Topic 3: Determinants of TFP

The Solow model identified total factor productivity (TFP) as the key determinant of growth in the long run, but did not provide any explanation of what determines it. In the technical language used by macroeconomists, long-run growth in the Solow framework is determined by something that is *exogenous* to the model. In this handout, we will start by considering a particular model that makes TFP *endogeous*, meaning determined by the actions of the economic agents described in the model. We will then briefly describe a model of cross-country patterns for TFP. Finally, we will consider the role played by institutions in determining TFP.

Part I: The Romer Model

We will now describe a model of endogenous growth, due to Paul Romer ("Endogenous Technological Change," *Journal of Political Economy*, 1990) which starts by accepting the Solow model's result that technological progress is what determines long-run growth in output per worker. But, unlike the Solow model, Romer attemts to explain what determines technological progress.

TFP Growth as Invention of New Inputs

So what is this technology term A anyway? The Romer model takes a specific concrete view on this issue. Romer describes the aggregate production function as

$$Y = L_Y^{1-\alpha}\left(x_1^\alpha + x_2^\alpha + ... + x_A^\alpha\right) = L_Y^{1-\alpha}\sum_{i=1}^{A} x_i^\alpha \tag{1}$$

where L_Y is the number of workers producing output and the x_i's are different types of capital goods. The crucial feature of this production function is that diminishing marginal returns applies, not to capital as a whole, but separately to each of the individual capital goods (because $0 < \alpha < 1$).

If A was fixed, the pattern of diminishing returns to each of the separate capital goods would mean that growth would eventually taper off to zero. However, in the Romer model, A is not fixed. Instead, there are L_A workers engaged in R&D and this leads to the invention of new capital goods. This is described using a "production function" for the change in the number of capital goods:

$$\dot{A} = \gamma L_A^\lambda A^\phi \tag{2}$$

The change in the number of capital goods depends positively on the number of researchers (λ is an index of how slowly diminishing marginal productivity sets in for researchers) and also on the prevailing value of A itself. This latter effect stems from the "giants shoulders" effect.[1] For instance, the invention of a new piece of software will have relied on the previous invention of the relevant computer hardware, which itself relied on the previous invention of semiconductor chips, and so on. Note that Romer's original paper only examined the case $\lambda = 1, \phi = 1$, whereas we will not restrict λ and will generally consider the case $\phi < 1$.

As described in Romer's paper and in the Jones textbook, the model contains a full description of what determines the fraction of workers that work in the research section. The research sector gets rewarded with patents that allow it to maintain a monopoly in the product it invents; wages are equated across sectors, so the research sector hire workers up to point where their value to it is as high as it is to producers of final output. In keeping with the spirit of the Solow model, I'm going to just treat the share of workers in the research sector as an exogenous parameter (but will discuss later some of the factors that should determine this share). So, we have

$$L = L_A + L_Y \tag{3}$$

$$L_A = s_A L \tag{4}$$

And again we assume that the total number of workers grows at an exogenous rate n:

$$\frac{\dot{L}}{L} = n \tag{5}$$

One can define the aggregate capital stock as

$$K = \sum_{i=1}^{A} x_i \tag{6}$$

Again, I'll treat the savings rate as exogenous and assume

$$\dot{K} = s_K Y - \delta K \tag{7}$$

One observation that simplifies the analysis of the model is the fact that all of the capital goods play an identical role in the production process. For this reason, the demand for each is the same, implying that

$$x_i = \bar{x} \qquad i = 1, 2, ...A \tag{8}$$

[1] Stemming from Isaac Newton's observation "If I have seen farther than others, it is because I was standing on the shoulders of giants."

This means that the production function can be written as

$$Y = AL_Y^{1-\alpha}\bar{x}^{\alpha} \tag{9}$$

Note now that

$$K = A\bar{x} \Rightarrow \bar{x} = \frac{K}{A} \tag{10}$$

so output can be re-expressed as

$$Y = AL_Y^{1-\alpha}\left(\frac{K}{A}\right)^{\alpha} = (AL_Y)^{1-\alpha} K^{\alpha} \tag{11}$$

This looks just like the Solow model's production function. The TFP term is written as $A^{1-\alpha}$ as opposed to just A as it was in our first handout, but this makes no difference to the substance of the model.

