

Staggered Price Contracts and Inflation Persistence: Some General Results

Karl Whelan

Central Bank and Financial Services Authority of Ireland*

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Abstract

Despite their popularity as theoretical tools for illustrating the effects of nominal rigidities, some have questioned whether models based on staggered price contracts with rational expectations can match the persistence of the empirical inflation process. This paper presents some general results about this class of models. It is shown that these models do not have a problem matching high autocorrelations for inflation. However, they fail to explain a key feature of reduced-form Phillips-curve regressions: The positive dependence of inflation on its own lags. It is shown that staggered price contracting models instead predict that the coefficients on these lag terms should be negative.

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1 Introduction

Most macroeconomists agree that nominal rigidities play an important role in influencing real-world pricing behavior, and staggered contracting models based on rational expectations are commonly used to illustrate the macroeconomic effects of these nominal rigidities. There have, however, been questions about the ability of this class of models to match important aspects of macroeconomic data. Chari, Kehoe, and McGrattan (2000) have argued that these models cannot resolve the “persistence problem” underlying empirical business cycle dynamics for output. In addition, there has been some debate about whether staggered contracting models can match the persistence of the empirical inflation process. In particular, Fuhrer and Moore (1995) have questioned whether these models can match the observed high autocorrelation of inflation.

This paper has two goals. The first is to report some general findings that have not been presented before concerning the dynamics of the relationship between inflation and real activity under staggered pricing and rational expectations. The generality of the results is along a number of dimensions: The characterization of inflation dynamics derived here applies to all models that have time-dependent pricing rules while ruling out the possibility of infinite price durations. This class of models excludes the popular Calvo model, but it does include the so-called truncated-Calvo model in which there is a random probability of price change each period up to a maximum finite price duration. The results apply to Taylor-style models, in which all price contracts have the same duration, and can also be seen as approximating the inflation dynamics of state-dependent models in which the probability of price changes increase with duration.

The second goal of the paper is to clarify the dimensions along which staggered contracting models with rational expectations do and do not match the inflation persistence seen in the data. It is shown that staggered price contracting can reproduce high autocorrelations for inflation. However, it is argued that this is not a particularly useful definition of inflation persistence. Conversely, staggered contracting models fail to explain a statistical regularity that, it can reasonably be argued, provides a more useful definition of inflation persistence: The positive dependence of inflation on its own lagged values in reduced-form “Phillips curve” regressions. It is shown that, in general, these models instead predict that these lagged dependent variable coefficients should be *negative*. This result is particularly noteworthy given that models such as those based on Taylor-style contracts are commonly cited as potentially providing an explanation for the empirical pattern of positive coefficients

on lagged inflation in Phillips curve regressions.¹

Of course, this paper is hardly alone in testing the adequacy of rational expectations models incorporating nominal pricing rigidities. However, unlike many other papers in this area, such as Christiano, Eichenbaum, and Evans (2005), which combine a pricing model with other equations for the rest of the economy and then assess the full model's performance against metrics like ability to match empirical impulse responses, the approach taken here has the advantage of focusing only on the models' implications for the behavior of inflation. It should be pointed out, also, that the failure of staggered contracting models to match the evidence on inflation persistence is not necessarily evidence against models based on nominal rigidities. However, it does suggest that departures from a full rational expectations framework may be necessary, involving perhaps the incorporation of agents using rule-of-thumb expectations such as in Galí and Gertler (1999) or the modelling of imperfect information and learning such as in Orphanides and Williams (2005) and Sargent, Williams, and Zha (2004).

The contents are as follows. Section 2 reports some facts about inflation autocorrelations and reduced-form inflation regressions for the US and Euro area. Section 3 derives an analytical solution for the relationship between inflation and output for a general staggered contracting model. Section 4 then provides some examples of how the results apply to specific cases of well-known time-dependent pricing models, as well as approximating the dynamics of state-dependent pricing models. It is shown that, for each of these models, inflation depends negatively on its own lagged values once one has conditioned on economic fundamentals (meaning, past and expected future economic activity). Section 5 presents an alternative derivation of the results based on log-linearizations around steady-states with non-zero inflation.

Sections 6 and 7 then discuss various testable predictions of the staggered contracting models based on different assumptions about the determination of output. Section 6 uses a simple model with an exogenous output gap to illustrate how these models can match high autocorrelations while failing to match the evidence in reduced-form regressions. Section 7 derives the solution for the reduced-form process for inflation for the standard monetary model described in Chari, Kehoe, and McGrattan (2000) in which the output gap is determined by real money balances and money growth follows an AR(1) process. Fi-

¹See, for instance, Dotsey (2002), Galí, Gertler and Lopez-Salido (2005), page 248 of Goodfriend and King (1997), or page 3 of Eller and Gordon (2003).

nally, Section 8 discusses the models' problems with matching the evidence on inflation persistence in some more detail.

2 Evidence on Inflation Persistence

The concept of inflation persistence can be interpreted in different ways. However, probably the most common statistic cited to illustrate the persistence of inflation is the high value of its first-order autocorrelation coefficient. Table 1 reports these autocorrelations for quarterly GDP price inflation for the US and for the Euro Area.² They show first-order autocorrelations of almost 0.9 for both the US and the Euro area. Clearly, by this definition, inflation is indeed a persistent series.

A question worth posing about this fact, however, is whether it is in any way surprising. For instance, a wide range of theories about inflation, ranging from the simple to the sophisticated, suppose that inflationary pressures are determined by measures of economic slack such as the output gap or the unemployment rate. Table 1 shows that these measures are also quite persistent, with both output gaps having first-order autocorrelations of about 0.85. Indeed, for both the US and the Euro Area, the unemployment rate has a far higher autocorrelation coefficient than inflation.³ The table also reports autocorrelations for the labor share: Galí and Gertler (1999) have proposed this as an alternative driving variable for inflation.⁴ Again, these series are more autocorrelated than the corresponding inflation series.

In light of these results, it is hardly surprising that inflation autocorrelations are quite high, and matching this fact should not be considered too high a bar for a theoretical model. Table 1, however, still leaves open the question of the *source* of the high autocorrelation for inflation. Is this high autocorrelation simply driven by the autocorrelation imparted by the underlying driving variables, or does the persistence have some independent source? To address this question, Tables 2 and 3 report results for the US and Euro Area from

²The US GDP deflator was downloaded from the BEA's website, and the sample used was 1960:Q1 to 2003:Q2. The Euro-Area data are taken from the ECB's Area Wide Model database, described in Fagan, Henry, and Mestre (2001) and the sample used for this series was 1970:Q2 to 2002:Q4.

³The output gaps are defined by applying a Hodrick-Prescott filter to the log of real GDP.

⁴For the US, the labor share series was downloaded from the BLS website (www.bls.gov). For the Euro area, I follow Galí, Gertler and Lopez-Salido (2001) in defining this series as the ratio of wage compensation of employees to nominal GDP, where these variables are measured as *WIN* and *YEN* from the AWM database.

regressions of the form

$$\pi_t = \alpha + \rho(1)\pi_{t-1} + \sum_{k=1}^3 \psi_k \Delta\pi_{t-k} + \sum_{k=0}^3 \gamma_k y_{t-k} + \epsilon_t, \quad (1)$$

where y is either the output gap, the unemployment rate or the labor share. If the persistence of inflation came simply from the autocorrelations in the driving variables, then we would expect to find a low value of the parameter $\rho(1)$. However, these regressions each report large and extremely statistically significant values of $\rho(1)$ for both the US and Euro area, and for each of the selected driving variables.

Some researchers, such as Cogley and Sargent (2002), have argued that the lagged dependent variable effect has weakened over time in the US. This is verified to some extent in Table 2, which shows that estimates of $\rho(1)$ for the post-1983 sample are lower for each of the specifications than for the previous period, and lower again for the sample beginning in 1991. The Euro area results, in contrast, show little systematic tendency for lower values of $\rho(1)$ for the later samples, consistent with the results of O'Reilly and Whelan (2005). The point relevant here for our analysis is merely that while there may be some evidence for changes over time in the size of the lagged dependent variable effect, the effect is always estimated to be positive and highly statistically significant.⁵

These results show that inflation appears to have an *intrinsic* persistence or inertia that would result in a pattern of highly positively autocorrelated inflation, even if its driving variables were themselves only weakly autocorrelated. Indeed, one could argue that the pattern of positive dependence of inflation on its own lags documented in these regressions provides a useful definition of the concept of “inflation persistence” because it documents a phenomenon that is specific to the behavior of inflation, and does not depend solely on the exogenous *deus ex machina* of an autocorrelated driving variable.

The rest of this paper will show that models based on staggered price contracts, while consistent with positively autocorrelated inflation, are completely inconsistent with the pattern of intrinsic inflation persistence described by these reduced-form regressions.

⁵One could, of course, argue that each of these explanatory variables are potentially endogenous and thus that the equations should be estimated via instrumental variables. However, the pattern of estimates of ρ are very similar when the equations are estimated via two-stage least-squares.

3 Inflation Under Staggered Contracts: A General Solution

This section introduces a general model of staggered price setting, derives an analytical solution for the form of the aggregate inflation process, and discusses some of the properties of this solution.

3.1 The Model

The type of price stickiness considered here is of the general form assumed by Goodfriend and King (1997). Firms that set a price today must take into account that it will remain fixed for an interval of stochastic length. The interval is determined by a hazard function $(a_1, a_2, \dots, a_{n-1}, a_n)$ such that a_k is the probability that a price that has remained in place for k periods will be changed. We will assume that $a_n = 1$, so there is a maximum length of time that any price can remain fixed. This framework incorporates a number of popular models. For example, the hazard function $(a_1, a_2, \dots, a_{n-1}, a_n) = (0, 0, \dots, 0, 1)$ implies an n -period version of the familiar contracting model due to Taylor (1979). The hazard function $(a_1, a_2, \dots, a_{n-1}, a_n) = (\alpha, \alpha, \dots, \alpha, 1)$ is a “truncated” version of the familiar Calvo (1983) model. Thus we are allowing for the possibility of Calvo-style random price adjustment while excluding the possibility of price durations of arbitrarily long length, which is commonly seen as a weakness of the basic Calvo model.

