

# Consumption and Expected Asset Returns Without Assumptions About Unobservables

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## Abstract

If asset returns are predictable, then rational expectations and the arithmetic of budget constraints together imply that these predictable changes in returns should affect current consumption. This paper presents a new framework linking consumption, income, and observable assets to expectations of future asset returns. Relative to previous work on this topic, the framework proposed in this paper has a number of advantages including not relying on untestable assumptions concerning unobservable variables and not requiring estimation of unknown parameters to arrive at a forecasting variable.

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# 1 Introduction

That current consumption should reflect predictable information about future values of labor income is a central theme in macroeconomic theory. Less commonly discussed is the idea that current consumption may also reflect information about predictable future movements in asset returns. However, the question of whether asset returns are predictable over time is perhaps the key issue in modern financial economics: For example, it is widely accepted that news about dividend payments can explain only a small fraction of the fluctuations in stock prices, so theories based on rational investors have focused on the idea that these movements are largely related to news about future stock returns.<sup>1</sup> And, as Campbell and Mankiw (1989) have shown, if such predictable fluctuations in returns exist, then rational expectations and the arithmetic of budget constraints imply that these fluctuations should be reflected in current consumption.

These considerations suggest that the link between consumption spending and future asset returns should play an important role in empirical research in both macroeconomics and financial economics. However, an important drawback in assessing the relationship derived by Campbell and Mankiw is the fact that it involves an observed variable. Specifically, the Campbell-Mankiw relationship stems from a log-linear approximation for the evolution of a *total* wealth variable defined as the sum of observable household assets and the unobservable present value of future expected labor income. This relationship can be re-stated as relating the ratio of consumption to total wealth to expected future consumption growth and expected future returns on total wealth.

In light of the unobservability of some of the variables in this relationship, Martin Lettau and Sydney Ludvigson (2001) have operationalized the Campbell-Mankiw equation using a set of approximating assumptions that link the unobservable total wealth series to observable series on assets and labor income. These assumptions imply that a linear combination of the logs of consumption, assets, and labor income (whose parameters must be estimated) should be related to a discounted sum of expected future values of consumption growth, returns on observable assets, and returns on human capital. Lettau and Ludvigson show that an estimated linear combination of these variables—which they term *cay*—is a useful predictor of stock returns.

This paper introduces an alternative approach to modelling the behavior of consumption

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<sup>1</sup>See, for instance, Campbell (1991). Chapter 20 of Cochrane (2001) summarizes the extensive empirical literature on the prediction of variations over time in asset returns.

and expected asset returns. A key advantage of the approach introduced here is that it does not require any assumptions about unobservable variables because it focuses instead on the standard budget identity for *observable* assets. A log-linearized relationship is derived in which the log ratio of *excess consumption* (defined as consumption in excess of labor income) to observable assets is expressed as an expected discounted sum of future returns on household assets minus future growth rates of excess consumption. Specifically, the relationship derived takes the form

$$x - a_t \approx E_t \sum_{k=1}^{\infty} \rho_a^k (r_{t+k}^a - \Delta x_{t+k})$$

where  $x_t$  is the log of consumption minus labor income,  $a_t$  is the log of observable household assets,  $r_t^a$  is the return on these assets, and  $\rho_a$  is a known constant slightly less than one.

This relationship—essentially a log-linearized version of the traditional intertemporal household budget constraint—provides an analytically convenient methodology for assessing the idea that predictable fluctuations in asset returns may be reflected in current consumption in a manner consistent with rational expectations. Relative to the *cay* approach, the relationship also has a number of attractive features:

- It relies on only one log-linear approximation, involving the equation for the evolution of observable assets. And because all variables in this equation are observable, the approximation can be checked and confirmed to be highly accurate. This contrasts with the *cay* approach which relies on a number of approximating assumptions involving unobservable variables, the accuracy of which are very difficult to assess.
- It implies an approximate equality between one observable variable and an expected discounted sum of other observable variables. Thus, one can directly test whether the forecasting ratio has predictive power for the exact combination of variables predicted by the theory.
- Because the predictive variable here is a ratio of two observable variables, there is no need to estimate any parameters to construct it. This is a useful feature because a number of critiques of Lettau and Ludvigson’s finding of stock return predictability have focused on the process by which the parameters of the forecasting linear combination were estimated.<sup>2</sup>

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<sup>2</sup>Gourinchas and Rey (2005) is another paper that uses the Lettau-Ludvigson approach, applying it to

Our empirical results provide new evidence in favor of the idea that current values of consumption reflect information about predictable future movements in asset returns. The  $x_t - a_t$  ratio is shown to be a statistically significant predictor of discounted sums of future values of asset returns minus excess consumption growth, exactly as predicted by the model. And the ratio’s forecasting power, which is especially strong at long horizons, stems mainly (though not completely) from its ability to forecast future asset returns. Evidence is also presented that this ratio can provide statistically significant forecasts of stock returns and excess returns on stocks over various horizons, though this forecasting performance is not as strong as that of Lettau and Ludvigson’s *cay* variable. That said, there is little theoretical reason why the variable derived here *should* be used to forecast equity returns alone, because it is designed to forecast a combination of asset returns and changes in excess consumption, and also because the theoretically-appropriate measure of asset returns in this case is far broader than the return on stocks.

The contents are as follows. Section 2 describes previous work linking consumption with expected asset returns and Section 3 introduces our alternative approach. Section 4 describes the data. Section 5 documents strong confirmation of the theoretical prediction that the  $x_t - a_t$  ratio can forecast a combination of returns on household assets and changes in excess consumption. Section 6 narrows the focus to forecasts of stock returns and provides some direct comparisons with forecasts generated from the *cay* approach. Section 7 concludes.