Steady-State Growth in The Romer Model

One can use the same arguments as before to show that this economy converges to a steady-state growth path in which capital and output grow at the same rate. So, one can derive the steady-state growth rate as follows. Re-write the production function as

$$Y = (As_Y L)^{1-\alpha} K^{\alpha} \tag{12}$$

where

$$s_Y = 1 - s_A \tag{13}$$

Take logs and derivatives to get

$$\frac{\dot{Y}}{Y} = (1-\alpha)\left(\frac{\dot{A}}{A} + \frac{\dot{s_Y}}{s_Y} + \frac{\dot{L}}{L}\right) + \alpha\frac{\dot{K}}{K} \tag{14}$$

Now use the fact that the steady-state growth rates of capital and output are the same to derive that this steady-state growth rate is given by

$$\left(\frac{\dot{Y}}{Y}\right)^{*} = (1-\alpha)\left(\frac{\dot{A}}{A} + \frac{\dot{s_Y}}{s_Y} + \frac{\dot{L}}{L}\right) + \alpha\left(\frac{\dot{Y}}{Y}\right)^{*} \tag{15}$$

Finally, because the share of labour allocated to the non-research sector cannot be changing along the steady-state path (otherwise the fraction of researchers would eventually go to zero or become greater than one, which would not be feasible) we have

$$\left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}\right)^{*} = \frac{\dot{A}}{A} \tag{16}$$

The steady-state growth rate of output per worker equals the steady-state growth rate of A. The only difference from the Solow model is that writing the TFP term as $A^{1-\alpha}$ makes this growth rate $\frac{\dot{A}}{A}$ as opposed to $\frac{1}{1-\alpha}\frac{\dot{A}}{A}$.

The big difference relative to the Solow model is that the A term is determined within the model as opposed to evolving at some fixed rate unrelated to the actions of the agents in the model economy. To derive the steady-state growth rate in this model, note that the growth rate of the number of capital goods is

$$\frac{\dot{A}}{A} = \gamma \left(s_A L\right)^{\lambda} A^{\phi-1} \tag{17}$$

The steady-state of this economy features A growing at a constant rate. This can only be the case if the growth rate of the right-hand-side of (17) is zero. Taking logs and derivatives, this implies

$$\lambda \left(\frac{\dot{s}_A}{s_A} + \frac{\dot{L}}{L}\right) - (1-\phi)\frac{\dot{A}}{A} = 0 \tag{18}$$

Again, in steady-state, the growth rate of the fraction of researchers must be zero. So, along the model's steady-state growth path, the growth rate of the number of capital goods (and hence output per worker) is

$$\left(\frac{\dot{A}}{A}\right)^{*} = \frac{\lambda n}{1-\phi} \tag{19}$$

The long-run growth rate of output per worker in this model depends on positively on three factors:

- The growth rate of the number of workers n. The higher this, the faster the economy adds researchers. This may seem like a somewhat unusual prediction, but it is worth noting that both economic growth and population growth have accelerated over the past 200 years. There is also a well-known paper by Harvard's Michael Kremer, who surveys human history back to one million years BC and concludes that those civilisations with higher populations tended to have higher living standards.[2]

- The parameter λ, which describes the extent to which diminishing marginal productivity sets in as we add researchers.

[2]Michael Kremer, "Population Growth and Technological Change: One Million B.C. to 1990", *Quarterly Journal of Economics*, Vol. 108, No. 3. (Aug., 1993), pp. 681-716.

- The strength of the "standing on shoulders" effect, ϕ. The more past inventions help to boost the rate of current inventions, the faster the growth rate will be.

A Special Case

An important exception to these results is the original Romer model in which $\phi = 1$. In this case, the growth rate of the number of capital goods is

$$\frac{\dot{A}}{A} = \gamma L_A^{\lambda} \tag{20}$$

so there is no steady-state growth path. The growth rate increases over time in line with increases in the number of researchers. Charles Jones's textbook argues fairly convincingly that this is not a very plausible model. For instance, Jones points out that the number of researchers has grown considerably over the post-War period, so growth rates should have accelerated accordingly if the model was correct. But this has not happened.