It is assumed that the economy consists of imperfectly competitive firms who have demand functions derived from Dixit-Stiglitz-style preferences: There is a continuum of firms, such that firm i with price P_{it} has demand function

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t} \right)^{-\theta}, \quad (2)$$

where Y_t is total output and P_t is the aggregate price level. All firms in the model are identical, so each firm that sets a price at time t will set the same price. We label this new price set at time t as X_t . Thus, the aggregate price level consistent with the Dixit-Stiglitz preferences is defined as

$$P_t = \left(\sum_{k=0}^{n-1} s_{kt} X_{t-k}^{1-\theta} \right)^{\frac{1}{1-\theta}}. \quad (3)$$

where s_{kt} is the fraction of prices that are k periods old at time t .

In our analytical results we will assume that the economy has converged to a point at which the shares s_{kt} are constant over time. The value of these constant shares are derived

as follows. By definition, the shares evolve over time according to

$$s_{0t} = \sum_{i=1}^n a_i s_{i,t-1}, \quad (4)$$

$$s_{k,t} = (1 - a_k) s_{k-1,t-1} \quad k = 1, 2, \dots, n-1. \quad (5)$$

Defining $f_0 = 1$ and

$$f_k = \prod_{r=1}^k (1 - a_r), \quad (6)$$

then along a steady-state path, we can write the constant shares as $s_k = f_k s_0$. The fact that the shares sum to one thus defines them as

$$s_k = \frac{f_k}{\sum_{k=0}^{n-1} f_k} \quad (7)$$

As is standard, we will work with log-linear approximations to the model's key equations. Log-linearizing the price level equation around a zero inflation steady-state, we get

$$p_t = \sum_{k=0}^{n-1} s_k x_{t-k}. \quad (8)$$

The details of all log-linearizations are reported in Appendix A.

Given the market structure and the hazard function for price changes, firms that set a price at time t choose X_t to maximize the discounted sum of expected real profits over the life of the contract. Note that f_k (the probability that the price will last as far as period $t+k$) is directly proportional to s_k (the steady-state share of firms whose price was set k periods ago). Thus, the firm's problem can be written in terms of maximizing

$$E_t \left[\sum_{k=0}^{n-1} \beta^k s_k \left(Y_{t+k} P_{t+k}^{\theta-1} X_t^{1-\theta} - \frac{1}{P_{t+k}} C_{t+k} \left(Y_{t+k} P_{t+k}^{\theta} X_t^{-\theta} \right) \right) \right], \quad (9)$$

where β is the firm's discount rate and C_t is its nominal cost function at time t . Solving this problem yields the following formula for the optimal contract price

$$X_t = \frac{\theta}{\theta - 1} \frac{E_t \left(\sum_{k=0}^{n-1} s_k \beta^k Y_{t+k} P_{t+k}^{\theta-1} MC_{t+k} \right)}{E_t \left(\sum_{k=0}^{n-1} s_k \beta^k Y_{t+k} P_{t+k}^{\theta-1} \right)}. \quad (10)$$

where MC_t is the firm's nominal marginal cost at time t . Log-linearizing this expression around a constant output level and a zero inflation rate, this becomes

$$x_t = \frac{E_t \left[\sum_{k=0}^{n-1} \beta^k s_k m c_{t+k} \right]}{\sum_{k=0}^{n-1} \beta^k s_k} \quad (11)$$

where lower-case symbols corresponds to logged variables. Defining real marginal cost as

$$MC_t^r = \frac{MC_t}{P_t}, \quad (12)$$

and assuming a simple relationship between the log of real marginal cost and the output gap as derived, for instance, in Chapter 3 of Woodford (2003):

$$mc_t^r = \gamma y_t \quad (13)$$

the optimal contract price becomes

$$x_t = \frac{E_t \left[\sum_{k=0}^{n-1} \beta^k s_k (p_{t+k} + \gamma y_{t+k}) \right]}{\sum_{k=0}^{n-1} \beta^k s_k}. \quad (14)$$

Together, equations (8) and (14), determine the dynamics of inflation in the model.

3.2 Analytical Solution

We now show how one can derive an analytical solution for the relationship between inflation and output for this model. Our solution method starts by noting from equation (8) that aggregate inflation is a weighted average of current and past rates of change of the contract price:

$$\pi_t = \sum_{k=0}^{n-1} s_k \Delta x_{t-k}. \quad (15)$$

In light of this result, our strategy for deriving a solution for price inflation will involve first characterizing the behavior of the contract price.

The first step in solving for the process for the contract price is to substitute out the expected future price levels in terms of future and past contract prices to get

$$x_t = \frac{E_t \left[\sum_{k=0}^{n-1} \beta^k s_k \left(\sum_{r=0}^{n-1} s_r x_{t+k-r} + \gamma y_{t+k} \right) \right]}{\sum_{k=0}^{n-1} \beta^k s_k}. \quad (16)$$

This is a $2(n-1)$ th-order stochastic difference equation in x_t and its properties underlie the properties of aggregate price inflation in this model. The equation can be re-arranged to give

$$\left(\sum_{k=0}^{n-1} \beta^k s_k \right) x_t = \sum_{k=0}^{n-1} \beta^k s_k \sum_{r=0}^{n-1} s_r E_t x_{t+k-r} + \gamma \sum_{k=0}^{n-1} \beta^k s_k y_{t+k}, \quad (17)$$

The form of this difference equation can be simplified somewhat by making use of the following equality:

$$\begin{aligned} \sum_{k=0}^{n-1} \beta^k s_k \sum_{r=0}^{n-1} s_r E_t x_{t+k-r} &= \left(\sum_{k=0}^{n-1} \beta^k s_k^2 \right) x_t + (s_0 s_{n-1}) (x_{t-n+1} + \beta^{n-1} E_t x_{t+n-1}) \\ &+ (s_0 s_{n-2} + \beta s_1 s_{n-1}) (x_{t-n+2} + \beta^{n-2} E_t x_{t+n-2}) + \dots \\ &+ \left(s_0 s_1 + \beta s_1 s_2 + \dots + \beta^{n-2} s_{n-2} s_{n-1} \right) (x_{t-1} + \beta E_t x_{t+1}) \end{aligned}$$

In particular, defining the following polynomial

$$\sigma(x) = \sum_{k=1}^{n-1} \left(\sum_{r=0}^{n-k-1} \beta^r s_r s_{r+k} \right) x^k, \quad (18)$$

the contract price process can be re-written in terms of lag and forward operators as

$$E_t \left[\sigma(\beta F) - \left(\sum_{k=0}^{n-1} \beta^k s_k (1 - s_k) \right) + \sigma(L) \right] x_t = -\gamma Z_t. \quad (19)$$

where

$$Z_t = \sum_{k=0}^{n-1} \beta^k s_k y_{t+k}. \quad (20)$$

Some of the key properties of this process can then be derived from the following results.

Proposition: The $2(n-1)$ th-order polynomial equation

$$\left[\sigma(\beta x) - \left(\sum_{k=0}^{n-1} \beta^k s_k (1 - s_k) \right) + \sigma(x^{-1}) \right] x^{n-1} = 0 \quad (21)$$

has the following properties

- (a) If λ is a solution, then $(\beta\lambda)^{-1}$ is also a solution.
- (b) One and β^{-1} are both solutions.
- (c) The equation can be re-written as

$$s_0 s_{n-1} \beta^{n-1} (x-1) (x-\beta^{-1}) g(x) = 0 \quad (22)$$

where $g(x)$ is a $2(n-2)$ -th order polynomial with only positive coefficients.

Proof: (a) The fact that all of the coefficients of $\sigma(x)$ are positive means that the equation has a positive intercept and this rules out zero solutions. Equation (21) thus holds when the term inside the square brackets in this equation is zero. The required result comes

from noting that the term inside the square brackets is unchanged when λ is replaced with $(\beta\lambda)^{-1}$.

(b) Note from equation (17) that this polynomial can also be written as

$$\left[\left(\sum_{k=0}^{n-1} \beta^k s_k \right) \lambda - \sum_{k=0}^{n-1} \beta^k s_k \sum_{r=0}^{n-1} s_r \lambda^{k-r} \right] \lambda^{n-1} = 0 \quad (23)$$

Using the fact that $\sum_{r=0}^{n-1} s_r = 1$, we can see that $\lambda = 1$ is a solution to this equation. That β^{-1} is also a root follows directly from part (a).

(c) The $2(n-1)$ th-order polynomial on the left-hand-side of (21) can be expressed in terms of its roots as

$$\left[\sigma(\beta x) - \left(\sum_{k=0}^{2(n-1)} \beta^k s_k (1 - s_k) \right) + \sigma(x^{-1}) \right] \lambda^{n-1} = s_0 s_{n-1} \beta^{n-1} \left(\prod_{i=1}^{2(n-1)} (x - \lambda_i) \right) \quad (24)$$

(The term $s_0 s_{n-1} \beta^{n-1}$ is needed because this is the coefficient on the leading power of the polynomial.) The required result stems from a Lemma used in the proof of Descartes' Rule of Signs. The Lemma states that if a is a positive real then the polynomial $p(x)$ has at least one more sign change than $q(x)$ where this is defined by $p(x) = (x - a)q(x)$.⁶ We know that the polynomial has exactly two sign changes and also that 1 and β^{-1} are both roots. Together, these imply that the polynomial can be written in the form of equation (22), where $g(x)$ has no sign changes. Because the lead coefficient of the polynomial in (21) is positive, the lead coefficient of $g(x)$ must also be positive, so all of the coefficients in $g(x)$ are positive. Descartes' Rule of Signs, which states that the number of sign changes in a polynomial is an upper limit on the number of positive real solutions, also tells us that the roots of g are either negative reals or complex numbers. \square

With these results in hand, we can now discuss three aspects of the model: The existence of a stable solution, the form of the relationship between inflation and output, and restrictions placed on the dynamics of this relationship. We will take these in turn.