## 2 Previous Approaches

### 2.1 The Campbell-Mankiw Log-Linearized Budget Constraint

Campbell and Mankiw (1989) originally developed the log-linearized budget constraint in the context of the following equation for total wealth, which is defined as the sum of observable assets and human capital:

$$W_{t+1} = R_{t+1}^w(W_t - C_t). \quad (1)$$

Here,  $R_{t+1}^w$  is the gross return on total wealth. Labor income does not feature explicitly in the formula because it is interpreted as part of the “return” from this broad measure of the current account budget constraint to obtain a variable to forecast future returns on domestic and foreign assets and future trade deficits. The Gourinchas-Rey model also requires the estimation of coefficients to construct a forecasting variable.

wealth. Dividing across by  $W_t$  and taking logs, this equation becomes

$$\Delta w_{t+1} = r_{t+1}^w + \log(1 - \exp(c_t - w_t)) \quad (2)$$

where log variables are denoted with lowercase letters. The second term in this equation can be approximated using a first-order Taylor expansion around the sample average of  $c_t - w_t$ :

$$\log(1 - \exp(c_t - w_t)) \approx \log(1 - \exp(\bar{c} - \bar{w})) - \left( \frac{\bar{C}}{\bar{W} - \bar{C}} \right) (c_t - w_t - \bar{c} + \bar{w}). \quad (3)$$

This can be simplified to

$$\log(1 - \exp(c_t - w_t)) \approx k + (1 - \rho_w^{-1})(c_t - w_t). \quad (4)$$

where  $k$  is a constant and

$$\rho_w = \frac{\bar{W} - \bar{C}}{\bar{W}}. \quad (5)$$

Using this log-linearization, and dropping the constant term, the budget constraint can be re-written as

$$c_t - w_t \approx \frac{r_{t+1}^w - \Delta w_{t+1}}{\rho_w^{-1} - 1}. \quad (6)$$

This re-arranges to give

$$c_t - w_t \approx \rho_w (r_{t+1}^w - \Delta c_{t+1}) + \rho_w (c_{t+1} - w_{t+1}). \quad (7)$$

Solving forward via repeated substitution on  $c_{t+i} - w_{t+i}$  and imposing the condition that  $\lim_{i \rightarrow \infty} \rho_w^{-i} (c_{t+i} - w_{t+i}) = 0$ , one obtains

$$c_t - w_t \approx \sum_{k=1}^{\infty} \rho_w^k (r_{t+k}^w - \Delta c_{t+k}). \quad (8)$$

This equation holds *ex post*, but it should also hold if we replace actual future values with *ex ante* rational expectations. Taking the mathematical expectation of equation (8) conditional on time- $t$  information therefore yields the following expression for the consumption-wealth ratio:

$$c_t - w_t \approx E_t \sum_{k=1}^{\infty} \rho_w^k (r_{t+k}^w - \Delta c_{t+k}). \quad (9)$$

## 2.2 The *cay* Approach

Equation (9) demonstrates the generality of a link between current consumption behavior and unobserved expectations concerning future returns on a very broad definition of wealth. However, because this aggregate wealth variable  $W_t$  is unobservable, the equation does not directly suggest an empirical methodology for assessing this linkage. Lettau and Ludvigson (2001) have addressed this issue by modifying equation (9) based on assumptions about the unobserved human wealth series. First, they approximate the log of aggregate wealth as

$$w_t \approx \omega a_t + (1 - \omega)h_t, \quad (10)$$

where  $\omega$  is the average share of observable assets  $A$  in total wealth  $W$ . Second, the log return on aggregate wealth,  $r_{w,t}$ , is approximated by a weighted sum of the return on assets  $r_{a,t}$  and the return on human capital  $r_{h,t}$ :

$$r_t^w \approx \omega r_t^a + (1 - \omega)r_t^h. \quad (11)$$

Finally, the nonstationary component of human wealth is assumed to be captured by aggregate labor income  $Y_t$ , such that

$$h_t = \mu + y_t + z_t, \quad (12)$$

where  $\mu$  is a constant and  $z_t$  is a stationary zero-mean variable. Putting these pieces together (and again omitting uninteresting constants) yields the following expression:

$$cay_t \equiv c_t - \omega a_t - (1 - \omega)y_t \approx E_t \sum_{k=1}^{\infty} \rho_w^k [\omega r_{t+k}^a + (1 - \omega)r_{t+k}^h - \Delta c_{t+k}] + (1 - \omega)z_t. \quad (13)$$

This re-expresses the Campbell-Mankiw relationship with the unobservable variable  $h_t$  omitted. However, it is not quite ready for empirical usage because the parameter  $\omega$  also cannot be observed. Lettau and Ludvigson address this issue by arguing that the expected return on total wealth and expected consumption growth should both be stationary, and thus  $cay_t$  should be stationary as well. This reasoning implies the existence of a cointegrating relationship between log consumption, assets, and labor income. Under these assumptions, the parameter  $\omega$  can be superconsistently estimated using cointegration methods. Lettau and Ludvigson apply Stock and Watson's (1993) dynamic ordinary least squares methodology to estimate the parameters for their  $cay_t$  series. The constructed series is then shown to have forecasting power for returns on S&P 500 stock index, consistent with the hypothesis of the existence of systematic variations in expected stock returns.

### 2.3 Some Limitations of the *cay* Method

Before outlining the alternative approach adopted in this paper, it is worth pointing out some limitations on the use of the *cay* methodology to assess the link between consumption and expectations of future asset returns.

The first limitation is that the relationship of interest—between  $cay_t$  and expected future values of  $r_t^a$ ,  $r_t^h$ , and  $\Delta c_t$ —is an approximation, and the quality of the approximation is not known. The accuracy of the relationship described in equation (13) relies on three different sets of approximating assumptions about unobservable variables:

- The Campbell-Mankiw approximation, equation (3), whose accuracy cannot be assessed because it involves the unobservable human capital variable.
- The approximations introduced by Lettau and Ludvigson in equations (10) and (11). The accuracy of these approximations will depend on the stability over time of the ratio of observed assets to total wealth, and it seems possible that this ratio exhibits substantial variability.
- The approximation of  $\omega$  with a regression-based estimate. Lettau and Ludvigson argue that these estimates are superconsistent, but this rests on the assumed stationarity of expected returns and expected consumption growth. However, while these series clearly cannot have trends, it is quite possible that they experience mean breaks associated, for instance, with changes in the trend rate of productivity growth.

In addition, even if the model is correct, the relationship between  $cay_t$  and expectations of future macroeconomic variables will be obscured to the extent that fluctuations in  $cay_t$  are determined by movements in the unobservable variable  $z_t$ . A priori, it is unclear how much of the empirical variation in  $cay_t$  will be due to variations in  $(1 - \omega)z_t$ , but it seems likely that this unobserved term could contribute substantially to these fluctuations.

## 3 An Approach Based on Observable Assets

The Campbell-Mankiw approach provides a way of linking current consumption with expectations of an unobservable variable, namely the return on total wealth. However, the focus of the literature relating to the *cay* variable has been largely restricted to the question of whether returns on *observable* assets (and in particular, equities) are predictable, and this

approach has required numerous untestable assumptions about unobserved variables. This suggests that it may be worthwhile re-examining the issue of predictability of asset returns by starting from the budget constraint describing the evolution of observable assets, rather than from the Campbell and Mankiw equation for total wealth. In this section, I show that such an approach yields an alternative relationship that has a number of advantages over the *cay* approach.

