The Calvo Model of Price Rigidity

The form of price rigidity faced by the Calvo firm is as follows. Each period, only a random fraction $(1-\alpha)$ of firms are able to reset their price; all other firms keep their prices unchanged. When firms do get to reset their price, they must take into account that the price may be fixed for many periods.

Firms operate in an imperfectly competitive market, so that if there were no frictions they would set prices as a fixed markup over marginal cost. In most modern applications, the market structure of the economy follows that set out in a famous 1976 paper by Avinash Dixit and Joseph Stiglitz. The basic model has no capital, so all goods are consumption goods, and consumers seek to maximize a utility function over a continuum of differentiated goods given by

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}} \tag{1}$$

This leads to demand functions for the differentiated goods of the form

$$Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta} \tag{2}$$

where P_t is the aggregate price index defined by

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}} \tag{3}$$

The Calvo assumption about price stickiness implies that this price level can be re-written as

$$P_t = \left[(1-\alpha) X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{4}$$

which can be re-written as

$$P_t^{1-\theta} = (1-\alpha) X_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \tag{5}$$

We now know that to apply the solution techniques for rational expectations models, these type of equations need to be log-linearized. This is done as follows. If we log-linearize this equation around a zero-inflation steady-state such that

$$X_t^* = P_t^* = P_{t-1}^* = P^* \tag{6}$$

then we have

$$(P^*)^{1-\theta} (1 + (1-\theta) p_t) = (1-\alpha) (P^*)^{1-\theta} (1 + (1-\theta) x_t) + \alpha (P^*)^{1-\theta} (1 + (1-\theta) p_{t-1}) \tag{7}$$

which simplifies to

$$p_t = (1-\alpha)\, x_t + \alpha p_{t-1} \tag{8}$$

Optimal Pricing in the Calvo Model

How do firms that get to change their price at time t set this price? They model assumes that they select this optimal reset price to maximize the expected present discounted values of real profits over the course of this price contract, where β is the discount rate. This present value is given by

$$E_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k \left(Y_{t+k} P_{t+k}^{\theta-1} X_t^{1-\theta} - P_{t+k}^{-1} C\left(Y_{t+k} P_{t+k}^{\theta} X_t^{-\theta} \right) \right) \right] \tag{9}$$

where $C(.)$ is the nominal cost function. Note that α appears in the discount rate because α^k is the probability that this particular price contract is still in existence in k periods time. Differentiating this equation with respect to X_t we get the following first-order condition:

$$E_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k \left((1-\theta)\, Y_{t+k} P_{t+k}^{\theta-1} X_t^{-\theta} + \theta M C_{t+k} Y_{t+k} P_{t+k}^{\theta-1} X_t^{-\theta-1} \right) \right] = 0 \tag{10}$$

This can be solved to give the following solution for the optimal price

$$X_t = \frac{\theta}{\theta-1} \frac{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} M C_{t+k} \right)}{E_t \left(\sum_{k=0}^{\infty} (\alpha\beta)^k Y_{t+k} P_{t+k}^{\theta-1} \right)}. \tag{11}$$

This equation looks very complicated but is quite intuitive if it is inspected closely. Without pricing frictions, the optimal rule for an imperfectly competitive firm with elasticity of demand θ is to set prices as a markup over marginal cost, such that price equals $\frac{\theta}{\theta-1}$ times marginal cost. In this case, because the price is likely to be fixed for some period of time, this equation states that the optimal price is a markup over a weighted average of future marginal costs. The weight for each future marginal cost has two elements to it:

- The term $(\alpha\beta)^k$ which puts less weight on future marginal costs because of discounting and because the price being set now has lower probabilities of still being around in k periods time as k gets bigger.
- The term $Y_{t+k} P_{t+k}^{\theta-1}$ which represents aggregate factors affecting firm demand in the future. As Y_{t+k} goes up the firm will sell more; as the aggregate price level P_{t+k} goes up, the firm's relative price goes down and its demand increases. This factor will

probably somewhat offset the discounting term—if the firm is going to sell a lot more in k period's time than now, then it may put a bit more weight on marginal cost in that period.