Existence of a Stable Solution: For there to exist a unique stable process for contract

⁶See "Some Polynomial Theorems" by John Kennedy of Santa Monica College for a clear statement of this and other results related to Descartes' Rule of Signs. This paper can be downloaded at <http://homepage.smc.edu/kennedy-john/>

prices, equation (21) must have $n - 1$ roots on or inside the unit circle.⁷ If $\beta = 1$, a stable solution is guaranteed because part (b) above implies that for each root outside the unit circle, there must exist a corresponding root inside the unit circle and vice versa; thus, $n - 1$ stable roots are guaranteed.

The case in which $\beta < 1$ is a little more complex. For each root λ inside the unit circle, the corresponding root $\beta^{-1}\lambda^{-1}$ is outside the unit circle. Also, for each root outside the unit circle with modulus greater than β , the corresponding root $\beta^{-1}\lambda^{-1}$ will be inside the unit circle. So, as long as there are no roots with modulus between one and β^{-1} , there is guaranteed to be a unique stable equilibrium. If roots with modulus inside $(1, \beta^{-1})$ did exist, then we would have pairs of roots, λ and $\beta^{-1}\lambda^{-1}$, both of which were outside the unit circle, and we could not have $n - 1$ stable roots. However, in all of the calculations reported in this paper, this case did not arise, and the models had unique stable solutions.

The Relationship Between Inflation and Output: The form of the relationship between inflation and output can be derived as follows. Let $\lambda_1, \lambda_2, \dots, \lambda_{n-2}$ represent the $n - 2$ roots on or inside the unit circle in addition to one, and let $\lambda_{n-1} = 1$. The contract price process can now be written as

$$E_t \left[s_0 s_{n-1} \beta^{n-1} (F - 1)(F - \beta^{-1}) \left\{ \prod_{i=1}^{n-2} (F - \lambda_i) \left(F - \frac{1}{\beta \lambda_i} \right) \right\} L^{n-1} x_t \right] = -\gamma Z_t. \quad (25)$$

Using the general principle of solving stable roots backwards and unstable roots forward, we need to apply the $n - 1$ lag operators L^{n-1} to the roots on or inside the unit circle to leave only one possible non-explosive solution. This solution can be written as:

$$s_0 s_{n-1} \beta^{n-1} \left\{ \prod_{i=1}^{n-2} (1 - \lambda_i L) \right\} \Delta x_t = -\gamma E_t \left[\left\{ \prod_{i=1}^{n-1} \left(F - \frac{1}{\beta \lambda_i} \right) \right\}^{-1} Z_t \right]. \quad (26)$$

Note also that

$$\frac{1}{F - (\beta \lambda_i)^{-1}} = -\frac{\beta \lambda_i}{1 - \beta \lambda_i F} = -\beta \lambda_i \sum_{k=0}^{\infty} \beta^k \lambda_i^k F^k. \quad (27)$$

⁷This system can, of course, be written in “companion matrix” form, $Z_t = A E_t Z_{t+1}$ where the eigenvalues of the matrix A are the roots of equation (21). The well-known conditions for a unique stable solution, as described in Blanchard and Kahn (1980), are that the number of stable eigenvalues equal the number of predetermined variables in the system, which in this case is $n - 1$.

Thus, the contract price process is

$$\left\{ \prod_{i=1}^{n-2} (1 - \lambda_i L) \right\} \Delta x_t = \frac{\gamma}{s_0 s_{n-1}} E_t \left[\left(\prod_{i=1}^{n-2} (-\lambda_i) \right) \left(\sum_{k=0}^{\infty} \beta^k \lambda_1^k F^k \right) \dots \left(\sum_{k=0}^{\infty} \beta^k \lambda_{n-1}^k F^k \right) Z_t \right]. \quad (28)$$

Letting

$$\tilde{\gamma} = \frac{\gamma}{s_0 s_{n-1}} \quad (29)$$

and

$$\delta(L) = \left\{ \prod_{i=1}^{n-2} (1 - \lambda_i L) \right\}, \quad (30)$$

we obtain the solution for the rate of change of the new contract price as

$$\delta(L) \Delta x_t = \tilde{\gamma} \sum_{k=0}^{\infty} \kappa_k E_t Z_{t+k}. \quad (31)$$

Now let

$$\alpha(L) = \left(\sum_{j=0}^{n-1} s_j L^j \right), \quad (32)$$

be the operator that translates contract prices into the aggregate price level. Aggregate price inflation can then be written as

$$\pi_t = \alpha(L) \Delta x_t, \quad (33)$$

and aggregate price inflation is given by:

$$\delta(L) \pi_t = \tilde{\gamma} \alpha(L) \left[\sum_{k=0}^{\infty} \kappa_k E_t Z_{t+k} \right]. \quad (34)$$

Because $\delta_0 = 1$, this can be re-written as

$$\pi_t = \psi(L) \pi_{t-1} + \tilde{\gamma} \alpha(L) \left[\sum_{k=0}^{\infty} \kappa_k E_t Z_{t+k} \right]. \quad (35)$$

Aggregate price inflation is a function of two factors. The first factor is current and past expectations about the future paths of the driving variable y_t . The second factor is inflation's own lagged values: From equation (30), we see that as long as contracts are longer than two periods in length, ($n > 2$), then inflation will be directly affected by its own lags.

Restrictions on $\psi(L)$: The nature of the lagged dependent variable effect on current inflation is determined by the properties of the roots of the polynomial equation (21). To see this, consider some specific cases.

First consider the case in which $n = 3$, so there is a single lagged inflation term in $\psi(L)$. In this case, (21) is a fourth-order difference equation. Two of its roots are one and β^{-1} and part (c) above ensures that its two other roots are negative reals. Descartes' Rule of Signs rules out positive real roots for $g(x)$, and the order of the difference equation rules out complex roots. To see this, note that complex solutions to this equation must come in fours: If $\lambda_1 = c + di$ is a solution, then so its complex conjugate $\lambda_2 = c - di$, and both $\beta^{-1}\lambda_1^{-1}$ and $\beta^{-1}\lambda_2^{-1}$ are also solutions. So, in this case, we have

$$\delta(L) = 1 - \lambda_1 L \tag{36}$$

where λ_1 is negative. Hence, the coefficient on the first and only lag in $\psi(L)$ is negative.

Now consider the case in which $n = 4$, so that there are two lagged inflation terms in $\psi(L)$. Again, it turns out that these terms must be negative. The polynomial in (21) is of order six for this case. So, as well as one and β^{-1} , there are four additional roots, which are the roots of $g(x)$. Again, Descartes' Rule of Signs implies that these roots are either negative reals or complex roots, and we can rule out the possibility of complex roots with positive real components. This is because if $\lambda_1 = c + di$ is a root where $c > 0$, then $\lambda_2 = c - di$ is also a root and $\beta^{-1}\lambda_1^{-1}$ and $\beta^{-1}\lambda_2^{-1}$ must also have positive real components. Thus the negative of sum of these four roots must be negative. However, this would imply that the coefficient on x^3 in $g(x)$ would be negative, which contradicts result (c) above. This means that for $n = 4$, we have

$$\delta(L) = (1 - \lambda_1 L)(1 - \lambda_2 L) \tag{37}$$

where λ_1 and λ_2 are either negative reals or complex conjugates with negative real components. These conditions ensure that both of the lag coefficients in $\psi(L)$ are negative.

For higher values of n , this type of reasoning cannot rule out the possibility of one or more of the coefficients in $\psi(L)$ being positive. However, the properties of the roots of the characteristic equation derived suggest that negative coefficients should dominate: For instance, the sum of the roots must always be negative. In fact, as we will see, for all of the cases examined here, all of the lag coefficients turn out to be negative.

4 Examples

We now consider three types of staggered contracting models (the Taylor model, the truncated Calvo model, and models based on increasing hazards) and provide numerical examples of the negative effect of lagged inflation on current inflation implied by these models.

4.1 Taylor Contracting

Perhaps the most familiar staggered contracting model is the one pioneered by John Taylor (1979). This model assumes that all price contracts last exactly n periods, implying a hazard function of $(a_1, a_2, \dots, a_{n-1}, a_n) = (0, 0, \dots, 0, 1)$. The unique stable distribution of price duration shares is $s_k = \frac{1}{n}$ for $k = 0, 1, \dots, n - 1$. Worth noting is that for the case $n = 2$, the contract price solution (28) for this model reduces to

$$\Delta x_t = \beta E_t \Delta x_{t+1} + 2\gamma (y_t + E_t y_{t+1}), \quad (38)$$

and thus aggregate price inflation is

$$\pi_t = \beta E_t \pi_{t+1} + \gamma (y_{t-1} + 2y_t + E_t y_{t+1}) + \nu_t \quad (39)$$

where ν_t is an expectational error. This result generalizes to the case of discounting a result previously presented by Roberts (1995), which is that two-period Taylor contracting implies a type of New-Keynesian Phillips curve specification for inflation.

Moving beyond the two-period model, however, the solution for inflation features lagged dependent variable terms. The results in the previous section established that for three- and four-period models, these lagged dependent variable coefficients must be negative. The upper panel of Table 4 establishes the size of these coefficients for the Taylor model, and also that the lag coefficients are all negative for models featuring longer maximum contract durations.⁸ The results in the table correspond to a value of $\beta = 0.99$ (implying an annual discount rate of about four percent if the period is interpreted as a quarter) but the calculations are not very sensitive to variations in this parameter. For example, for the case $\beta = 0.99$, the four-period Taylor model has an inflation process of the form

$$\pi_t = -0.430\pi_{t-1} - 0.121\pi_{t-2} + \tilde{\gamma}\alpha(L) \left[\sum_{k=0}^{\infty} \kappa_k E_t Z_{t+k} \right]. \quad (40)$$

⁸While the table reports results for a maximum of $n = 6$, unreported calculations confirm the negative lag result for all higher values of n tried. All of the numerical calculations reported in the paper were derived using the numerical solution algorithm for rational expectations models of Binder and Pesaran (1995).