### 3.1 The Excess Consumption to Assets Ratio

Our approach starts with the textbook household budget constraint. This equation describes the evolution of total household assets as

$$A_{t+1} = R_{t+1}^a (A_t + Y_t - C_t), \quad (14)$$

where  $R_{t+1}^a$  is the gross return on these assets and, as before,  $Y_t$  is labor income and  $C_t$  is outlays on consumption. Dividing across by  $A_t$  and taking logs we get

$$\Delta a_{t+1} = r_{t+1}^a + \log \left( 1 - \frac{C_t - Y_t}{A_t} \right). \quad (15)$$

Now define *excess consumption* as

$$X_t = C_t - Y_t. \quad (16)$$

With this definition in hand, the budget identity can be expressed as

$$\Delta a_{t+1} = r_{t+1}^a + \log (1 - \exp(x_t - a_t)), \quad (17)$$

which is identical in form to equation (2), with  $x$  replacing  $c$  and  $a$  replacing  $w$ . As in equation (3), the log term can be approximated as

$$\log (1 - \exp(x_t - a_t)) \approx \log (1 - \exp(\bar{x} - \bar{a})) - \left( \frac{\bar{X}}{\bar{A} - \bar{X}} \right) (x_t - a_t - \bar{x} + \bar{a}). \quad (18)$$

The same sequence of algebraic steps used to derive equation (9) can now be applied to derive

$$x_t - a_t \approx E_t \sum_{k=1}^{\infty} \rho_a^k (r_{t+k}^a - \Delta x_{t+k}), \quad (19)$$

where

$$\rho_a = \frac{\bar{A} - \bar{X}}{\bar{A}}. \quad (20)$$

In other words, applying the same methodology as before to the budget constraint for observable assets, we obtain the prediction that the ratio of excess consumption to assets equals a discounted sum of expected future returns on assets minus expected future growth rates of excess consumption.

Equation (19) is the key equation that we will examine in the rest of the paper. It may seem a little unintuitive because it features unfamiliar variables such as the growth rate of the variable we have termed excess consumption. However, this is simply a different way of writing the standard textbook intertemporal budget constraint. To see this, recall that the intertemporal budget constraint can be obtained from applying repeated substitution to (14) and imposing a transversality condition to obtain:

$$\sum_{k=0}^{\infty} \frac{C_{t+k}}{\left(\prod_{m=0}^k R_{t+m}^a\right)} = A_t + \sum_{k=0}^{\infty} \frac{Y_{t+k}}{\left(\prod_{m=0}^k R_{t+m}^a\right)}. \quad (21)$$

In other words, the present discounted value of consumption expenditures equals current assets plus the present discount value of labor income. This can be re-written as

$$A_t = \sum_{k=0}^{\infty} \frac{-X_{t+k}}{\left(\prod_{m=0}^k R_{t+m}^a\right)}. \quad (22)$$

In other words, the current value of assets equals the present discount value of future excess consumption. Our forward-looking equation (19) is simply a log-linearized version of this relationship.

Another way to look at this relationship is to note that the series  $x_t - a_t$  represents the fraction of assets that households are willing to “eat into” each period for consumption purposes. Thus, a high value of  $x_t - a_t$  indicates either a high expected future returns on assets or a future retrenchment towards a slower pace of eating into assets, or indeed that both of these outcomes should be expected.

### 3.2 Advantages of the $x - a$ Approach

Equation (19) has a number of useful features as a vehicle for examining the link between current macroeconomic variables and expected future asset returns.

**Verification of Accuracy of Log-Linear Approximation:** Despite its popularity as a theoretical tool, the accuracy of the Campbell-Mankiw log-linearized approximation to the

total wealth budget constraint is unknown because it involves a variable, human wealth, that cannot be observed. Indeed, Campbell (1993) constructed a theoretical example in which the approximation is poor if the intertemporal elasticity of substitution is sufficiently high. In contrast, the log-linear approximation required in our case—equation (18)—is one in which one observable variable is used to approximate another. Hence, the accuracy of this approximation can be checked and, as we discuss below, for our empirical implementation it turns out to be extremely accurate.

**An Observable Forecast Variable:** An important difference between the  $x - a$  and  $cay$  approaches is that  $x_t - a_t$  can be directly constructed from the observable series on consumption, labor income, and assets, while  $cay_t$  depends on unknown coefficients that must be estimated. This is a useful feature for a number of reasons. First, it removes an additional source of approximating uncertainty by allowing us to work with exactly the forecasting variable predicted by the theory rather than an empirical proxy for it. Second, because our forecasting variable does not rely on econometric coefficient estimates, the empirical results from this approach cannot be criticized on the basis of the estimation methodology used to obtain the coefficients.

This latter point is important in light of some of the discussions that have surrounded Lettau and Ludvigson’s finding that their  $cay$  series was useful in forecasting stock returns. For example, Brennan and Xia (2005) argue that the apparent forecasting power of  $cay$  largely stems from its incorporation of full-sample information in the form of the estimated full-sample coefficients used to construct the series; in other words, that the forecasting power comes from a form of “look-ahead bias” introduced by the procedure used to construct the forecasting variable. While Lettau and Ludvigson (2005) dispute this critique, it can be noted that this criticism does not apply at all to the forecasting approach suggested here. In addition, Hahn and Lee (2001) critique the original  $cay$  series for not allowing for changes over time in the cointegrating vector defining these coefficients.

**Observability of Forecasted Variables:** While  $cay_t$  has been used to forecast future asset returns, the exact series that it is supposed to forecast according to equation (13) cannot be observed. This is because neither  $\omega$  or  $r_t^h$  are observable. In addition, the discount rate  $\rho_w$ , used to construct the weighted sum of future variables, also cannot be observed because it involves the sample average of human wealth (see equation 5). In

contrast, the series whose expected future values should be captured by  $x_t - a_t$  is  $r_t^a - \Delta x_t$ , which is observable. The variable  $\rho_a$  used to construct the discounted sum can also be calculated.

**Accuracy of Forecasting Relationship:** The relationship between  $x_t - a_t$  and the expected discounted sum described in equation (19) is exact apart from a single log-linearizing approximation error. This contrast with the *cay* equation (13), in which *cay* $_t$  depends not only on a present value of unobserved variables, but also on the unobserved series  $z_t$ , which describes the ratio of labor income to human capital. Because the relationship being examined in our approach is not obscured by this additional error term, the forecasting variable used here is, *a priori*, a cleaner indicator of the variables being forecasted.