Log-Linearizing the Optimal Pricing Rule

To get the optimal pricing equation into a linear form that we can use in our computer programs, we log-linearize the first-order condition (10) around a zero inflation steady-state with constant output, the first term inside the big curly bracket becomes

$$(1-\theta) Y_{t+k} P_{t+k}^{\theta-1} X_t^{-\theta} \approx (1-\theta) Y^* (P^*)^{\theta-1} (X^*)^{-\theta} (1 + y_{t+k} + (\theta-1) p_{t+k} - \theta x_t) \quad (12)$$

while the second term becomes

$$\theta MC_{t+k} Y_{t+k} P_{t+k}^{\theta-1} X_t^{-\theta-1} \approx \theta MC^* Y^* (P^*)^{\theta-1} (X^*)^{-\theta-1} (1 + mc_{t+k} + y_{t+k} + (\theta-1) p_{t+k} - (1+\theta) x_t) \quad (13)$$

One can then use the fact that in steady-state

$$X^* = \left(\frac{\theta}{\theta-1}\right) MC^* \quad (14)$$

to then simplify this to

$$E_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k (x_t - mc_{t+k}) \right] = 0 \quad (15)$$

which simplifies further to

$$x_t = (1-\alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^k E_t mc_{t+k} \quad (16)$$

The terms in aggregate output and the price level drop out and the equation says that the log-price is a weighted average of expected future logs of marginal cost.

The New-Keynesian Phillips Curve

So, the dynamics of pricing in the Calvo model can be summarized by the two equations

$$p_t = (1-\alpha) x_t + \alpha p_{t-1} \quad (17)$$

$$x_t = (1-\alpha\beta) \sum_{k=0}^{\infty} (\alpha\beta)^k E_t mc_{t+k} \quad (18)$$

The first equation can be written in the form required by solution algorithms such as Binder-Pesaran. The second equation, which involves an infinite sum, can not. However, it describes the standard solution to a first-order stochastic difference equation, and thus it can be "reverse engineered" to be written in that format as

$$x_t = (1 - \alpha\beta)\, mc_t + (\alpha\beta)\, E_t x_{t+1} \tag{19}$$

Equations (17) and (19) can be put on the computer and combined with a model of marginal cost, this can be solved to describe prices and inflation in this model.

It turns, however, that there is also a neat *analytical* solution that sheds useful insight on how inflation behaves in this model. This is derived as follows. First, note that equation (17) can be re-arranged to express the reset price as a function of the current and past aggregate price levels

$$x_t = \frac{1}{1-\alpha}\left(p_t - \alpha p_{t-1}\right) \tag{20}$$

Substituting this into equation (19) we get

$$\frac{1}{1-\alpha}\left(p_t - \alpha p_{t-1}\right) = (1 - \alpha\beta)\, mc_t + \frac{\alpha\beta}{1-\alpha}\left(E_t p_{t+1} - \alpha p_t\right) \tag{21}$$

Multiplying across by $1 - \alpha$ and collecting terms, this becomes

$$p_t \left(1 + \alpha^2 \beta\right) - \alpha p_{t-1} = \alpha\beta E_t p_{t+1} + (1-\alpha)(1-\alpha\beta)\, mc_t \tag{22}$$

We can divide both sides by α and then subtract βp_t from both sides to get

$$\left(\frac{1 - \beta\alpha + \alpha^2\beta}{\alpha}\right) p_t - p_{t-1} = \beta E_t \pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} mc_t \tag{23}$$

where

$$\pi_t = p_t - p_{t-1} \tag{24}$$

is the inflation rate. Now we can use the fact that

$$\frac{1 - \beta\alpha + \alpha^2\beta}{\alpha} = 1 + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \tag{25}$$

To obtain

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(mc_t - p_t\right) \tag{26}$$

Or, defining real marginal cost as

$$mc_t^r = mc_t - p_t \tag{27}$$

the model implies

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} mc_t^r \tag{28}$$

In other words, inflation is a function of expected inflation and real marginal cost. This equation is known as the New-Keynesian Phillips Curve (NKPC).