For the case $\beta = 1$, this process becomes

$$\pi_t = -0.428\pi_{t-1} - 0.120\pi_{t-2} + \tilde{\gamma}\alpha(L) \left[\sum_{k=0}^{\infty} \kappa_k E_t Z_{t+k} \right]. \quad (41)$$

These calculations show that the prediction of negative coefficients on the lagged inflation terms is not just a theoretical curiosity: Realistic calibrations of a standard model predict quite large negative coefficients on the lagged inflation terms.

4.2 Truncated Calvo Model

The Taylor model's prediction that all completed price spells should be of the same duration is clearly at odds with the evidence of significant heterogeneity in price durations.⁹ An alternative approach that fits within our framework and is consistent with this evidence is the truncated Calvo model. This model implies a hazard function of the form $(a_1, a_2, \dots, a_{n-1}, a_n) = (\alpha, \alpha, \dots, \alpha, 1)$.

The middle two panels of Table 4 report the $\psi(L)$ coefficients generated by two different truncated Calvo models, one with $\alpha = 0.5$ and one with $\alpha = 0.25$. (Again, a value of $\beta = 0.99$ is used.) For comparison purposes, while the average completed price spell in the Taylor example always equals n , the truncated Calvo model with $\alpha = 0.25$ has an average duration that varies from 1.8 periods to 2.7 periods as n goes from three to six, while the average duration varies from 1.5 periods to 1.9 periods for the case in which $\alpha = 0.5$. The results in the table show that the model's lagged dependent variable coefficients do not vary much across the different models. For a given value of n , the truncated Calvo and Taylor models each imply similar patterns of values for the parameters.

4.3 Upward-Sloping Hazard Functions

Finally, the bottom panel of Table 4 reports calculations for a set of hazards consistent with an increasing probability of a price change as duration rises. This type of hazard function makes more intuitive sense than the Calvo or Taylor formulations because it recognizes that a price set today is more likely to still be suitable next quarter than it is next year when economic conditions have changed more. Indeed, the type of hazards reported in Table 4, in which the probability of a price change is very low at first but rises rapidly after the first few periods, is modelled on the typical pattern generated by models of state-dependent

⁹See, for instance, Bils and Klenow (2004).

pricing such as Dotsey, King, and Wolman (1999, henceforth DKW). Despite the different assumptions underlying these models, the pattern of negative lagged inflation coefficients turns out to be quite similar to those generated by the Taylor and, in particular, truncated Calvo models.

One interesting implication of these calculations is that our result concerning negative lagged inflation coefficients also applies to local approximations to the inflation dynamics generated by state-dependent pricing models of the DKW variety. The local nature of this characterization stems from the fact that for different inflation rates, the DKW model will generate different average upward-sloping hazards. However, these results show that, for each specific hazard, inflation dynamics will be characterized by negative lagged inflation coefficients. That the results only approximate the dynamics of state-dependent models stems from the fact that we have not accounted, in either the price-level equation or the optimal contract equation, for the fact that hazard functions may deviate from their steady-state values in state-dependent models.¹⁰ However, DKW show that the terms describing these deviations contribute little to inflation dynamics in these models, so our equations likely still constitute a good local approximation.¹¹

5 Positive Steady-State Inflation

All of the derivations presented so far have been based on approximating dynamics around a zero-inflation steady-state. However, it is known that the log-linearized dynamics of sticky-price models become more complicated when the relevant equations are approximated around steady-states involving positive inflation.¹² The results in this section demonstrate that our conclusions about inflation dynamics in staggered contracting models still obtain when one approximates around positive inflation steady-states.

The two equations that are approximated to give the log-linearized version of the model are the definition of the price level, equation (3), and the optimal contract price condition, equation (10). Appendix A demonstrates that the price level equation can be log-linearized

¹⁰See Bahkshi, Kahn, and Rudolf (2004) for a characterization of price dynamics in state-dependent models that accounts for this factor.

¹¹See page 676 of Dotsey, King, and Wolman (1999) for a discussion of this issue.

¹²See Ascari (2004).

around a steady-state path with a constant gross inflation rate $\frac{P_t}{P_{t-1}} = \Pi$, to give

$$p_t = \sum_{i=0}^{n-1} \mu_k x_{t-k}, \quad (42)$$

where

$$\mu_k = \frac{s_k \Pi^{(\theta-1)k}}{\sum_{i=0}^{n-1} s_k \Pi^{(\theta-1)k}}. \quad (43)$$

When one approximates around a zero-inflation steady-state, so that $\Pi = 1$, this reduces to equation (8).

Appendix A also shows that the log-linearized optimal contract price equation now takes the form

$$x_t = E_t \sum_{k=0}^{n-1} \left(\lambda_k m c_{t+k} + (\lambda_k - \phi_k) \left(y_{t+k} + (\theta - 1) \sum_{r=1}^k \pi_{t+r} \right) \right), \quad (44)$$

where

$$\lambda_k = \frac{(\beta \Pi^\theta)^k s_k}{\sum_{k=0}^{n-1} (\beta \Pi^\theta)^k s_k}, \quad (45)$$

$$\phi_k = \frac{(\beta \Pi^{\theta-1})^k s_k}{\sum_{k=0}^{n-1} (\beta \Pi^{\theta-1})^k s_k}. \quad (46)$$

In general, it is difficult to examine the properties of this system using analytical methods. However, close inspection suggests that the terms $\lambda_k - \phi_k$ should be relatively small. On average these terms equal zero, each of the individual terms are zero when $\Pi = 1$, and even when steady-state inflation is positive the definitions of λ_k and ϕ_k are very similar ensuring that their differences should be small. Indeed, calculations based on a wide range of realistic parameter values show that these terms are essentially negligible; this finding has also been reported before in the context of the state-dependent model of Dotsey, King and Wolman (1999).¹³ Thus, the pricing equation is well approximated as

$$x_t = E_t \sum_{k=0}^{n-1} \lambda_k m c_{t+k}. \quad (47)$$

This simplification allows us to apply analytical solution methods in the same manner as before. The contract price equation can be re-expressed as

$$x_t = E_t \sum_{k=0}^{n-1} \lambda_k (p_{t+k} + \gamma y_{t+k}),$$

¹³Again, see page 676 of Dotsey, King, and Wolman (1999).

$$= E_t \sum_{k=0}^{n-1} \lambda_k \sum_{r=0}^{n-1} \mu_k x_{t+k-r} + \gamma E_t \sum_{k=0}^{n-1} \lambda_k y_{t+k}. \quad (48)$$

And, following similar patterns as before, the first term on the right-hand-side of this equation can be re-written in terms of lag and lead polynomials as

$$E_t \sum_{k=0}^{n-1} \lambda_k \sum_{r=0}^{n-1} \mu_k x_{t+k-r} = E_t \left[\left\{ \sum_{r=0}^{n-1} \lambda_k \mu_k + \omega(L) + \omega^*(F) \right\} x_t \right], \quad (49)$$

where

$$\omega(z) = \sum_{k=1}^{n-1} \left(\sum_{r=0}^{n-k-1} \lambda_r \mu_{r+k} \right) z^k, \quad (50)$$

$$\omega^*(z) = \sum_{k=1}^{n-1} \left(\sum_{r=0}^{n-k-1} \lambda_{r+k} \mu_r \right) z^k. \quad (51)$$

Note now that

$$\lambda_{r+k} = \beta^k \Pi^{k\theta} \left(\frac{s_{r+k}}{s_r} \right) \lambda_r \quad (52)$$

$$\mu_{r+k} = \Pi^{k(\theta-1)} \left(\frac{s_{r+k}}{s_r} \right) \mu_r \quad (53)$$

So, the coefficients in the lag and lead operators are related by the expression

$$\lambda_{r+k} \mu_r = (\beta \Pi)^k \lambda_r \mu_{r+k} \quad (54)$$

This allows us to re-write the contract price process as

$$E_t \left[\left\{ \omega(\beta \Pi F) - \left(1 - \sum_{r=0}^{n-1} \lambda_k \mu_k \right) + \omega(L) \right\} x_t \right] = -\gamma Z_t. \quad (55)$$

where

$$Z_t = E_t \sum_{k=0}^{n-1} \lambda_k y_{t+k} \quad (56)$$

Considerations related to the existence and properties of solutions to this equation are similar to the zero-inflation steady-state case, with $\beta \Pi$ playing the same role as β did in the analysis of Section 3. Thus, in terms of existence, a unique non-explosive solution is guaranteed in the case where $\beta \Pi = 1$. Beyond this case, there is a unique solution provided the characteristic polynomial has no roots with modulus between one and $(\beta \Pi)^{-1}$. Calculations with various parameter values and versions of the three models from the previous

section all indicated the existence of unique stable solutions as long as the steady-state inflation rate was below very high levels. Specifically, unique stable solutions existed for values of Π up to at least 1.05. Interpreting the unit of time as one quarter, this implies the existence of a unique stable solution for values of annualized inflation of up to about 20 percent.

In terms of the properties of the model's solutions, one is again a solution of the characteristic equation for contract price process, so again there is a solution for aggregate inflation of the same form as equation (35). And, as before, there are two sign changes so the theoretical arguments guaranteeing negative $\psi(L)$ coefficients for $n = 3$ and $n = 4$ are still valid. More generally, numerical calculations show that there are only very small changes in the $\psi(L)$ coefficients as we move away from the zero inflation steady state towards steady states with higher inflation. Table 5 provides an example of this, displaying these coefficients for the truncated Calvo model (with $\alpha = 0.25$) for values of Π ranging from 1 to 1.05; the standard value of $\theta = 10$ was used to represent preferences, but the results were not very sensitive to this parameter. We conclude from these calculations that the generality of our results does not depend on the zero-inflation steady-state assumption.

6 Autocorrelations versus Intrinsic Persistence

In Section 2 it was noted that, while related, there were conceptual differences between the idea of inflation persistence as high autocorrelations and the idea of intrinsic persistence generated by a positive lagged dependent variable effect. Here, we use a simple example to illustrate how staggered contracting models can match high autocorrelations for inflation, while failing to match the empirical evidence on intrinsic persistence.

