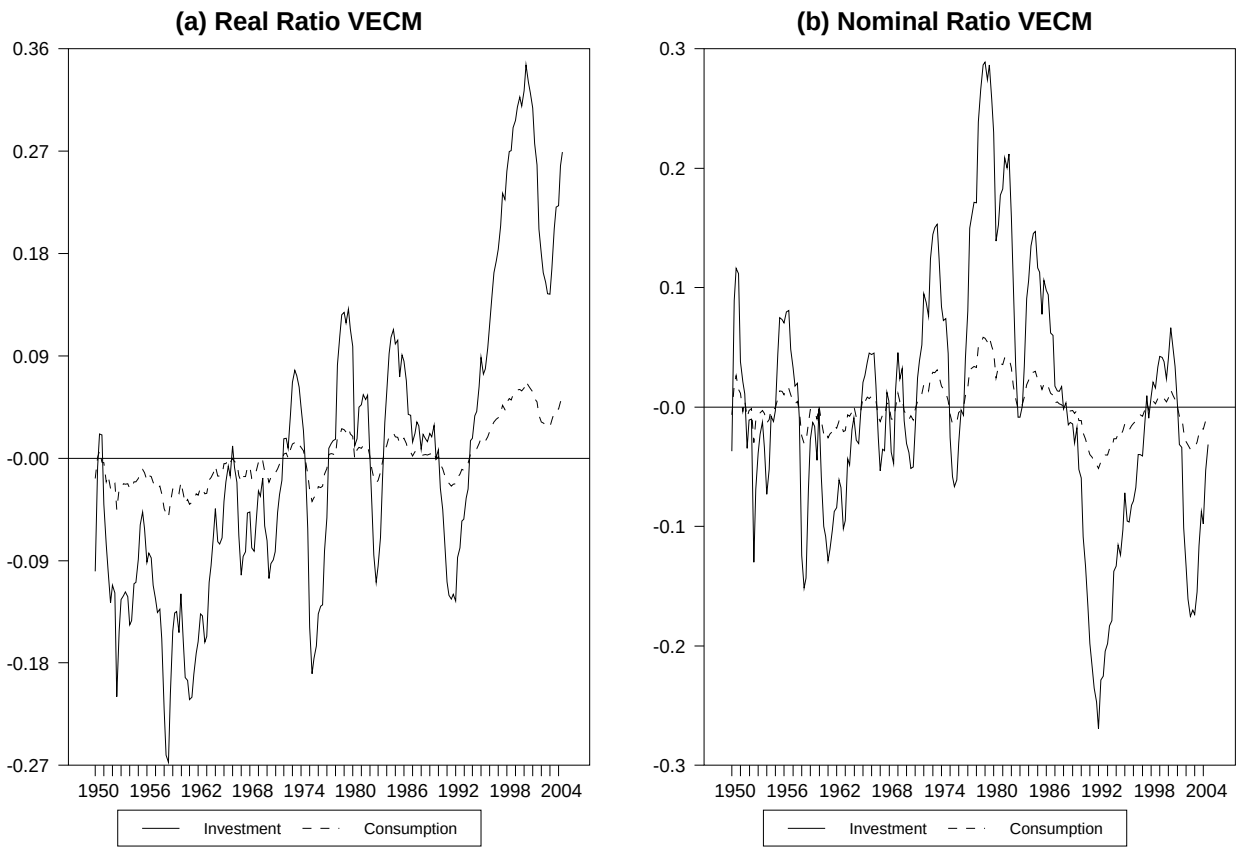
# Figure 1

*Ratios of Investment to Consumption (with Sample Averages)*



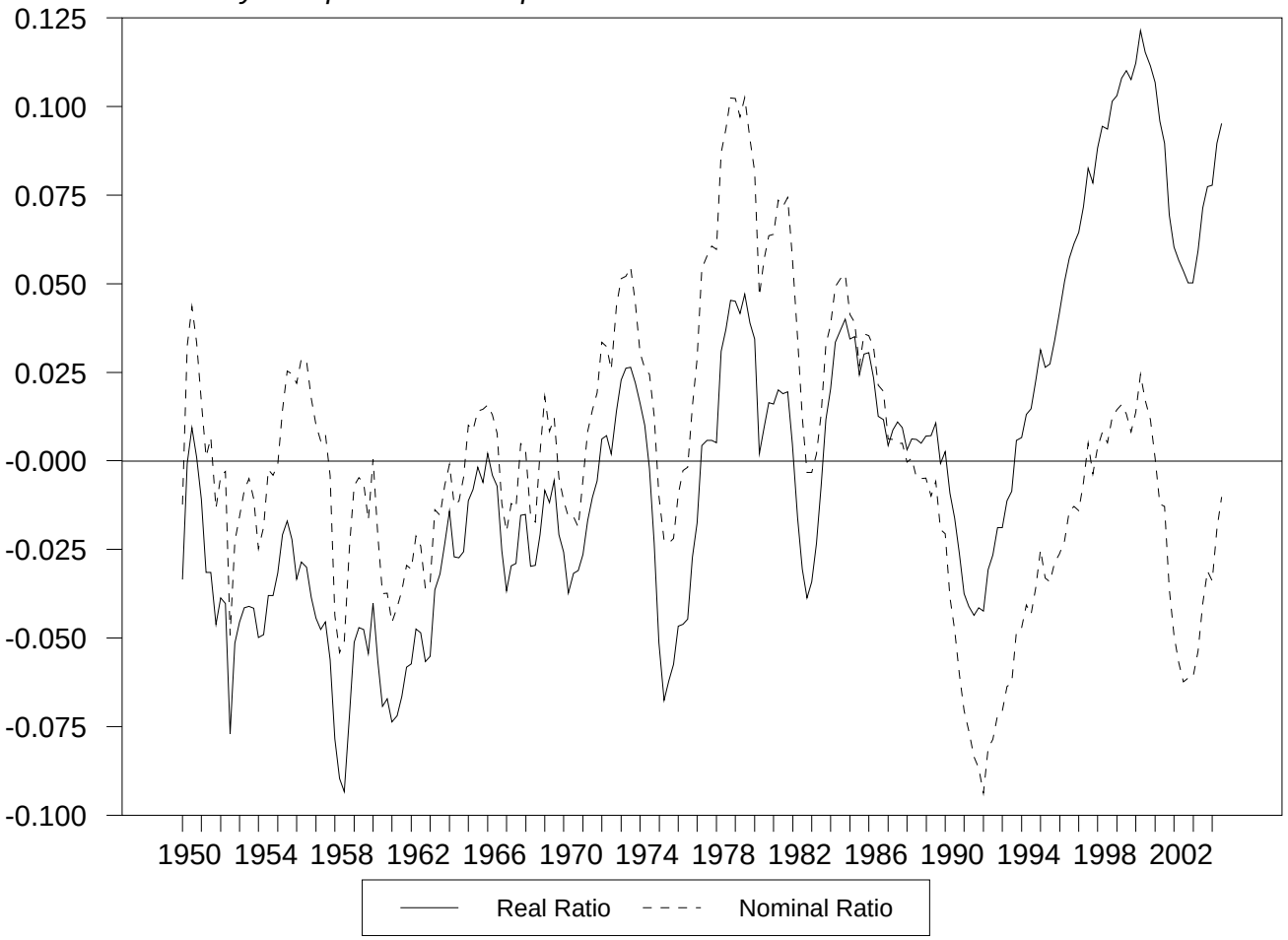
# Figure 2

*Transitory Components from Real- and Nominal-Ratio-Based VECMs*