The Steady-State Level of Output Per Worker

One can use the same arguments as in our first handout to decompose output per worker into a capital-output ratio component and a TFP component. In other words, one can re-arrange equation (11) to get

$$\frac{Y}{L_Y} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A \tag{21}$$

and use the fact that $L_Y = (1 - s_A)L$ to get

$$\frac{Y}{L} = (1 - s_A)\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A \tag{22}$$

Note that the s_A term reflects the reduction in the production of goods and services due to a fraction of the labour force being employed as researchers. One can also use the same arguments to show that, along the steady-state growth path the capital-output ratio is

$$\left(\frac{K}{Y}\right)^{*} = \frac{s_K}{n + \frac{\lambda n}{1-\phi} + \delta} \tag{23}$$

(The $\frac{\lambda n}{1-\phi}$ here takes the place of the $\frac{g}{1-\alpha}$ in the first handout's expression for the steady-state capital-output ratio because this is the new formula for the growth rate of output per worker). Finally, we can also figure out the level of A along the steady-state growth path as follows. Along the steady-state path, we have

$$\frac{\dot{A}}{A} = \gamma \left(s_A L\right)^{\lambda} A^{\phi-1} = \frac{\lambda n}{1-\phi} \tag{24}$$

This latter equality can be re-arranged as

$$A^{*} = \left(\frac{\gamma\left(1-\phi\right)}{\lambda n}\right)^{\frac{1}{1-\phi}} \left(s_A L\right)^{\frac{\lambda}{1-\phi}} \tag{25}$$

So, along the steady-state growth path, output per worker is

$$\left(\frac{Y}{L}\right)^{*} = \left(1-s_A\right)\left(\frac{s_K}{n+\frac{\lambda n}{1-\phi}+\delta}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\gamma\left(1-\phi\right)}{\lambda n}\right)^{\frac{1}{1-\phi}} \left(s_A L\right)^{\frac{\lambda}{1-\phi}} \tag{26}$$

Convergence Dynamics for A

We noted already that the arguments showing that the capital-output ratio tends to converge towards its steady-state are the same here as in the Solow model. What about the A term? How do we know, for instance, that A always reverts back eventually to the path given by equation (25)? To see that this is the case, let

$$g_A = \frac{\dot{A}}{A} = \gamma\left(s_A L\right)^{\lambda} A^{\phi-1} \tag{27}$$

Taking logs and derivatives of this we get

$$\frac{\dot{g}_A}{g_A} = \lambda\left(\frac{\dot{s}_A}{s_A}+n\right) - \left(1-\phi\right) g_A \tag{28}$$

One can use this equation to show that g_A will be falling whenever

$$g_A > \frac{\lambda n}{1-\phi} + \frac{\lambda}{1-\phi}\frac{\dot{s}_A}{s_A} \tag{29}$$

So, apart from periods when the share of researchers is changing, the growth rate of A will be declining whenever it is greater than its steady-state value of $\frac{\lambda n}{1-\phi}$. The same argument works in reverse when g_A is below its steady-state value. Thus, the growth rate of A displays convergent dynamics, always tending back towards its steady-state value. And equation (25) tells us exactly what the *level* of A has to be if the growth rate of A is at its steady-state value.

Optimal R&D?

We haven't discussed the various factors that may determine the share of the labour force allocated to the research sectors, s_A. However, in equation (26) we have diagnosed two separate offsetting effects that s_A has on output: A negative one caused by the fact the researchers don't actually produce output, and a positive one due to the positive effect of the share of researchers on the level of technology.

It is a relatively simple calculus problem to figure out the level of s_A that maximises the level of output per worker along the steady-state growth path. In other words, one can can differentiate equation (26) with respect to s_A, set equal to zero, and solve to obtain that this optimizing share of researchers is

$$s_A^{**} = \frac{\frac{\lambda}{1-\phi}}{1 + \frac{\lambda}{1-\phi}} = \frac{\lambda}{1 - \phi + \lambda} \tag{30}$$

When one fills in the model to determine s_A endogenously, does the economy generally arrive at this optimal level? No. The reason for this is that research activity generates *externalities* that affect the level of output per worker, but which are not taken into account by private individuals or firms when they make the choice of whether or not to conduct research. Looking at the "ideas" production function, equation (2), one can see both positive and negative externalities:

- A positive externality due to the "giants shoulders" effect. Researchers don't take into account the effect their inventions have in boosting the future productivity of other researchers. The higher is θ, the more likely it is that the R&D share will be too low.
- A negative externality due to the fact that $\lambda < 1$, so diminishing marginal productivity applies to the number of researchers.

