# MA Macroeconomics
# 12. Endogenous Technological Change: The Romer Model

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# A Model of Technological Change

- The Solow model identified technological progress or improvements in total factor productivity (TFP) as the key determinant of growth in the long run, but did not provide any explanation of what determines it.
- In the technical language used by macroeconomists, long-run growth in the Solow framework is determined by something that is *exogenous* to the model.
- Now we will consider a particular model that makes technological progress *endogeous*, meaning determined by the actions of the economic agents described in the model.
- The model, due to Paul Romer ("Endogenous Technological Change," *Journal of Political Economy*, 1990) starts by accepting the Solow model's result that technological progress is what determines long-run growth in output per worker.
- But, unlike the Solow model, Romer attempts to explain what determines technological progress.

# Multiple Types of Capital Good

- So what is this technology term $A$ anyway?
- The Romer model takes a specific concrete view on this issue. Romer describes the aggregate production function as

$$Y = L_Y^{1-\alpha} \left( x_1^\alpha + x_2^\alpha + .... + x_A^\alpha \right) = L_Y^{1-\alpha} \sum_{i=1}^{A} x_i^\alpha$$

  where $L_Y$ is the number of workers producing output and the $x_i$'s are different types of capital goods.
- The crucial feature of this production function is that diminishing marginal returns applies, not to capital as a whole, but separately to each of the individual capital goods (because $0 < \alpha < 1$).
- In this model, $A$ is the number of different types of capital inputs.
- If $A$ was fixed, the pattern of diminishing returns to each of the separate capital goods would mean that growth would eventually taper off to zero. However, in the Romer model, $A$ is not fixed.

# TFP Growth as Invention of New Inputs

- There are $L_A$ workers engaged in R&D and this leads to the invention of new capital goods.
- This is described using a "production function" for the change in the number of capital goods:

$$\dot{A} = \gamma L_A^{\lambda} A^{\phi}$$

- The change in the number of capital goods depends positively on the number of researchers ($\lambda$ is an index of how slowly diminishing marginal productivity sets in for researchers) and also on the prevailing value of $A$ itself.
- The positive effect of the level of $A$ stems from a "giants shoulders" effect. For instance, the invention of a new piece of software will have relied on the previous invention of the relevant computer hardware, which itself relied on the previous invention of semiconductor chips, and so on.

# Allocation of Labour

- The total amount of labour is split between producing output and working in the research sector. We assume that a fraction $s_A$ of the labour force works in the research sector.

$$\begin{aligned} L &= L_A + L_Y \\ L_A &= s_A L \end{aligned}$$

- We will take $s_A$ as given (Romer's full model provides an explanation of the factors that determine this share).
- And again we assume that the total number of workers grows at an exogenous rate $n$:

$$\frac{\dot{L}}{L} = n$$

# Simplifying the Aggregate Production Function

- Define the aggregate capital stock as

$$K = \sum_{i=1}^{A} x_i$$

- Again, we'll treat the savings rate as exogenous and assume

$$\dot{K} = s_K Y - \delta K$$

- All of the capital goods play an identical role in the production process, so demand from producers for each of these capital goods is the same. So for all $i$, we have $x_i = \bar{x}$

$$K = A\bar{x} \Rightarrow \bar{x} = \frac{K}{A}$$

- This means that the production function can be written as

$$Y = AL_Y^{1-\alpha}\bar{x}^{\alpha} = Y = AL_Y^{1-\alpha}\left(\frac{K}{A}\right)^{\alpha} = (AL_Y)^{1-\alpha} K^{\alpha}$$

# Similarity With Solow Model

- The production function just derived:

$$Y = (AL_Y)^{1-\alpha} K^{\alpha}$$

looks very similar to the Solow model's production function. The only difference is that the TFP term is written as $A^{1-\alpha}$ as opposed to just $A$.

- However, this makes little difference to the substance of the model. We can still write output per number of output-sector workers in terms of technology and the capital-output ratio, only the technology term is $A$ as opposed to $A^{\frac{1}{1-\alpha}}$.

$$\frac{Y}{L_Y} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A$$

- And the arguments about the behaviour of the capital-output ratio are just the same, so it converges to

$$\left(\frac{K}{Y}\right)^{*} = \frac{s_K}{n+g+\delta}$$

Here $g$ takes place of $\frac{g}{1-\alpha}$ in the formula for the equilibrium capital-output ratio because the TFP term grows at rate $(1-\alpha)\,g$ instead of $g$.