**A Possible Drawback?** Before moving on to describe our data and empirical results, a potential drawback of the method should be mentioned, which is that we cannot rule out the possibility that labor income may exceed consumption during some periods. In this case, excess consumption ( $X_t$ ) is negative and thus its logged value ( $x_t$ ) does not exist, so the method could not be implemented. Two points can be made on this issue. The first is that for the US data series used in this study, which rely on a standard definition of labor income, the negativity problem never arises. The second is that one can derive essentially the same relationship as the one examined here, focusing not on  $X_t = C_t - Y_t$  but instead on  $X_t^* = C_t - Y_t + \theta A_t$  where  $\theta$  is defined to be large enough to ensure that  $X_t^*$  is always positive. Appendix A shows that this approach results in a forecasting equation involving a ratio whose fluctuations are driven by  $C_t$ ,  $Y_t$ , and  $A_t$  in exactly the same manner described here. Thus, the framework can be applied with little substantive change even if one has some periods in which labor income exceeds consumption.

## 4 Data

### 4.1 Definitions and Sources

Before describing our choice of data series in more detail, we first note that the assets described in equation (14) have a market value that is measured based on current transactions prices. Thus, by necessity, all data on asset valuations are *nominal* data, and the evolution equation that describes changes in nominal assets features nominal asset returns

as well as nominal consumption and income. However, because we are primarily interested in the behavior of *real* consumption and asset returns, we work instead with an equation describing the evolution of real assets. Calculations reported in Appendix B show that this can be derived from the asset evolution equation underlying the nominal data as long as each of the series in equation (14) are defined relative to the same deflator. In other words, the price indexes used to define the real series of  $A_t$ ,  $Y_t$ , and  $C_t$  must be the same, and the real asset return  $R_t^a$  must be defined relative to the rate of inflation described by this price index.<sup>3</sup>

With this in mind, our empirical counterparts of these real series are each defined relative to the deflator for total personal consumption expenditures. This series was obtained from NIPA Table 2.3.4 available on the BEA website ([www.bea.gov](http://www.bea.gov)) and, as with all data used in this paper, it is quarterly in frequency.<sup>4</sup> In keeping with this choice, our series for  $C$  is total real personal consumption expenditures.

Our objective in constructing a dataset is to come as close as possible to obtaining empirical series that are consistent with the equation for the evolution of total observable household assets, equation (14). Thus, we want to have the broadest possible measure of observable assets. To this end, our measure of total household assets is based on the Federal Reserve Board's Flow of Funds net worth series, as published on Table B.100 of the Flow of Funds accounts. Because our measure of consumption includes outlays on durable goods and the Flow of Funds net worth series includes the value of the stock of consumer durables, consistency with the theoretical budget constraint (14) requires that we subtract the value of consumer durables from the net worth series to arrive at the theoretically-correct series for  $A_t$ , which can be done because the Flow of Funds data include a line on the value of the stock of durables.<sup>5</sup>

The BEA does not publish an official measure of labor income, so our measure was constructed using data from NIPA Table 2.1 according to a standard procedure. Specifically, labor income was defined as in Lettau and Ludvigson (2001) as wages and salaries *plus* transfer payments *plus* other labor income *minus* personal contributions for social insurance *minus* labor taxes. Labor taxes are defined by imputing a share of personal tax and nontax

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<sup>3</sup>See Palumbo, Rudd, and Whelan (2006) for a more detailed discussion of this issue.

<sup>4</sup>This and all other NIPA-related series used in the paper were downloaded during September 2005 and were originally published on August 31, 2005.

<sup>5</sup>These data were downloaded from the Federal Reserve Board's website ([www.federalreserve.gov](http://www.federalreserve.gov)) and were originally published on June 9, 2005.

payments to labor income, with the share calculated as the ratio of wage and salaries to the sum of wage and salaries, proprietors' income, and rental, dividend, and interest income.

## 4.2 Some Features of the Data

Before reporting our principal results, we first describe a few relevant features of our data. First, note that our empirical series for  $A_t$ ,  $Y_t$ , and  $C_t$  directly imply a time series for the gross rate of return on all household assets, defined by inverting equation (14) as

$$R_{t+1}^a = \frac{A_{t+1}}{A_t + Y_t - C_t}. \quad (23)$$

Figure 1 shows the time series for the log of this gross return,  $r_{t+k}^a$ , along with the log of the gross real return on the stock market as measured by the value-weighted CRSP return.<sup>6</sup> The latter series is charted because forecasting stock returns has been the focus of much the existing research in this area, and this issue is examined later in Section 6. The figure shows that these two series differ substantially in their volatility, but that the returns are quite highly correlated (the correlation coefficient is 0.89) implying that equity markets play a key role in determining the variability of the overall return on assets. Thus, while our theory about the consequences of predictable asset returns applies to the broad return measure, one might also expect it to apply (if not quite as well) to forecasting equity returns. This prediction turns out to be confirmed by the data.

Figures 2 and 3 confirm two features of our data that were briefly mentioned earlier. First, Figure 2 shows the ratio of excess consumption to assets and confirms that it is always positive. Even at its lowest point, the positive gap between quarterly consumption and labor income is about two percent of assets. Second, Figure 3 shows that the log-linear approximation of equation (18) is extremely accurate. The figure compares the empirical series given by the left-hand-side of (18) with the approximation given by the right-hand-side: The empirical series and the approximation lie on top of each other for most of the sample, and even the largest approximation errors are small: The correlation between the two series is 0.996. These calculations ensure that our key equation (19) is essentially a direct consequence of the standard household budget constraint and rational expectations, with no additional assumptions being made.

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<sup>6</sup>The value-weighted CRSP return series was downloaded from Professor Kenneth French's website at <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french>

## 5 Results

Our key theoretical relationship—equation (19)—states that the ratio of excess consumption to assets should equal a discounted sum of expected future values for asset returns minus excess consumption growth. If asset returns and excess consumption growth are inately unpredictable, for instance if they are drawn from an *iid* distribution, then the theory implies that the  $x_t - a_t$  ratio should also be unpredictable. However, Figure 2 tells us that this ratio is positively autocorrelated and displays clear low-frequency swings. This raises the question of whether these swings are indeed related to predictable future movements in  $r_t^a - \Delta x_t$ .