Whether there is too little or too much research in the economy relative to the optimal level depends on the strength of these various externalities. However, using empirical estimates of the parameters of equation (2), Charles Jones and John Williams have calculated that it is far more likely that the private sector will do too little research relative to the social optimum.[3]

To give some insight into this result, note that the steady-state growth rate in this model is $\frac{\lambda n}{1-\phi}$, so $\frac{\lambda}{1-\phi}$ is the ratio of the growth rate of output per worker to the growth rate of population. Suppose this equals one, so growth in output per worker equals growth in population—a not unreasonable assumption. In this case $\frac{\lambda}{1-\phi} = 1$ and the optimal

[3] Charles I. Jones and John C. Williams, "Too Much of a Good Thing? The Economics of Investment in R&D", *Journal of Economic Growth*, March 2000, Vol. 5, No. 1, pp. 65-85.

share of researchers is one-half. Indeed, for any reasonable steady-state growth rate, the optimal share of researchers is very high, so it is hardly surprising that the economy does not automatically generate this share.

This result points to the potential for policy interventions to boost the rate of economic growth by raising the number of researchers. For instance, laws to strength patent protection may raise the incentives to conduct R&D. This points to a potential conflict between macroeconomic policies aimed at raising growth and microeconomic policies aimed at reducing the inefficiencies due to monopoly power: Some amount of monopoly power for patent-holders may be necessary if we want to induce a high level of R&D and thus a high level of output.

Part II: Cross-Country Technology Diffusion

The Romer model should not be thought of as a model of growth in any one particular country. No country uses only technologies that were invented in that country; rather, products invented in one country end up being used all around the world. Thus, the model is best thought of as a very long-run model of the world economy. How then should long-run growth rates be determined for individual countries? By itself, the Romer model has no clear answer, but it suggests a model in which ability to learn about the usage of new technologies should plays a key role in determining output per worker.

We will now describe such a model. A model like this is described in the Bernard and Jones paper on the reading list. The mathematics of model are also formally equivalent to a well-known model of Nelson and Phelps (*AER*, 1966), though the application there is different, their subject being the diffusion of technological knowledge over time within an individual country. Our version of the model comes from Chapter 18 of Daron Acemoglu's textbook.

We will assume that there is a lead country that has technology level, A, that grows at rate g, so

$$\frac{\dot{A}_t}{A_t} = g \tag{31}$$

All other countries in the world, indexed by j, have technology levels given by $A_j < A$. These technology levels change according to

$$\dot{A}_{jt} = \sigma_j (A_t - A_{jt}) + \lambda_j A_{jt} \tag{32}$$

where $\lambda_j < g$. The parameter σ_j can be interpreted as a measure of how good country j is at learning about the technologies being applied in the advanced countries. Note that while technically this is a model with endogenous technology for country i, it does not specify the exact factors that determine σ_j. We impose the condition $\lambda_j < g$ so that the country j cannot only grow faster than the frontier through the learning that comes from having lower technology than the frontier.

Dynamics of Technology

Equation is known as a first-order differential equation. It can be solved to illustrate how A_j changes over time. To do this, we will first draw some terms together to re-write it as

$$\dot{A}_{jt} + (\sigma_j - \lambda_j) A_{jt} = \sigma_j A_t \tag{33}$$

We can derive a solution using the following steps. First note that the frontier technology can be written as

$$A_t = A_0 e^{gt} \tag{34}$$

This is because

$$\frac{de^{gt}}{dt} = ge^{gt} \tag{35}$$

So our differential equation can be re-written as

$$\dot{A}_{jt} + (\sigma_j - \lambda_j) A_{jt} = \sigma_j A_0 e^{gt} \tag{36}$$

Looked at this way, we can see that one possible solution for an A_{jt} process that will satisfy this equation is something of the form De^{gt} where D is some unknown coefficient. Let's figure out what D must be. It must satisfy

$$gDe^{gt} + (\sigma_j - \lambda_j) De^{gt} = \sigma_j A_0 e^{gt} \tag{37}$$

Canceling the e^{gt} terms, we see that

$$D = \frac{\sigma_j A_0}{\sigma_j + g - \lambda_j} \tag{38}$$

So, this solution takes the form

$$A^p_{jt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j} \right) A_0 e^{gt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j} \right) A_t \tag{39}$$

In addition, however, we would also have a solution if we added on any series a_t^g that satisfied

$$\dot{A}_{jt}^g + (\sigma_j - \lambda_j) A_{jt}^g = 0 \tag{40}$$

Again using the properties of the exponential function, this equation is satisfied by anything of the form

$$A_{jt}^g = A_0^g e^{-(\sigma_j - \lambda_j)t} \tag{41}$$

where A_0^g is some arbitrary parameter. So, the full solution for technology in country j is

$$A_{jt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) A_t + A_0^g e^{-(\sigma_j - \lambda_j)t} \tag{42}$$