# Steady-State Growth in The Romer Model

- Re-write the production function as

$$Y = (A s_Y L)^{1-\alpha} K^{\alpha}$$

  where $s_Y = 1 - s_A$

- Usual method for calculating growth rates give us

$$\frac{\dot{Y}}{Y} = (1-\alpha)\left(\frac{\dot{A}}{A} + \frac{\dot{s_Y}}{s_Y} + \frac{\dot{L}}{L}\right) + \alpha \frac{\dot{K}}{K}$$

- Now use the fact that the steady-state growth rates of capital and output are the same:

$$\left(\frac{\dot{Y}}{Y}\right)^* = (1-\alpha)\left(\frac{\dot{A}}{A} + \frac{\dot{s_Y}}{s_Y} + \frac{\dot{L}}{L}\right) + \alpha \left(\frac{\dot{Y}}{Y}\right)^*$$

- Because the share of labour allocated to the non-research sector cannot be changing along the steady-state path, we have

$$\left(\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}\right)^* = \frac{\dot{A}}{A}$$

# How Fast is Steady-State Growth?

- The big difference relative to the Solow model: $A$ is determined within the model as opposed to evolving at some exogenous fixed rate.
- Recall that
$$\dot{A} = \gamma L_A^{\lambda} A^{\phi}$$
- Growth rate of $A$ is
$$\frac{\dot{A}}{A} = \gamma \left(s_A L\right)^{\lambda} A^{\phi - 1}$$
- Steady-state of this economy features $A$ growing at a constant rate. So the growth rate of the growth rate is zero, right?
- Use our usual method to calculate growth rate of right-hand-side of previous equation.
$$\lambda \left( \frac{\dot{s}_A}{s_A} + \frac{\dot{L}}{L} \right) - (1 - \phi) \frac{\dot{A}}{A} = 0$$
- In steady-state, growth rate of the fraction of researchers $\left(\frac{\dot{s}_A}{s_A}\right)$ must be zero.
$$\left(\frac{\dot{A}}{A}\right)^{*} = \frac{\lambda n}{1 - \phi}$$

# Factors Determining Steady-State Growth Rate

- The long-run growth rate of output per worker in this model depends on positively on three factors:
  1. The parameter $\lambda$, which describes the extent to which diminishing marginal productivity sets in as we add researchers.
  2. The strength of the "standing on shoulders" effect, $\phi$. The more past inventions help to boost the rate of current inventions, the faster the growth rate will be.
  3. The growth rate of the number of workers $n$. The higher this, the faster the economy adds researchers. This may seem like a somewhat unusual prediction, but it holds well if one takes a very long view of world economic history. Prior to the industrial revolution, growth rates of population and GDP per capita were very low. The past 200 years have seen both population growth and economic growth rates increases.

# World Economic History

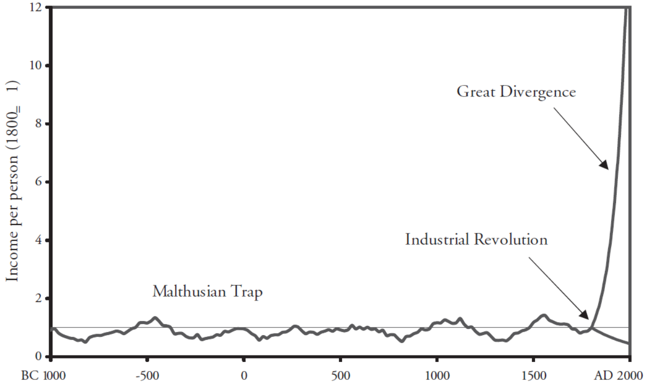
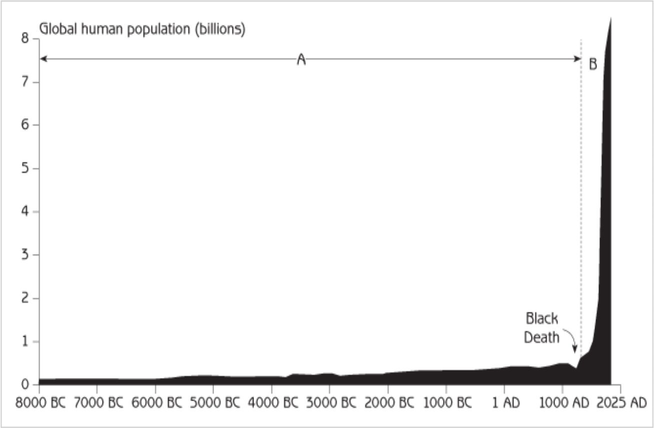