Table 1 addresses this question by reporting results from regressions of the form

$$\sum_{k=1}^N \rho_a^k (r_{t+k}^a - \Delta x_{t+k}) = \gamma (x_t - a_t) + \epsilon_{t+N} \quad (24)$$

for various values of  $N$ . These regressions assess the relationship between the *realized* discounted sums  $\sum_{k=1}^N \rho_a^k (r_{t+k}^a - \Delta x_{t+k})$ , observable at time  $t + N$ , and the value of the excess consumption to assets ratio from  $N$  periods earlier. Specifically, the tables report the  $t$ -statistics and  $R^2$  from these regressions. The  $t$ -statistics are based on Newey-West HAC-consistent standard errors calculated using a bandwidth of  $N - 1$ , thus controlling for the effects of autocorrelated errors of order  $N - 1$  induced by the dependent variable being a form of moving average. The data used for these regressions start at 1952:1 and end at 2005:1, but the effective sample of the regression is limited by the size of  $N$ . For example, when  $N = 40$ , the effective sample ends in 1995:1. Recall also from equation (20) that  $\rho_a$  is defined as one minus the ratio of the sample average of excess consumption to the sample average of the value of assets, and this is calculated as 0.991.<sup>7</sup> This value is consistent with the highly accurate log-linearized approximation shown in Figure 3, but regressions run using discounted sums constructed from alternative reasonable values of  $\rho_a$  give very similar results to those reported here.

The results in Table 1 provide strong confirmation that current consumption—in the form of  $x_t - a_t$ —contains useful predictive information about future values of  $r_t - \Delta x_t$ .

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<sup>7</sup>This calculation adjusts for the fact that NIPA consumption and labor income measures are reported on an annualized basis. Thus, the excess consumption series constructed from NIPA sources is divided by four to arrive at the correct figure for the average reduction in assets per quarter due to consumption in excess of labor income. The chart in Figure 2 sticks with the usual conventions in reporting the series for excess consumption on an annualized basis.

The  $t$ -statistics in the first row are significant at the five percent level for all values of  $N$ , and become more so as the forecast horizon is extended out. This type of long-horizon predictability is consistent with equation (19) because it predicts a relationship between  $x_t - a_t$  and expectations of an infinite-horizon discounted sum. For our longest horizon regression, forty quarters, the ratio of excess consumption to assets explains a striking 60 percent of the subsequent realized discounted sum. In addition, as predicted, the estimated value of  $\gamma$  gets closer to one as we increase the horizon.

An advantage of our approach is that one can check the exact predictive relationship implied by theory. While  $cay_t$  is supposed to contain information about future values of  $\omega r_t^a + (1 - \omega)r_t^h - \Delta c_t$  (see equation 13) in practice  $\omega$  and  $r_t^h$  cannot be observed, so this exact combination of variables cannot be computed. In contrast,  $r_t - \Delta x_t$  can be computed, and the results in the bottom two panels of Table 1 suggest that one obtains substantially stronger long-horizon predictive relationship by exactly following our theory's predictions and looking at this combination of variables, rather than examining only the return on assets.

A priori, one would expect that  $x_t - a_t$  would contain some information that could be helpful in separate forecasting regressions for  $r_t$ , but that it would be a noisier indicator for this series than for  $r_t - \Delta x_t$ . John Cochrane (2006) has made a related point in the context of the Campbell-Shiller formula relating the dividend-price ratio to expected future values of dividend growth and returns. He notes that because the dividend-price ratio should be a function of expectations of *both* of these variables, one needs to be careful in interpreting null hypotheses from regressions that focus on the ratio's ability to forecast only one of the variables.

The results confirm this conjecture. The  $x_t - a_t$  ratio is a statistically significant predictor of household asset returns at both short and long horizons. However, apart from the case  $N = 1$ , the  $t$ -statistics and measures of fit are higher in the top rows of the table than in the middle rows: In the case  $N = 40$ , the  $R^2$  is 0.37 when one forecasts asset returns alone, compared with 0.60 when one forecasts the linear combination  $r_t - \Delta x_t$ . In addition, as would be expected from the use of a noisy indicator, the estimates of  $\gamma$  are smaller for these regressions.

The bottom panel shows that the improved forecasting performance exhibited in the top panel does not stem from excess consumption growth being highly forecastable on its own. In fact, the opposite is the case: The  $x_t - a_t$  ratio fails to be a statistically significant

predictor of all of the discounted sums of future values of  $\Delta x_t$ . Taken together, these calculations point to predictable movements in asset returns as the primary factor in the predictive relationship suggested by our approach, but they also indicate that allowing for the possibility there are some predictable future patterns for consumption and labor income substantially improves the fit of the relationship, which is in line with the model’s predictions.

Because the discounted-sum regressions reported in Table 1 are somewhat unusual in the literature on long-horizon predictability of asset returns, Table 2 repeats the same set of regressions but this time without discounting; thus, for instance, the dependent variables in the returns regressions are simply the  $N$ -quarter cumulative returns. The results are essentially identical, which is hardly surprising given that the discount factor used in Table 1 is 0.991 and thus very close to one.

## 6 Forecasting Stock Returns

The theoretical relationships discussed in this paper—both the Campbell-Mankiw relationship and the one derived here for observable assets—clearly focus on the potential information in current consumption regarding returns on a *broad* concept of household assets. However, the finance profession has focused principally on predicting returns on stocks, and it was noted earlier that the return on total household assets is highly correlated with the rate of return on the stock market. So, one might expect that the  $x_t - a_t$  ratio has some ability to forecast stock returns, and Table 3 confirms that this is the case.

The upper panel of the table shows that the ratio is a highly statistically significant predictor of cumulated stock returns at all of the horizons shown apart from forty quarters. However, the evidence for predictability is somewhat weaker than for the return on total household assets, a pattern that is consistent with the theory outlined above. For instance, at a forty-quarter horizon, the  $x_t - a_t$  ratio explains 15 percent of stock returns, compared with 34 percent of the return on total household assets. The bottom panel of the table shows that evidence for forecastability of the excess return on stocks over one-month treasury bills (the subject of much of Lettau and Ludvigson’s analysis) is stronger than for stock returns alone, but still weaker than for the return on total household assets.

One obvious question raised by these results is whether the  $x_t - a_t$  ratio forecasts equity returns better than the  $ca y_t$  variable adopted by Lettau and Ludvigson. Table 4 shows that

it does not. For both total and excess stock returns (the results in the upper and middle panels of the table) and for all of the horizons examined, one can reject the hypothesis that  $x_t - a_t$  adds explanatory power to a regression containing  $cay_t$ , where this series was downloaded from Martin Lettau's website. In contrast,  $cay_t$  is a significant predictor of these return series for all horizons examined apart from forty quarters.