The example is based on the assumption that the output gap is determined by an AR(1) process

$$y_t = \rho y_{t-1} + \epsilon_t, \tag{57}$$

where ϵ_t is assumed to be white noise. In this case, all of the expectational variables, $E_t Z_{t+k}$, reduce to being multiples of y_t . This simplification means that there is no connection between lags of inflation and the expectational terms in equation (35), so that the coefficients on the lagged terms in the reduced-form representation are the same as in the structural representation.

So, in this case, the four-period Taylor contracting model with $\beta = 0.99$, and $\rho = 0.9$

has a solution that reduces to

$$\pi_t = -0.430\pi_{t-1} - 0.121\pi_{t-2} + 10.179\gamma\alpha(L)y_t, \quad (58)$$

where, in this case,

$$\alpha(L) = \frac{1}{4} \sum_{j=0}^3 L^j, \quad (59)$$

is the simple moving average operator.

Simulating this process, the first-order autocorrelation coefficient for inflation is 0.965. Thus, the model produces an inflation series that is more autocorrelated than its driving variable.¹⁴ This may be a little surprising given the negative coefficients on the lagged dependent variables. This can be explained, however, by noting that the model predicts inflation is an *ARMA*(2, 3) series, with driving variable y_t . While, *ceteris paribus*, the *AR* component acts to make π_t less autocorrelated than y_t , the *MA* component tends to make it more so.

One formal way to explain this result is to compute the spectral properties of the filter that transforms y_t into π_t . In other words, we can analyze how the application of the filter

$$f(L) = \frac{10.179}{4} \frac{\gamma(1 + L + L^2 + L^3)}{1 + 0.430L + 0.121L^2}, \quad (60)$$

tends to promote the role of certain frequencies over others. Normalizing γ as $\frac{4}{10.179}$ for convenience, the spectral transformation of y_t implied by this filter is

$$f(e^{i\omega}) f(e^{-i\omega}) = \frac{4 + 6 \cos \omega + 4 \cos 2\omega + 2 \cos 3\omega}{1.199 + 0.964 \cos \omega + 0.121 \cos 2\omega}, \quad (61)$$

where the numerator here describes the effect of the *MA* component and the denominator describes the effect of the *AR* component. As Figure 1 shows, on its own the effect of the *AR* component of the filter is to increase the role of higher-frequency cycles (the left panel), but the effect of the *MA* component is to increase the role of lower-frequency cycles (the middle panel). When the two components are put together (the right panel), we see that the combined effect produces a downward-sloping spectral transformation, implying that the inflation series will exhibit more low-frequency variation, and thus higher autocorrelations, than the driving variable.

These examples show that staggered contracting models do not have difficulty generating high autocorrelations for inflation, in contrast to the claims of Fuhrer and Moore

¹⁴This result is not affected by the value of γ chosen.

(1995). Thus, these results support the findings of Guerrieri (2002) who shows that Taylor contracting models can match the high inflation autocorrelations seen in the data. However, at the same time, they also show that it is possible for the models to completely fail to capture a key element of the empirical inflation process that perhaps better describes what is meant by inflation persistence, i.e. the positive dependence of inflation on its own lagged values.

7 A Simple Monetary Model of Output

7.1 From Structural-Form to Reduced-Form Relationships

We have shown that staggered contracting models imply a structural relationship of the form

$$\pi_t = \psi(L)\pi_{t-1} + \tilde{\gamma}\alpha(L) \sum_{k=0}^{\infty} \kappa_k E_t Z_{t+k}.$$

in which the coefficients in the $\psi(L)$ lag polynomial are all negative. On the face of it, this seems to strongly contradict the evidence from the regressions reported in Section 2. However, an important caveat to this interpretation is that the negative coefficients in this representation depend on the inclusion of unobservable expectational variables, while the evidence in Section 2 relates to reduced-form regressions relating inflation to its own lags and to current and lagged values of the relevant driving variable.

The example of an autoregressive output gap in the previous section got around this problem by assuming that lagged values of inflation contain no information about future output beyond what is already contained in current or lagged values of output, i.e. that there was no Granger causality going from inflation to output. In this case, the reduced form and structural coefficients on the lagged inflation terms are identical. In reality, however, this lack of causality may not be a reasonable assumption.

This suggests one potential route for reconciling the contracting models with the evidence in Section 2. If lags of inflation acted as positive leading indicators for the driving variable y_t , then this relationship could still potentially be consistent with positive coefficients on lagged inflation in a *reduced-form* regressions. Put formally, suppose this positive leading indicator role took the form of

$$\sum_{k=0}^{\infty} \kappa_k E_t Z_{t+k} = v(L) \pi_{t-1} + \zeta(L) y_t, \tag{62}$$

where the coefficients in the $v(L)$ polynomial were positive. In this case, the reduced-form relationship would be

$$\pi_t = [\psi(L) + \gamma\alpha(L)v(L)]\pi_{t-1} + \gamma\alpha(L)\zeta(L)y_t, \quad (63)$$

and it is possible that the positive coefficients in the $\gamma\alpha(L)v(L)$ polynomial could sufficiently outweigh the negative coefficients in the $\psi(L)$ polynomial to produce the positive coefficients seen in the estimated reduced-form relationships.

We now consider a standard monetary model with endogenously-determined output in which this positive causality is present, and examine whether such a model is likely to be consistent with the reduced-form evidence.

7.2 The Model

Here we consider the case in which the output gap is determined by real money balances

$$y_t = m_t - p_t, \quad (64)$$

and money growth evolves according to an AR(1) process:

$$\Delta m_t = \rho_m \Delta m_{t-1} + \epsilon_t^m. \quad (65)$$

These assumptions have previously been considered in conjunction with a staggered contracting model in the work of Chari, Kehoe, and McGrattan (2000) and we can refer the reader to their paper for more formal derivations of this log-linearized equation for output.¹⁵

Before deriving the implications for the reduced-form characterization of inflation in this case, we first note that this model contains exactly the positive causality linkages that could, potentially, imply positive coefficients on lagged inflation in a reduced-form regression. To see this, note that output growth in this model is determined by

$$\Delta y_t = \rho_m \Delta m_{t-1} - \pi_t + \epsilon_t. \quad (66)$$

Substituting in $\Delta m_{t-1} = \Delta y_{t-1} + \pi_{t-1}$ and equation (35)'s structural representation for inflation, we obtain

$$\Delta y_t = \rho_m \Delta y_{t-1} + (\rho_m - \psi(L))\pi_{t-1} - \tilde{\gamma}\alpha(L) \sum_{k=0}^{\infty} \kappa_k E_t Z_{t+k}. \quad (67)$$

¹⁵Adding a positive intercept to the money growth equation so that inflation is positive on average does not change the analysis here.

Because the coefficients in the $\psi(L)$ polynomial are negative, this implies that there will be positive causality from lagged inflation to output growth in this model: High lagged inflation tends to reduce inflation today and thus boost real money growth. It turns out, however, that this effect does not appear to be enough to reconcile this model with the reduced-form evidence.

An analytical solution for the reduced-form inflation process for this model can be obtained as follows. Appendix B shows that the solution for the optimal contract price in this case takes the form

$$x_t = \sum_{k=1}^{n-1} \tau_k x_{t-k} + \left(1 - \sum_{k=1}^{n-1} \tau_k\right) m_t + \varphi \Delta m_t. \quad (68)$$

where $0 < \sum_{k=1}^{n-1} \tau_k < 1$, the τ_k 's are independent of the value of ρ , and φ depends on both ρ and γ . This implies a solution for the price level of form

$$p_t = \sum_{k=1}^{n-1} \tau_k p_{t-k} + \left(1 - \sum_{k=1}^{n-1} \tau_k\right) \alpha(L) m_t + \varphi \alpha(L) \Delta m_t \quad (69)$$

Finally, substituting $m_t = y_t + p_t$ and re-arranging, we obtain a reduced-form Phillips curve in terms of inflation and the output gap of the form

$$\pi_t = \sum_{k=1}^{n-1} a_k \pi_{t-k} + \sum_{k=0}^n b_k y_{t-k} \quad (70)$$

In this reduced-form representation, the coefficients on lagged inflation are different from those in the structural representation of the same model (that is, from the coefficients in equation 35) and in theory they can be positive. However, numerical calculations show that these theoretical reduced-form inflation processes do not come close to matching those obtained from regressions.

For example, consider again the four-period Taylor model. Setting $\gamma = 0.50$, $\beta = 1$, and $\rho_m = 0.66$ (the value consistent with a quarterly AR(1) regression for M1 growth), one obtains the following inflation process:¹⁶

$$\pi_t = -0.50\pi_{t-1} - 0.08\pi_{t-2} + 0.28\pi_{t-3} + 0.58y_t + 0.30(y_{t-1} + y_{t-2} + y_{t-3}) - 0.28y_{t-4} \quad (71)$$

¹⁶This estimate of ρ is based on a sample of 1959:3 to 2004:2. The data were downloaded from the Federal Reserve Board's website. Chari, Kehoe, and McGrattan (2000) also estimate a regression for M1 growth and report a similar coefficient value of 0.57.

Though the sum of the lagged inflation coefficients in this case is slightly less negative than in the structural representation for this model (-0.30 relative to -0.55), it is clear that this process does not look anything like the pattern of large positive coefficients reported in Section 2. Again, though, the model does succeed in generating an inflation series that is autocorrelated, and more so than the output gap: In this case, inflation has an autocorrelation coefficient of 0.88, compared with 0.83 for the output gap.

Table 5 reports the reduced-form lagged inflation coefficients for the four-period Taylor model obtained under a range of different values of γ and ρ_m . The γ parameter in these calculations varies from 0.1 to 3.0, representing a range in which real marginal cost can be either far less variable or far more variable than the output gap. Our estimate of the money growth autocorrelation coefficient of 0.66 has a standard error of 0.056, so this suggests 0.5 to 0.8 as endpoints of a wide range of reasonable values for this parameter. The results show that the sums of the lag coefficients are almost all negative and none come close to matching even the smallest of the values on Table 2. In addition, the first lag coefficients are always highly negative, which fails to match the empirical pattern that this tends to be the most positive coefficient. Consider, for example, the full-sample regression for US GDP price inflation featuring the output gap. In this case, the sum of the coefficients is 0.94, and the first lag coefficient is 0.51 with a standard error of 0.09.