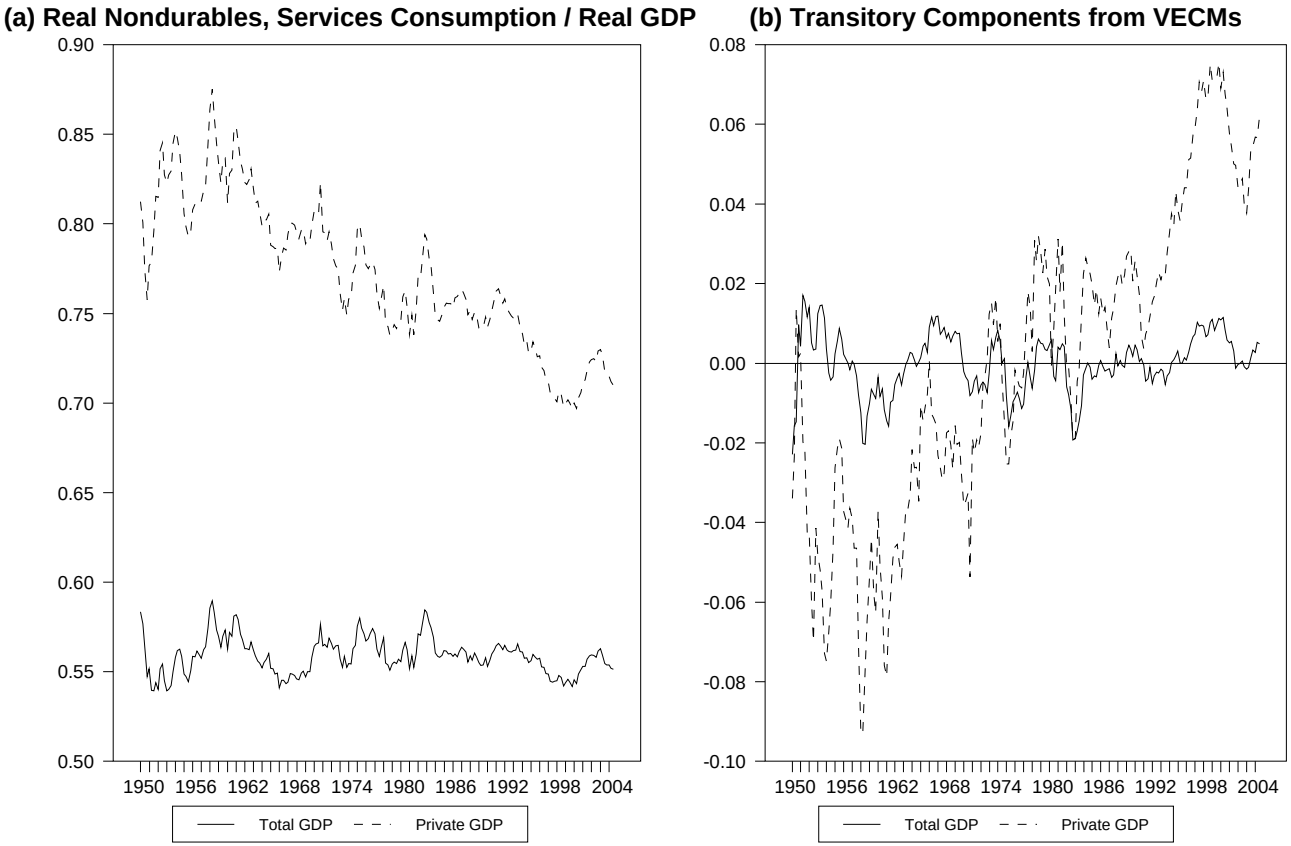
# Figure 3

*Transitory Components of Output from Real- and Nominal-Ratio-Based VECMs*

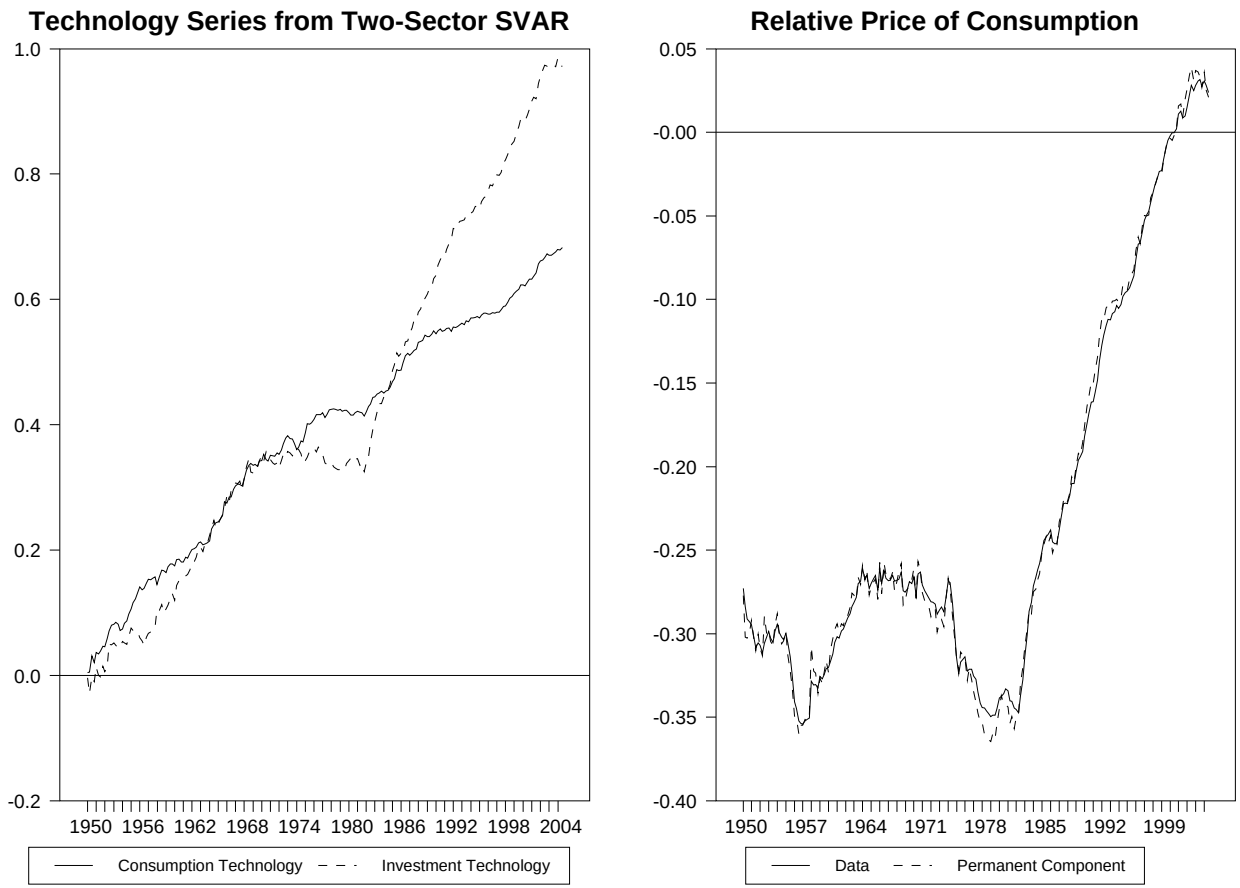


# Figure 4

*Comparison with Cochrane Results*

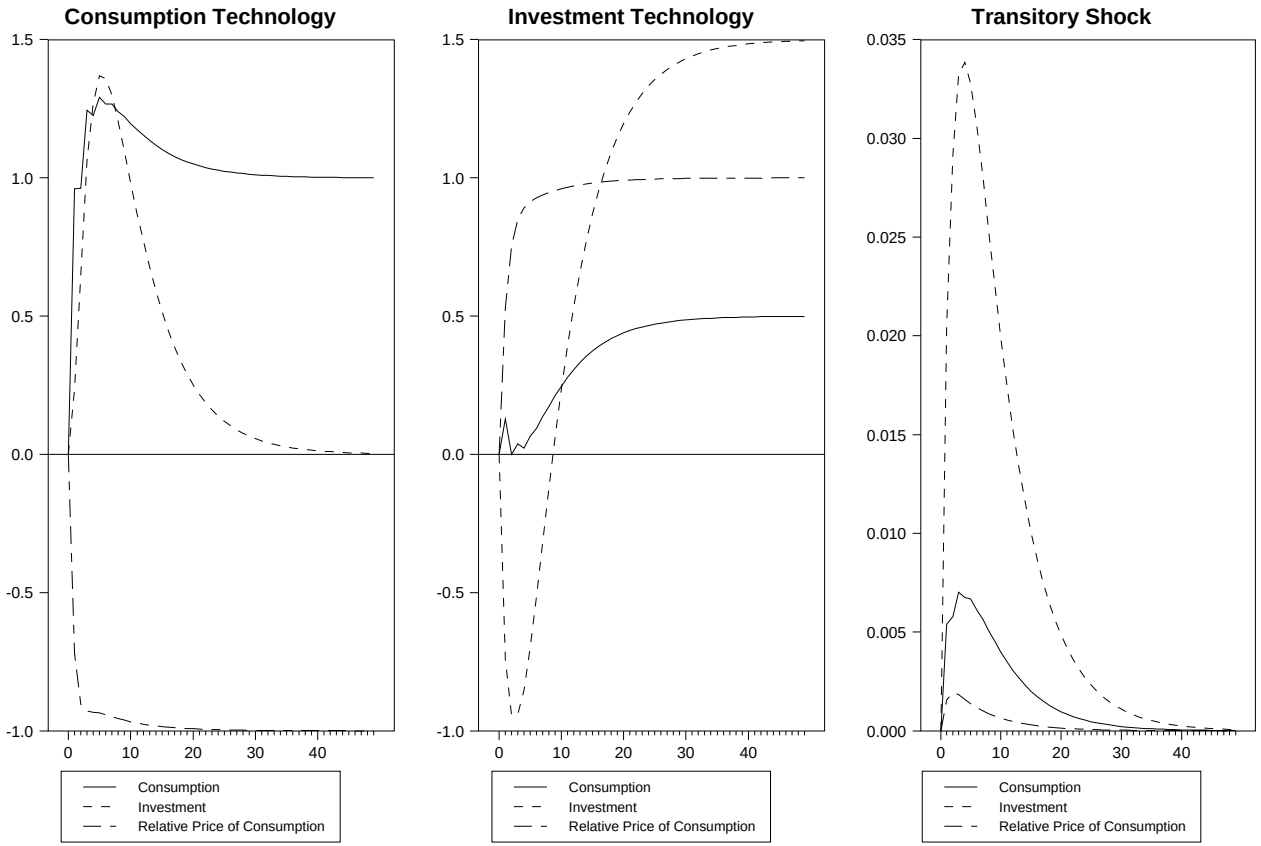


# Figure 5


Technology Series from Two-Sector SVAR
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1950 1956 1962 1968 1974 1980 1986 1992 1998 2004
Consumption Technology
Investment Technology
Relative Price of Consumption
0.05
-0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
-0.40
1950 1957 1964 1971 1978 1985 1992 1999
Data
Permanent Component

# Figure 6

*Impulse Responses from the Two-Sector Structural VAR*



# Figure 7

*Business-Cycle Fluctuations and their Technology-Driven Components*