An examination of the theoretical results in Sections 2 and 3 does not suggest any obvious reasons why  $cay_t$  performs so much better in forecasting equity returns. In theory, both variables should incorporate some information about future returns on total household assets as well as information about future labor income and consumption. However, equation (13) makes clear that  $cay_t$  also depends on the unobserved variables  $z_t$  (the ratio of human capital to labor income), that this theoretical relationship relies on a number of approximations whose accuracy is unknown, and that empirical implementations of it are subject to the sampling error associated with estimating the  $\omega$  parameter. So, *a priori*, one might expect that the  $x_t - a_t$  ratio could be a cleaner measure of expected future asset returns. In practice, this expectation is not confirmed by the data.

That said, it should still be kept in mind that the theory outlined here implies that it is the combination of variables  $r_t - \Delta x_t$  that should be forecasted by the  $x_t - a_t$  ratio, and this prediction is strongly supported by the data. It is interesting to note, for instance, that the results in the bottom panel show that the  $cay_t$  variable generally adds little explanatory power to this ratio when one attempts to forecast the full combination of variables suggested by our theory.

## 7 Conclusions

This paper has presented a simple re-formulation of the household intertemporal budget constraint and shown how it can be used to assess the relationship between current consumption spending and future returns on household assets. Specifically, it is shown that the ratio of excess consumption (consumption minus labor income) to household assets should be a function of expectations of future asset returns and future growth rates of excess consumption. Empirical implementation of the model strongly confirms the model's prediction that this ratio reflects long-horizon expectations of future values for asset returns and excess consumption.

The paper's empirical results reinforce the conclusions of Lettau and Ludvigson (2001)

that current consumption contains information about future asset returns. There are, however, some important differences between their work and the approach taken here. In particular, we have emphasized that current consumption reflects *both* information about future asset returns and information about the future behavior of income and consumption, whereas Lettau and Ludvigson stress only the information about asset returns. In addition, the framework developed here has a number of practical advantages because it does not rely on untestable assumptions about unobserved variables or require estimation of unknown parameters to arrive at a forecasting variable. In this sense, the results here are less open to some of the important critiques that have been levelled at the *cay* approach.

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## A Dealing with Negative Excess Consumption

This appendix shows that a methodology almost identical to that used in this paper can be employed in the case where some of the observations on consumption are less than labor income. Suppose that there are some values of  $X_t = C_t - Y_t$  that are negative. Now choose a number  $\theta$  large enough so that  $X_t^* = C_t - Y_t + \theta A_t$  is always positive. In this case, equation (15) can be re-written as

$$\Delta a_{t+1} = r_{t+1}^a + \log \left( 1 - \theta - \frac{X_t^*}{A_t} \right).$$

This can be re-expressed as

$$\Delta a_{t+1} = r_{t+1}^a + \log (1 - \theta - \exp(x_t^* - a_t))$$

where  $x_t^*$  (the log of  $X_t^*$ ) is always defined because  $X_t^*$  is always positive. The last term in this expression can be approximated as

$$\log (1 - \theta - \exp(x_t^* - a_t)) \approx c + \frac{\bar{X} + \theta \bar{A}}{\bar{A} - \bar{X}} (x_t^* - a_t)$$

Now applying the same steps as before, we can obtain an intertemporal budget constraint in terms of  $x_t^*$  and  $a_t$ .

$$x_t^* - a_t \approx E_t \sum_{k=1}^{\infty} \rho_a^k (r_{t+k}^a - \Delta x_{t+k}^*),$$

where

$$\rho_a = \frac{\bar{A} - \bar{X}}{(1 - \theta)\bar{A}}$$

## B Derivation of the Real Budget Constraint

Households start each period with a stock of nominal assets,  $\tilde{A}_t$ , and the nominal labor income flow,  $\tilde{y}_t$ , for that period. These can be used to make purchases of consumption goods,  $\tilde{C}_t$ , or invested; assets carried forward receive a gross nominal rate of return equal to  $\tilde{R}_{t+1}^a$ . The resulting budget constraint can therefore be written as:

$$\tilde{A}_{t+1} = \tilde{R}_{t+1}^a (\tilde{A}_t + \tilde{Y}_t - \tilde{C}_t).$$

Consumer utility depends on *quantities* consumed, so macroeconomists tend to re-express the budget constraint in terms of real consumption. To do this, we need to deflate both sides by the aggregate consumption deflator,  $P_{t+1}^C$ :

$$\frac{\tilde{A}_{t+1}}{P_{t+1}^C} = \frac{P_t^C \tilde{R}_{t+1}^a}{P_{t+1}^C} \cdot \frac{\tilde{A}_t + \tilde{Y}_t}{P_t^C} - \frac{P_t^C \tilde{R}_{t+1}^a}{P_{t+1}^C} C_t.$$

Defining inflation as

$$\pi_{t+1} = \frac{P_{t+1}^C}{P_t^C} - 1$$

and defining real wealth, real income, and the real gross interest rate by

$$A_t = \frac{\tilde{A}_t}{P_t^C}$$

$$Y_t = \frac{\tilde{Y}_t}{P_t^C}$$

$$R_{t+1}^a = \frac{1 + i_{t+1}}{1 + \pi_{t+1}}$$

yields the following representation of the budget constraint in terms of real variables:

$$A_{t+1} = R_{t+1}^a (A_t + Y_t - C_t).$$

as required.

Table 1: Predictive Regressions for Discounted Sums

This table reports results from quarterly regressions of the form

$$\sum_{k=1}^N \rho_a^k Z_{t+k} = \gamma (x_t - a_t) + \epsilon_{t+N}$$

for various definitions of  $Z_t$ .  $r_t$  stands for the return on all household assets,  $x$  is the log of consumption minus labor income.  $\rho_a$  is calculated from equation (20) as 0.991. The  $t$ -statistics were calculated using Newey-West standard errors with bandwidth parameter  $N - 1$ . The sample is 1952:1-2005:1.