These results also turn out to be repeated for the other contracting models considered here across a wide range of realistic parameter values, and including versions of the models based on positive inflation steady-states. In addition, calculations reported in Appendix B show that the reduced-form lagged inflation coefficients reported here are not changed by generalizing the model by adding a stochastic monetary velocity shock.

7.3 Estimates of Hybrid Calvo Models

The approach taken in this paper has been to compare the reduced-form inflation processes implied by theoretical models with the evidence from empirical regressions for such specifications. An advantage of this approach is that it provides a relatively transparent way to illustrate the empirical shortcomings of staggered contracting models. It is worth noting, however, that some other recent papers have discussed the implications of staggered contracting models for another type of regression estimation, namely GMM estimation of the so-called “hybrid” Calvo model proposed by Galí and Gertler (1999):

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t-1} + \psi y_t. \quad (72)$$

This equation is consistent with a Calvo-style model in which a fraction of firms adopt backward-looking rules of thumb when setting prices. In the context of this model, positive estimates of γ_b are considered evidence for the existence of backward-looking agents. However, using simulated data from truncated Calvo specifications, Dotsey (2002) and Bakhshi, Burriel-Llombart, Khan, and Rudolf (2003) both show that one can obtain positive values of γ_b from GMM estimation of this equation. Thus, they warn against interpreting significant positive estimates of γ_b as evidence for backward-looking price-setters, since the truncated Calvo models do not incorporate such behavior.

The findings of Dotsey and Bakhshi et al can be replicated using the class of models considered here. For instance, simulating the four-period Taylor model with $\gamma = 0.50$, $\beta = 1$, and $\rho = 0.66$ (the values that generate inflation equation 71) and estimating the equation via GMM using four lags of both inflation and the output gap, we obtain estimates of $\hat{\gamma}_b = 0.48$, $\hat{\gamma}_f = 0.64$, and $\hat{\psi} = -0.05$.¹⁷ These estimates of γ_b and γ_f are close to those reported in a number of empirical studies, and the finding of a negative coefficient on the driving variable is also reported by Bakhshi et al. While the specific estimates obtained depend on the values of the underlying parameters chosen, these exercises do invariably produce positive estimates for the γ_b coefficient.

These results confirm the cautionary warnings Dotsey and Bakhshi et al concerning the interpretation of tests of the hybrid Calvo model. However, one should be cautious in interpreting the estimates of γ_b generated by these simulated data as an important piece of evidence in favor of the Taylor or truncated Calvo models. For example, in the case of the estimates just reported, the behavior of inflation in the underlying model is fully described by (71), and this equation's implications for inflation dynamics are strongly contradicted by the evidence from reduced-form regressions.

In addition, it is worth keeping in mind that the estimates of the hybrid Calvo equation in these simulation exercises are driven purely by the fact that, in the simulated economies, this equation badly mis-specifies the dynamics of inflation, so the estimated coefficients are driven by the correlations with the omitted variables such as the additional lags of output and inflation. And as one moves closer to the correct underlying specification of the model's dynamics, one can overturn the positive estimates on the lagged inflation term as well as

¹⁷These estimates were based on taking the average of 10000 simulations, each based on a sample of 10000.

on the $E_t\pi_{t+1}$ term. For example, consider the case of GMM estimation of

$$\pi_t = \gamma_b\pi_{t-1} + \gamma_f E_t\pi_{t-1} + \sum_{k=0}^4 \psi_k y_{t-k}. \quad (73)$$

This specification adds in the additional lags of the output gap that belong in the correct model specification. Again simulating the case with $\gamma = 0.50$, $n = 4$, and $\rho = 0.66$, and estimating using $(\pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_{t-4}, y_t, y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4})$ as instruments, one now obtains $\hat{\gamma}_b = -0.95$ and $\hat{\gamma}_f = -2.66$. Overall, it could be argued that the complex interpretational issues raised by these exercises help to underscore the advantages of the simpler assessment procedure adopted in this paper based on deriving predictions for the properties of reduced-form equations.

8 Causality Tests

The results in the last section tell us that the causal linkages between inflation and output in a standard monetary model do not lead to an overturning of the prediction that staggered contracting models should imply negative coefficients on lagged inflation in reduced-form regressions. However, this cannot rule out the possibility that, in reality, these linkages are strong enough to overturn this prediction, and thus the theoretical results of the preceding section are misleading. This suggests a final route to checking whether the contracting models may be consistent with the evidence, which is to assess whether the relevant positive Granger-causality patterns from inflation to output are evident in the data.

From an a priori perspective it is, of course, also possible that the correct model implies a *negative* causal relationship from inflation to the output gap, and—if staggered contracting models considered here were the correct models of pricing—then this would imply reduced-form lag coefficients that should be more negative than those reported in Table 4. This point is worth noting because realistic structural models embedding a staggered contracting specification for pricing often contain a policy rule in which the central bank targets a particular value of inflation. And a policy rule of this form implies that high values of inflation trigger higher interest rates and thus will tend to dampen future output gaps, suggesting a Granger causality relationship with the wrong sign for reconciling Taylor-style models with the reduced-form evidence.

With these considerations in mind, one can see from the results reported on Table 7 that the positive causality argument does not appear to work well in practice. The table reports

results from a series of Granger Causality tests for US data, which test for causality running from inflation to each of the three driving variables discussed earlier (the output gap, the unemployment rate, and the labor share). These results show little evidence of causal relationships of the correct signs to allow for reconciliation of the staggered contracting models with the reduced-form evidence.

Full-sample tests reject the hypothesis that inflation Granger causes the output gap or the labor share. There is evidence of causation running from inflation to the unemployment rate, but this relationship has the wrong sign for reconciling the staggered contracting models with the evidence: Inflation appears to positively cause the unemployment rate, so a high lagged inflation rate should have an even more negative effect on current inflation than is indicated by the negative “intrinsic persistence” described by the $\psi(L)$ polynomial.

Because of the possible (or perhaps likely) changes over time in the reduced-form relationships between inflation and other macroeconomic variables, the table also reports results for the other samples reported for the earlier reduced-form regressions.¹⁸ The findings of no causal relationships from inflation to the output gap or labor share, and an incorrectly-signed relationship from inflation to the unemployment rate, turn out to be robust across each of the sub-samples. Similar results (not reported) for the Euro area also point against the Granger causality argument.

9 Conclusions

Staggered price contracting models are commonly used to illustrate the macroeconomic effects of nominal rigidities. This paper has focused on the ability of this approach to match the empirical evidence on inflation persistence. Some of the previous research on this issue has focused on whether the model can capture the high autocorrelations seen in the inflation data. We have shown here that staggered contracting models have no problem matching these autocorrelations: These models generally produce an inflation series whose autocorrelations are higher than those of the already-highly-autocorrelated driving variables, such as the output gap.

More importantly, though, the paper presents new results that illustrate staggered contracting’s implications for an alternative aspect of inflation persistence or inertia, namely

¹⁸The date 1984:1 was chosen because the Volcker disinflation had fully taken effect at this point, and any transitional dynamics associated with learning about a new policy regime may have been worked out. The results here are generally robust, however, across a wide range of sub-samples.

the positive dependence of inflation on its own lags. This feature of inflation, while closely related to high autocorrelations, represents a distinct definition of inflation persistence or inertia, and it is possible for a model to match one version of inflation persistence and not the other.

It is quite commonly assumed that staggered contracting models can provide a micro-foundation for the type of inflation inertia implied by the positive dependence on lag terms seen in inflation regressions. However, this paper shows that staggered contracting models actually imply that these lag coefficients should be negative. This appears to present a serious problem for matching the contracting approach with the data. For while there are ongoing debates about the magnitude and stability of the lagged dependent variable effects on inflation, there is no evidence in favor of the predictions derived here of a pattern of negative coefficients on these variables.

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A Log-Linearizations

This appendix derives the log-linearized equations used in the paper. In light of the results for the non-zero-inflation steady-state reported in Section 5, the approach taken here is to report the log-linearizations for this general case. The equations used for the zero-inflation steady-state can then be obtained as special cases.

A.1 Price Level Equation

The price level consistent with the Dixit-Stiglitz market structure is

$$P_t = \left(\sum_{i=0}^{n-1} s_k X_{t-k}^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

Dividing across by P_t , this becomes

$$\sum_{i=0}^{n-1} s_k \left(\frac{X_{t-k}}{P_t} \right)^{1-\theta} = 1$$

We log-linearize this equation around a steady-state path along which gross inflation, $\frac{P_t}{P_{t-1}}$, is constant and equal to Π . Along this path, all nominal variables such as the new contract price all grow at the same rate. Thus, the log-linearized price level equation is

$$\sum_{i=0}^{n-1} (1-\theta) s_k \left(\frac{X^*}{P^*} \right)^{1-\theta} \Pi^{(\theta-1)k} (x_{t-k} - p_t) = 0$$

This simplifies to give the equation used in the text

$$p_t = \frac{\sum_{i=0}^{n-1} s_k \Pi^{(\theta-1)k} x_{t-k}}{\sum_{i=0}^{n-1} s_k \Pi^{(\theta-1)k}} = \sum_{i=0}^{n-1} \tau_k x_{t-k}$$

A.2 Optimal Pricing Equation

The first-order condition determining the optimal contract price is

$$E_t \left(\sum_{k=0}^{n-1} \beta^k s_k Y_{t+k} P_{t+k}^{\theta-1} X_t \right) = \frac{\theta}{\theta-1} E_t \left(\sum_{k=0}^{n-1} \beta^k s_k Y_{t+k} P_{t+k}^{\theta-1} MC_{t+k} \right)$$