Expressed as a ratio relative to the technolgical frontier this can be written as

$$\frac{A_{jt}}{A_0} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) + \frac{A_0^g}{A_0} e^{-(\sigma_j + g - \lambda_j)t} \tag{43}$$

Because $\lambda_j < g$, the combined parameter $\sigma_j + g - \lambda_j > 0$. This means that the second term in (43) tends towards zero. So, over time, as this term disappears, the country converges to a steady-state growth rate in which its technology level is a constant ratio $\frac{\sigma_j}{\sigma_j + g - \lambda_j}$ of the frontier level. Note that this ratio depends positively on the country's learning parameter σ_j and also on λ_j, which represents the amount of growth in technology it can generate independent of the learning or catch-up mechanism.

Discussion

This solution illustrates an important point. The follower countries never actually catch up in this steady-state: The term $\frac{\sigma_j}{\sigma_j + g - \lambda_j}$ is less than one, so that (in the steady-state) the follower countries always have levels of technology that are below that of the leader. This makes sense if you think about it: The follower countries can only experience growth in technology if there is a gap between their level of technology and the leader. So, to have a steady-state in which everyone's technology is growing, the followers must all have technology levels below that of the leader.

The model shows that for most countries, it is not their ability to invent new capital goods that is key to growth, but rather their ability to learn from those countries that are more technologically advanced. In theory, it may also be able to account for the sort of "growth miracles" that are occasionally observed: If a country can increase its value of σ_j via education or science-related policies, its position in the steady-state distribution of income may move upwards substantially, with the economy then going through a phase of rapid growth.

Part III: Institutions and Efficiency

What determines these huge differences in total factor productivity across countries? One answer is provided by the Romer model. According to this model, these differences must be due to variations in the extent to which countries have adopted the latest technologies. However, though interesting, the Romer model is perhaps too mechanistic in its view of what generates cross-country differences in TFP. While technology adoption is almost certainly a factor in differences in TFP, this still leaves open the question of what drives the pace of technology adoption in poorer countries.

Also, TFP is really just a measure of the efficiency with which an economy makes use of its resources and there are a whole range of other factors that can affect this. For example:

- Crime: Time spent on crime does not produce output. Neither do resources devoted to protecting inviduals and firms from crime.
- Bureaucratic Inefficiency and Corruption: Satisfaction of bureaucratic requirements and bribing of officials can be important diversions of resources in poor economies.
- Restrictions on Market Mechanisms: Protectionism, price controls, and central planning can all lead to resources being allocated in an inefficient manner.

There are now quite a large number of papers that stress that understanding differences in *institutions* provides the key to understanding TFP differences across countries. Hall and Jones coined the term *social infrastructure* to describe the social institutions that affect incentives to produce and invest. Using a simple empirical proxy for social infrastructure, they concluded that these institutions are crucial determinants of TFP. In Chapter 7 his book, Jones also describes how social infrastructure is a key determinant of the accumulation of factors.

One of the problems faced in assessing the linkage between TFP and institutions is *endogeneity*. Do countries get rich because they have good institutions or do countries have good institutions because they are rich? The latter linkage certainly exists. Rich European countries have substantial incentives to keep good institutions that promote productive efficiency because they would have lot to lose if their markets ceased to work well; these incentives may be substantial weaker in the world's poorer countries.

This simultaneity presents an econometric problem to anyone wishing to estimate the effects of institutions on output per worker. This problem can be addressed by obtaining good *instruments*: Truly exogenous variables that turn out to be correlated with measures of institutions. Hall and Jones uses variables like distance from the equator. Acemoglu, Johnson, and Robinson (*AER*, 2001) use information on settler mortality in different European colonies. They argue that countries where mortality for initial settlers was low were places where Europeans were more likely to settle and set up good institutions, with the reverse working when settler mortality was high. With this variable as an instrument, they find a very strong effect of certain measures of institutions on output per worker.

These findings indicate the limits of the type of mechanical growth theory that we have studied. Countries can certainly get richer by devoting more resources to factor accumulation and by combining their factors in a more efficient manner. But what can be done if a country's institutions are curbing investment and leading to an inefficient allocation of resources? Growth theory has little to say about how institutions affect accumulation or productive efficiency. That said, the findings are part of a growing field that emphasises the importance of understanding how institutions affect economic outcomes, and this is likely to be an interesting field of research in the future.