	Forecast Horizon						
	1	4	8	12	20	24	40
<hr/> <hr/>							
$Z_t = r_t - \Delta x_t$							
$\gamma$	0.07	0.17	0.27	0.37	0.50	0.56	0.89
$t$ -statistics	2.07	3.16	3.20	3.38	4.03	4.27	5.88
$R^2$	0.04	0.11	0.18	0.25	0.35	0.40	0.60
$Z_t = r_t$							
$\gamma$	0.03	0.08	0.16	0.21	0.26	0.29	0.38
$t$ -statistics	2.72	2.14	2.46	2.64	2.44	2.65	3.27
$R^2$	0.03	0.08	0.13	0.18	0.20	0.24	0.37
$Z_t = -\Delta x_t$							
$\gamma$	0.04	0.08	0.11	0.15	0.23	0.27	0.50
$t$ -statistics	1.41	1.24	1.01	1.03	1.18	1.24	1.93
$R^2$	0.02	0.03	0.03	0.04	0.06	0.08	0.17
<hr/> <hr/>							

Table 2: Long-Horizon Regressions for Non-Discounted Sums

This table reports results from quarterly regressions of the form

$$\sum_{k=1}^N \rho_a^k Z_{t+k} = \gamma (x_t - a_t) + \epsilon_{t+N}$$

for various definitions of  $Z_t$ .  $r_t$  stands for the return on all household assets,  $x$  is the log of consumption minus labor income. The  $t$ -statistics were calculated using Newey-West standard errors with bandwidth parameter  $N - 1$ . The sample is 1952:1-2005:1.

	Forecast Horizon						
	1	4	8	12	20	24	40
<hr/>							
$Z_t = r_t - \Delta x_t$	<hr/>						
$\gamma$	0.07	0.17	0.28	0.39	0.54	0.61	1.04
$t$ -statistics	2.07	3.15	3.18	3.35	3.97	4.21	6.02
$R^2$	0.04	0.10	0.18	0.24	0.34	0.39	0.59
$Z_t = r_t$	<hr/>						
$\gamma$	0.03	0.09	0.16	0.23	0.28	0.31	0.44
$t$ -statistics	2.72	2.14	2.47	2.64	2.34	2.52	3.18
$R^2$	0.04	0.08	0.13	0.18	0.19	0.22	0.34
$Z_t = -\Delta x_t$	<hr/>						
$\gamma$	0.04	0.08	0.11	0.15	0.23	0.27	0.50
$t$ -statistics	1.41	1.24	1.00	1.02	1.18	1.24	1.99
$R^2$	0.02	0.03	0.03	0.04	0.06	0.08	0.17
<hr/>							

Table 3: Long-Horizon Regressions for Stock Returns

This table reports results from quarterly regressions of the form

$$\sum_{k=1}^N \rho_a^k Z_{t+k} = \gamma (x_t - a_t) + \epsilon_{t+N}$$

where  $Z_t$  is either real stock returns ( $r_t^s$ ) or the excess return on stocks over one-month treasury bills ( $r_t^s - r_t^f$ ). The  $t$ -statistics were calculated using Newey-West standard errors with bandwidth parameter  $N - 1$ . The sample is 1952:1-2005:1.

	Forecast Horizon						
	1	4	8	12	20	24	40
$Z_t = r_t^s$	0.08	0.29	0.55	0.75	0.96	1.08	1.18
$t$ -statistics	2.10	2.04	2.59	2.50	2.08	2.09	1.89
$R^2$	0.02	0.07	0.14	0.18	0.19	0.21	0.15
$Z_t = r_t^s - r_t^f$	0.08	0.28	0.54	0.73	0.91	1.03	1.23
$t$ -statistics	2.02	2.02	2.73	2.83	2.49	2.58	2.65
$R^2$	0.02	0.07	0.14	0.19	0.20	0.23	0.21

Table 4: Forecasting with  $x_t - a_t$  and  $cay_t$

The table reports  $t$ -statistics and  $R^2$  from quarterly regressions of the form

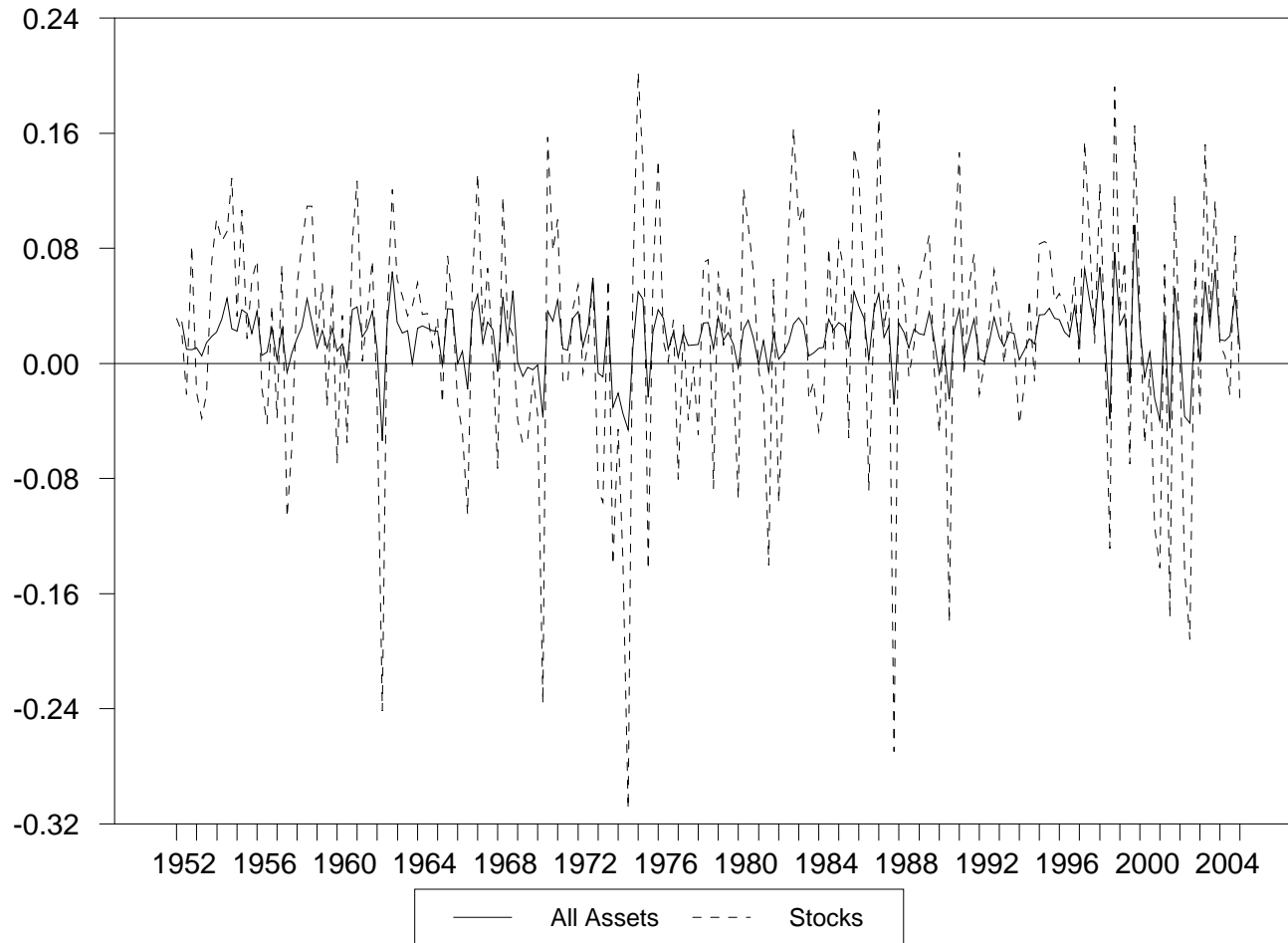
$$\sum_{k=1}^N \rho_a^k Z_{t+k} = \gamma_{xa} (x_t - a_t) + \gamma_{cay} cay_t + \epsilon_{t+N}$$