In a steady-state with a constant ratio of contract price to marginal cost, a constant output level of Y^* , and a positive gross inflation rate of Π , we have

$$Y^* P_t^{\theta-1} X_t^* \left(\sum_{k=0}^{n-1} (\beta \Pi^{\theta-1})^k s_k \right) = \frac{\theta}{\theta-1} Y^* P_t^{\theta-1} MC_t^* \left(\sum_{k=0}^{n-1} (\beta \Pi^\theta)^k s_k \right)$$

So, the steady-state markup over contract prices over marginal cost in this economy is

$$\frac{X_t^*}{MC_t^*} = \frac{\theta}{\theta - 1} \frac{\sum_{k=0}^{n-1} (\beta\Pi^\theta)^k s_k}{\sum_{k=0}^{n-1} (\beta\Pi^{\theta-1})^k s_k}$$

Now log-linearize both of sides of the optimality condition around this steady inflation path, taking into account that nominal marginal cost grows at the same rate as inflation along the steady-state path:

$$\begin{aligned} & Y^* P_t^{\theta-1} X_t^* E_t \left(\sum_{k=0}^{n-1} (\beta\Pi^{\theta-1})^k s_k \left(y_{t+k} + (\theta - 1) \sum_{r=1}^k \pi_{t+r} + x_t \right) \right) \\ &= Y^* P_t^{\theta-1} MC_t^* \left(\frac{\theta}{\theta - 1} \right) E_t \left(\sum_{k=0}^{n-1} (\beta\Pi^\theta)^k s_k \left(y_{t+k} + (\theta - 1) \sum_{r=1}^k \pi_{t+r} + mc_{t+k} \right) \right) \end{aligned}$$

Canceling terms, this is

$$\begin{aligned} & X_t^* E_t \left(\sum_{k=0}^{n-1} (\beta\Pi^{\theta-1})^k s_k \left(y_{t+k} + (\theta - 1) \sum_{r=1}^k \pi_{t+r} + x_t \right) \right) \\ &= MC_t^* \left(\frac{\theta}{\theta - 1} \right) E_t \left(\sum_{k=0}^{n-1} (\beta\Pi^\theta)^k s_k \left(y_{t+k} + (\theta - 1) \sum_{r=1}^k \pi_{t+r} + mc_{t+k} \right) \right) \end{aligned}$$

Using the expression for the ratio of the new contract price to marginal cost along the steady-state path, this becomes

$$\begin{aligned} & E_t \left(\sum_{k=0}^{n-1} (\beta\Pi^{\theta-1})^k s_k \left(y_{t+k} + (\theta - 1) \sum_{r=1}^k \pi_{t+r} + x_t \right) \right) \\ &= \left(\frac{\sum_{k=0}^{n-1} (\beta\Pi^{\theta-1})^k s_k}{\sum_{k=0}^{n-1} (\beta\Pi^\theta)^k s_k} \right) E_t \left(\sum_{k=0}^{n-1} (\beta\Pi^\theta)^k s_k \left(y_{t+k} + (\theta - 1) \sum_{r=1}^k \pi_{t+r} + mc_{t+k} \right) \right) \end{aligned}$$

This can be written more compactly if we define

$$\begin{aligned} \phi_k &= \frac{(\beta\Pi^{\theta-1})^k s_k}{\sum_{k=0}^{n-1} (\beta\Pi^{\theta-1})^k s_k} \\ \lambda_k &= \frac{(\beta\Pi^\theta)^k s_k}{\sum_{k=0}^{n-1} (\beta\Pi^\theta)^k s_k} \end{aligned}$$

Given these definitions, the optimal contract price can be re-written as

$$E_t \left(\sum_{k=0}^{n-1} \phi_k \left(y_{t+k} + (\theta - 1) \sum_{r=1}^k \pi_{t+r} + x_t \right) \right) = E_t \left(\sum_{k=0}^{n-1} \lambda_k \left(y_{t+k} + (\theta - 1) \sum_{r=1}^k \pi_{t+r} + mc_{t+k} \right) \right)$$

This solves to give

$$x_t = E_t \sum_{k=0}^{n-1} \left(\lambda_k mc_{t+k} + (\lambda_k - \phi_k) \left(y_{t+k} + (\theta - 1) \sum_{r=1}^k \pi_{t+r} \right) \right)$$

Again, the contract price depends on a weighted average of expected future nominal marginal costs. But, now there are some additional terms in y_{t+k} and π_{t+k} , each multiplied by $\lambda_k - \phi_k$. An examination of the definitions of λ_k and ϕ_k reveals that they are very similar—the only difference is that one is a set of weights with each term featuring $\Pi^{(\theta-1)k}$ while the other has terms featuring $\Pi^{\theta k}$ —and that they are identical when steady-state inflation is zero. So, it is not surprising that calculations with a wide range of parameter values show the $\lambda_k - \phi_k$ coefficients are negligible in size. This finding was also reported by Dotsey, King, and Wolman (1999). These calculations thus tell us that the equation

$$x_t = E_t \sum_{k=0}^{n-1} \lambda_k mc_{t+k}$$

provides a very good approximation to the optimal pricing rule.

B Money Growth Model

Here, we derive the analytical solution for the money growth model discussed in Section 7. In particular, the solution is derived for the more general case of the model in which, in addition to the money growth shock, there is also a stochastic shock to monetary velocity:

$$\begin{aligned} y_t &= m_t - p_t + v_t, \\ v_t &= \rho_v v_{t-1} + \epsilon_t^v, \\ \Delta m_t &= \rho_m \Delta m_{t-1} + \epsilon_t^m. \end{aligned}$$

The optimal contract price is

$$x_t = \frac{E_t \left[\sum_{k=0}^{n-1} \beta^k s_k (p_{t+k} + \gamma y_{t+k}) \right]}{\sum_{k=0}^{n-1} \beta^k s_k}$$

Substituting in the definition of output, this becomes

$$x_t = \frac{E_t \left[\sum_{k=0}^{n-1} \beta^k s_k ((1-\gamma)p_{t+k} + \gamma(m_{t+k} + v_{t+k})) \right]}{\sum_{k=0}^{n-1} \beta^k s_k}$$

This re-arranges to give

$$\begin{aligned} \left(\sum_{k=0}^{n-1} \beta^k s_k \right) x_t &= E_t \left[\sum_{k=0}^{n-1} \beta^k s_k ((1-\gamma)p_{t+k} + \gamma(m_{t+k} + v_{t+k})) \right] \\ &= (1-\gamma) E_t \left[\sum_{k=0}^{n-1} \beta^k s_k \sum_{r=0}^{n-1} s_r E_t x_{t+k-r} \right] + \gamma E_t \sum_{k=0}^{n-1} \beta^k s_k (m_{t+k} + v_{t+k}) \end{aligned}$$

Grouping terms together, this becomes

$$E_t \left[\left\{ \sigma(\beta F) - \frac{\left(\sum_{k=0}^{n-1} \beta^k s_k (1 - (1-\gamma)s_k) \right)}{1-\gamma} + \sigma(L) \right\} x_t \right] = \frac{-\gamma}{1-\gamma} (X_t^m + X_t^v)$$

where σ is defined in equation (18) and

$$\begin{aligned} X_t^m &= \sum_{k=0}^{n-1} \beta^k s_k E_t m_{t+k} \\ X_t^v &= \sum_{k=0}^{n-1} \beta^k s_k E_t v_{t+k} \end{aligned}$$

As with the other models considered here, a stable solution exists if there are $n-1$, and this is guaranteed as long as there are no roots between one and β^{-1} . Calculations with a wide range of realistic parameter values always produced a unique stable solution. As before, let $\lambda_1, \lambda_2, \dots, \lambda_{n-1}$ represent the $n-1$ roots inside the unit circle, the solution can be re-written as

$$E_t \left[s_0 s_{n-1} \beta^{n-1} \left\{ \prod_{i=1}^{n-1} \left(F - \lambda_i \right) \left(F - \frac{1}{\beta \lambda_i} \right) \right\} L^{n-1} x_t \right] = \frac{-\gamma}{1-\gamma} (X_t^m + X_t^v)$$

Solving the unstable roots forward, the solution takes the form

$$\left\{ \prod_{i=1}^{n-1} (1 - \lambda_i L) \right\} x_t = \frac{-\gamma}{(1-\gamma) s_0 s_{n-1}} E_t \left[\left(\prod_{i=1}^{n-1} (-\lambda_i) \right) \left(\sum_{k=0}^{\infty} \beta^k \lambda_1^k F^k \right) \dots \left(\sum_{k=0}^{\infty} \beta^k \lambda_{n-1}^k F^k \right) (X_t^m + X_t^v) \right]$$

Note that the roots of the contract process polynomial—and thus the lag coefficients in the contract price solution—are not affected by the parameters determining the two stochastic shocks in the model, but depend only on the pricing hazard and γ .

The AR(1) process for velocity implies

$$E_t v_{t+k} = \rho_v^k v_t$$

so all expected future values of velocity can be written in terms of the current value

$$X_t^v = \left[\sum_{k=0}^{n-1} (\beta \rho_v)^k s_k \right] v_t = \psi v_t$$

Similarly we have

$$\begin{aligned} E_t m_{t+k} &= m_t + E_t (\Delta m_{t+1} + \Delta m_{t+2} + \dots + \Delta m_{t+k}) \\ &= m_t + \left(\rho_m + \rho_m^2 + \dots + \rho_m^k \right) \Delta m_t \end{aligned}$$

Taken together, all of these results imply a solution for the optimal contract price of the form

$$x_t = \sum_{k=1}^{n-1} \tau_k x_{t-k} + \nu m_t + \varphi \Delta m_t + \psi v_t.$$

Finally, the coefficient on m_t is pinned down by the fact that the money supply is an $I(1)$ process, and that m_t and p_t are cointegrated by definition (because velocity is stationary). Thus, the long-run coefficient on m_t in the contract price equation must be one. This requires that $\nu = 1 - \sum_{k=1}^{n-1} \tau_k$. This implies the solution given as equation (68) in the text (with the addition of the velocity term):

$$x_t = \sum_{k=1}^{n-1} \tau_k x_{t-k} + \left(1 - \sum_{k=1}^{n-1} \tau_k \right) m_t + \varphi \Delta m_t + \psi v_t.$$

Note also that following the same steps as in Section 7.2, one arrives at a reduced-form inflation process of the form

$$\pi_t = \sum_{k=1}^{n-1} a_k \pi_{t-k} + \sum_{k=0}^n b_k y_{t-k} + \sum_{k=0}^n c_k v_{t-k}$$

where the reduced-form coefficients on inflation are the same whether the velocity shock is included in the model are not.