**Figure 1.1** World economic history in one picture. Incomes rose sharply in many countries after 1800 but declined in others.

# The History of Global Population



# What Is $A$ Along Steady-State Path?

- We can figure out more than just how fast $A$ grows along the steady-state path.
- Along the steady-state path, we have

$$\frac{\dot{A}}{A} = \gamma \left( s_A L \right)^{\lambda} A^{\phi - 1} = \frac{\lambda n}{1 - \phi}$$

- So, the steady-state level $A^*$ is determined by

$$A^{\phi - 1} = \frac{\lambda n}{1 - \phi} \left( \gamma \left( s_A L \right)^{\lambda} \right)^{-1}$$

- This last equation can be re-arranged as

$$A^* = \left( \frac{\gamma \left( 1 - \phi \right)}{\lambda n} \right)^{\frac{1}{1 - \phi}} \left( s_A L \right)^{\frac{\lambda}{1 - \phi}}$$

# Convergence Dynamics for $A$

- We can also show that $A$ always reverts back to its steady-state path. To see that this is the case, let

$$g_A = \frac{\dot{A}}{A} = \gamma \left(s_A L\right)^{\lambda} A^{\phi - 1}$$

- Calculate the growth rate of $g_A$ as follows

$$\frac{\dot{g}_A}{g_A} = \lambda \left( \frac{\dot{s}_A}{s_A} + n \right) - (1 - \phi) g_A$$

- One can use this equation to show that $g_A$ will be falling whenever

$$g_A > \frac{\lambda n}{1 - \phi} + \frac{\lambda}{1 - \phi} \frac{\dot{s}_A}{s_A}$$

- Apart from periods when the share of researchers is changing, the growth rate of $A$ will be declining whenever it is greater than its steady-state value of $\frac{\lambda n}{1-\phi}$ and increasing when it is below this rate.

# The Steady-State Level of Output Per Worker

- For the same reasons as before, we have

$$\frac{Y}{L_Y} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A$$

- Use the fact that $L_Y = (1 - s_A)L$ to get

$$\frac{Y}{L} = (1 - s_A)\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} A$$

- And now that we know the determinants of steady-state growth rate, $g$, we can substitute that into the formula for the steady-state capital-output ratio:

$$\left(\frac{K}{Y}\right)^{*} = \frac{s_K}{n + \frac{\lambda n}{1-\phi} + \delta}$$

# The Most Complicated Equation in the Course!

- Combine this with the formula for the steady-state level of $A$ and we get

$$A^* = \left(\frac{\gamma(1-\phi)}{\lambda n}\right)^{\frac{1}{1-\phi}} (s_A L)^{\frac{\lambda}{1-\phi}}$$

- And we get this very complicated-looking expression for output per worker on the steady-state path:

$$\left(\frac{Y}{L}\right)^* = (1-s_A)\left(\frac{s_K}{n+\frac{\lambda n}{1-\phi}+\delta}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\gamma(1-\phi)}{\lambda n}\right)^{\frac{1}{1-\phi}} (s_A L)^{\frac{\lambda}{1-\phi}}$$