where  $Z_t$  is either real stock returns ( $r_t^s$ ), the excess return on stocks over one-month treasury bills ( $r_t^s - r_t^f$ ), or  $r_t - \Delta x_t$  where  $r_t$  is the return on all household assets and  $x$  is the log of consumption minus labor income. The  $t$ -statistics were calculated using Newey-West standard errors with bandwidth parameter  $N - 1$ . The sample is 1952:1-2005:1. Data on  $cay_t$  were taken from Martin Lettau's website.

	Forecast Horizon						
	1	4	8	12	20	24	40
<hr/>							
$Z_t = r_t^s$	<hr/>						
$t_{xa}$	0.65	0.65	1.05	1.14	0.79	0.70	0.61
$t_{cay}$	3.56	3.83	4.14	4.39	5.12	4.34	1.92
$R^2$	0.08	0.23	0.38	0.47	0.45	0.44	0.26
<hr/>							
$Z_t = r_t^s - r_t^f$	<hr/>						
$t_{xa}$	0.59	0.62	1.06	1.25	0.96	0.95	1.13
$t_{cay}$	3.48	3.76	3.97	3.97	4.20	4.43	1.43
$R^2$	0.07	0.22	0.37	0.47	0.44	0.42	0.28
<hr/>							
$Z_t = r_t - \Delta x_t$	<hr/>						
$t_{xa}$	1.27	2.17	2.53	2.72	3.30	3.81	7.69
$t_{cay}$	2.26	1.76	0.71	0.60	-0.04	-1.54	-2.74
$R^2$	0.06	0.13	0.18	0.25	0.34	0.40	0.63
<hr/>							

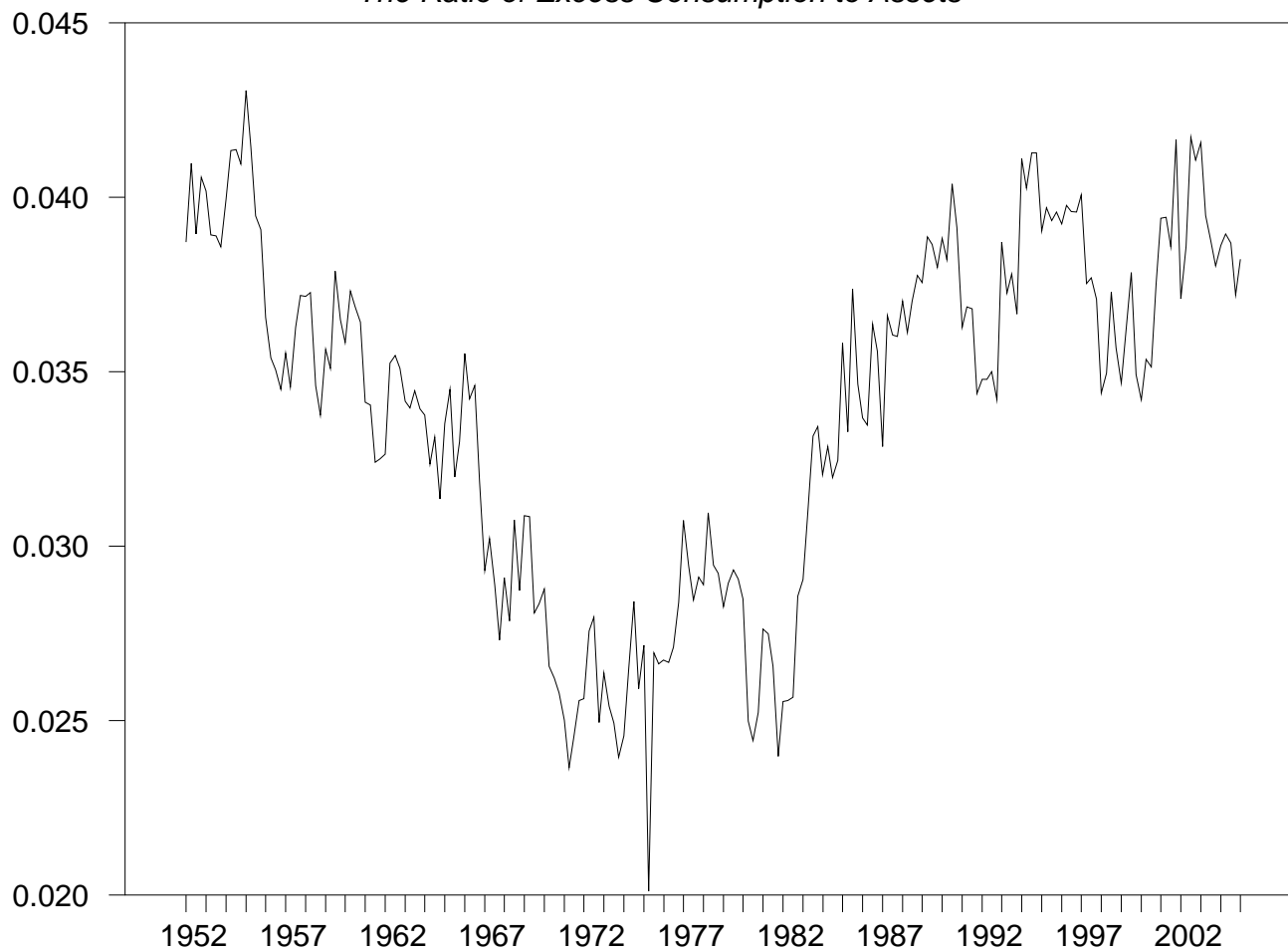
# Figure 1

*Returns on all Household Assets and on the Stock Market*



**Figure 2**

*The Ratio of Excess Consumption to Assets*



**Figure 3**

*Accuracy of the Log-Linear Approximation*