Table 1: First-Order Autocorrelations

	<i>United States</i>	<i>Euro Area</i>
Inflation	0.892	0.872
Output Gap	0.862	0.856
Unemployment Rate	0.975	0.998
Labor Share	0.912	0.993

Notes: Sample for US results in 1960:1-2003:2, for the Euro Area results is 1970:2-2002:4

Table 2: Reduced-Form Regressions for US GDP Price Inflation

	Driving Variables			
	None	Output Gap	Unemployment	Labor Share
<u>1960:1-2003:2</u>				
Estimated $\rho(1)$	0.940 (0.046)	0.938 (0.040)	1.033 (0.045)	0.927 (0.044)
Driving Variable p -value	<i>NA</i>	0.000	0.000	0.040
<u>1960:1-1983:4</u>				
Estimated $\rho(1)$	0.920 (0.051)	0.900 (0.046)	1.021 (0.049)	0.919 (0.048)
Driving Variable p -value	<i>NA</i>	0.000	0.000	0.049
<u>1984:1-2003:2</u>				
Estimated $\rho(1)$	0.819 (0.092)	0.781 (0.091)	0.941 (0.117)	0.704 (0.102)
Driving Variable p -value	<i>NA</i>	0.042	0.047	0.010
<u>1991:1-2003:2</u>				
Estimated $\rho(1)$	0.582 (0.165)	0.714 (0.171)	0.817 (0.242)	0.580 (0.159)
Driving Variable p -value	<i>NA</i>	0.014	0.516	0.501

Notes: These results relate to regressions of the form $\pi_t = \alpha + \rho(1)\pi_{t-1} + \sum_{k=1}^3 \psi_k \Delta\pi_{t-k} + \sum_{k=0}^3 \gamma_k y_{t-k} + \epsilon_t$, where y_t is the driving variable listed in the column headings. Figures in brackets are Newey-West standard errors.

Table 3: Reduced-Form Regressions for Euro Area GDP Price Inflation

	Driving Variables			
	None	Output Gap	Unemployment	Labor Share
<u>1970:2-2002:4</u>				
Estimated $\rho(1)$	0.960 (0.038)	0.976 (0.035)	0.884 (0.066)	0.891 (0.111)
Driving Variable p -value	<i>NA</i>	0.000	0.038	0.502
<u>1970:2-1983:4</u>				
Estimated $\rho(1)$	0.675 (0.156)	0.853 (0.123)	0.800 (0.147)	0.939 (0.261)
Driving Variable p -value	<i>NA</i>	0.000	0.333	0.174
<u>1984:1-2002:4</u>				
Estimated $\rho(1)$	0.832 (0.062)	0.877 (0.057)	0.832 (0.077)	0.486 (0.133)
Driving Variable p -value	<i>NA</i>	0.180	0.078	0.010
<u>1991:1-2002:4</u>				
Estimated $\rho(1)$	0.836 (0.131)	0.754 (0.129)	0.914 (0.198)	0.515 (0.270)
Driving Variable p -value	<i>NA</i>	0.067	0.130	0.012

Notes: These results relate to regressions of the form $\pi_t = \alpha + \rho(1)\pi_{t-1} + \sum_{k=1}^3 \psi_k \Delta\pi_{t-k} + \sum_{k=0}^3 \gamma_k y_{t-k} + \epsilon_t$, where y_t is the driving variable listed in the column headings. Figures in brackets are Newey-West standard errors.

Table 4: Lagged Inflation Coefficients in $\psi(L)$ Polynomial

	ψ_1	ψ_2	ψ_3	ψ_4
<hr/> <hr/>				
Taylor ($\beta = 0.99$)				
$n = 3$	-0.269			
$n = 4$	-0.430	-0.121		
$n = 5$	-0.533	-0.235	-0.068	
$n = 6$	-0.606	-0.328	-0.147	-0.044
<hr/>				
Truncated Calvo ($\beta = 0.99, \alpha = 0.25$)				
$n = 3$	-0.263			
$n = 4$	-0.415	-0.113		
$n = 5$	-0.510	-0.216	-0.061	
$n = 6$	-0.574	-0.296	-0.127	-0.036
<hr/>				
Truncated Calvo ($\beta = 0.99, \alpha = 0.5$)				
$n = 3$	-0.235			
$n = 4$	-0.355	-0.086		
$n = 5$	-0.419	-0.151	-0.037	
$n = 6$	-0.455	-0.192	-0.069	-0.017
<hr/>				
Upward-Sloping Hazards ($\beta = 0.99$)				
(0.1, 0.4, 1)	-0.232			
(0.1, 0.2, 0.7, 1)	-0.342	-0.058		
(0.1, 0.2, 0.3, 0.7, 1)	-0.453	-0.153	-0.026	
(0.1, 0.2, 0.3, 0.4, 0.8, 1)	-0.507	-0.213	-0.065	-0.009
<hr/> <hr/>				

Notes: Refers to coefficients in equation (35) for various different hazard functions. n is the maximum possible contract length in the Taylor or truncated Calvo models, β is the discount rate, α is the probability of price changes in truncated Calvo model before period n . All equations obtained by log-linearizing around zero-inflation steady state.

Table 5: Truncated Calvo ($\alpha = 0.25$) Lagged Inflation Coefficients for Various Π

	ψ_1	ψ_2	ψ_3	ψ_4
<hr/>				
$\Pi = 1$				
$n = 3$	-0.263			
$n = 4$	-0.415	-0.113		
$n = 5$	-0.510	-0.216	-0.061	
$n = 6$	-0.574	-0.296	-0.127	-0.036
<hr/>				
$\Pi = 1.01$				
$n = 3$	-0.265			
$n = 4$	-0.421	-0.116		
$n = 5$	-0.520	-0.224	-0.064	
$n = 6$	-0.588	-0.310	-0.135	-0.039
<hr/>				
$\Pi = 1.03$				
$n = 3$	-0.265			
$n = 4$	-0.424	-0.117		
$n = 5$	-0.526	-0.228	-0.065	
$n = 6$	-0.597	-0.319	-0.141	-0.041
<hr/>				
$\Pi = 1.05$				
$n = 3$	-0.261			
$n = 4$	-0.414	-0.113		
$n = 5$	-0.513	-0.217	-0.061	
$n = 6$	-0.580	-0.302	-0.130	-0.037
<hr/>				

Notes: Refers to coefficients in equation (35) for truncated Calvo model with $\beta = 0.99$ is the discount rate, and for log-linear approximations around various steady-state gross inflation rates $\frac{P_t}{P_{t-1}} = \Pi$.

Table 6: Reduced-Form Inflation Coefficients for Money Growth Model

	λ_1	λ_2	λ_3	$\sum_{k=1}^3 \lambda_k$
$\gamma = 0.1$				
$\rho_m = 0.50$	-0.48	-0.13	0.09	-0.52
$\rho_m = 0.66$	-0.45	-0.07	0.16	-0.36
$\rho_m = 0.80$	-0.40	0.02	0.27	-0.11
$\gamma = 0.2$				
$\rho_m = 0.50$	-0.50	-0.14	0.12	-0.52
$\rho_m = 0.66$	-0.47	-0.07	0.20	-0.33
$\rho_m = 0.80$	-0.41	0.03	0.33	-0.04
$\gamma = 0.3$				
$\rho_m = 0.50$	-0.52	-0.15	0.14	-0.53
$\rho_m = 0.66$	-0.48	-0.07	0.24	-0.31
$\rho_m = 0.80$	-0.42	0.03	0.38	-0.01
$\gamma = 0.5$				
$\rho_m = 0.50$	-0.55	-0.16	0.17	-0.54
$\rho_m = 0.66$	-0.50	-0.08	0.28	-0.30
$\rho_m = 0.80$	-0.44	0.04	0.44	-0.03
$\gamma = 1.0$				
$\rho_m = 0.50$	-0.59	-0.19	0.21	-0.56
$\rho_m = 0.66$	-0.55	-0.09	0.35	-0.29
$\rho_m = 0.80$	-0.49	0.02	0.54	-0.07
$\gamma = 3.0$				
$\rho_m = 0.50$	-0.70	-0.27	0.31	-0.65
$\rho_m = 0.66$	-0.65	-0.16	0.50	-0.32
$\rho_m = 0.80$	-0.60	-0.03	0.72	0.10

Notes: Refers to coefficients in equation (70) for various values of γ (elasticity of real marginal cost with respect to output) and ρ_m (autocorrelation of money growth).

Table 7: Granger Causality Tests for US GDP Price Inflation

	Driving Variables:		
	Output Gap	Unemployment	Labor Share
<u>1960:1-2003:2</u>			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	-0.021 (0.032)	0.031 (0.011)	0.024 (0.022)
p -value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.618	0.007	0.623
<u>1960:1-1983:4</u>			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	-0.018 (0.039)	0.038 (0.014)	0.011 (0.024)
p -value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.569	0.011	0.773
<u>1984:1-2003:2</u>			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	0.007 (0.063)	0.072 (0.026)	0.024 (0.076)
p -value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.952	0.039	0.958
<u>1991:1-2003:2</u>			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	-0.198 (0.108)	0.111 (0.043)	-0.043 (0.093)
p -value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.249	0.018	0.938

Notes: These results relate to regressions of the form $y_t = \alpha + \sum_{k=1}^4 \rho_k y_{t-k} + \sum_{k=1}^4 \beta_k \pi_{t-k} + \epsilon_t$, where y_t is the driving variable listed in the column headings and π_t is inflation. Figures in brackets are Newey-West standard errors

Figure 1

Why Taylor Contract Inflation is More Autocorrelated Than Output

