Simple Analytics of the Debt-GDP Ratio

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February 23, 2012

The stock of government debt evolves each period through the fiscal deficit, DEF, adding to the existing stock of debt, D_t, as follows

$$D_t = D_{t-1} + DEF_{t-1}$$

Let's assume that the deficit equals d percent of nominal GDP, Y_t

$$DEF_t = dY_t$$

and that nominal GDP grows at a constant rate g:

$$Y_t = (1+g)Y_{t-1}$$

The debt-GDP ratio evolves according to

$$\begin{aligned} \frac{D_t}{Y_t} &= \frac{D_{t-1}}{Y_t} + d\frac{Y_{t-1}}{Y_t} \\ &= \frac{D_{t-1}}{Y_{t-1}}\frac{Y_{t-1}}{Y_t} + \frac{d}{1+g} \\ &= \left(\frac{1}{1+g}\right)\frac{D_{t-1}}{Y_{t-1}} + \frac{d}{1+g} \end{aligned}$$

This is a stable first-order difference equation that converges to a unique stable point $\left(\frac{D_t}{Y_t}\right)^*$ given by

$$\left(\frac{D_t}{Y_t}\right)^* = \frac{\frac{d}{1+g}}{1-\frac{1}{1+g}} = \frac{d}{g}$$

Consider some example figures, suppose nominal GDP grows at 4 percent ($g = 0.04$) and the average deficit was 1 percent of GDP ($d = 0.01$), then the stable debt-GDP ratio would be 0.25.