# Optimal R&D?

- We haven't discussed what determines $s_A$, the share of the labour force allocated to the research sectors.
- Increasing $s_A$ has two separate offsetting effects that $s_A$ has on output: A negative one caused by the fact the researchers don't actually produce output, and a positive one due to the positive effect of the share of researchers on the level of technology.
- What would be the right level of $s_A$ to maximize output per worker?
- The really complicated equation for output per worker can be re-written as

$$\left(\frac{Y}{L}\right)^* = X\left(1 - s_A\right)\left(s_A\right)^Z$$

- Differentiate with respect to $s_A$, set equal to zero, and solve to obtain the optimizing share of researchers

$$s_A^{**} = \frac{Z}{1+Z} = \frac{\frac{\lambda}{1-\phi}}{1+\frac{\lambda}{1-\phi}} = \frac{\lambda}{1-\phi+\lambda}$$

# Optimal R&D?

- Filling in the model to determine $s_A$ endogenously, does the economy generally arrive at this optimal level? No.
- Research activity generates *externalities* that affect the level of output per worker, but which are not taken into account by private individuals or firms when they make the choice of whether or not to conduct research.
  - A positive externality due to the "giants shoulders" effect. Researchers don't take into account the effect their inventions have in boosting the future productivity of other researchers.
  - A negative externality due to the fact that $\lambda < 1$, so diminishing marginal productivity applies to the number of researchers.
- Whether there is too little or too much research in the economy relative to the optimal level depends on the strength of these various externalities.
- Let $\frac{\lambda}{1-\phi} = 1$ (so growth in output per worker equals growth in population). In this case, the optimal share of researchers is one-half.
- Policy interventions to boost the rate of economic growth by raising the number of researchers, e.g. strengthening patent protection.

# Trade-offs in the Romer Model

1. **Present versus Future**:
   - Governments could incentivise people to go into education and research with the hope of inventing new technologies that will raise productivity over time.
   - However, these people will then not be producing goods and services, so it means lower output today.
2. **Competition versus Growth**:
   - In general, Romer's model points to outcomes in which there is too little R&D activity.
   - People who invent a great new product can influence future inventions but usually do not receive the full stream of profits from these future inventions.
   - Laws to strengthen patent protection may raise the incentives to conduct R&D.
   - This points to a potential conflict between policies aimed at raising macroeconomic growth and microeconomic policies aimed at reducing the inefficiencies due to monopoly power.

# Past and Future of New Technologies

- Many of the facts about economic history back up Romer's model.
- Robert Gordon's paper (on the website) provides an excellent description of the various phases of technological invention.

1. **First Industrial Revolution (1750-1830)**
   - Inventions of the steam engine and cotton gin, lead to railroads and steamships. Took 150 years to have full impact.
2. **Second Industrial Revolution (1870-1900)**
   - Electric light, internal combustion engine, fresh running water to urban homes, sewers.
   - Telephone, radio, records, movies, electric machinery, consumer appliances, cars. The latter lead to suburbs, supermarkets, highways.
   - "Follow-up" inventions continued like television and air conditioning.
3. **Third Industrial Revolution (since 1960s)**
   - Electronic mainframe computers, 1960s.
   - Invention of the web and internet around 1995.

# Gordon on the Second Industrial Revolution

- Gordon believes that the inventions of the "second industrial revolution" made the biggest differences to standards of living.
- He describes life in 1870 as follows

  *"most aspects of life in 1870 (except for the rich) were dark, dangerous, and involved backbreaking work. There was no electricity in 1870. The insides of dwelling units were not only dark but also smoky, due to residue and air pollution from candles and oil lamps. The enclosed iron stove had only recently been invented and much cooking was still done on the open hearth. Only the proximity of the hearth or stove was warm; bedrooms were unheated and family members carried warm bricks with them to bed."*

  *But the biggest inconvenience was the lack of running water. Every drop of water for laundry, cooking, and indoor chamber pots had to be hauled in by the housewife, and wastewater hauled out. The average North Carolina housewife in 1885 had to walk 148 miles per year while carrying 35 tonnes of water."*

# Gordon's Thought Experiment

- To illustrate why he believes modern inventions don't match up with past improvements in terms of their ability to generate genuine improvements in living standards, Gordon offers the following thought experiment.

  "*You are required to make a choice between option A and option B. With option A you are allowed to keep 2002 electronic technology, including your Windows 98 laptop accessing Amazon, and you can keep running water and indoor toilets; but you can't use anything invented since 2002.*"

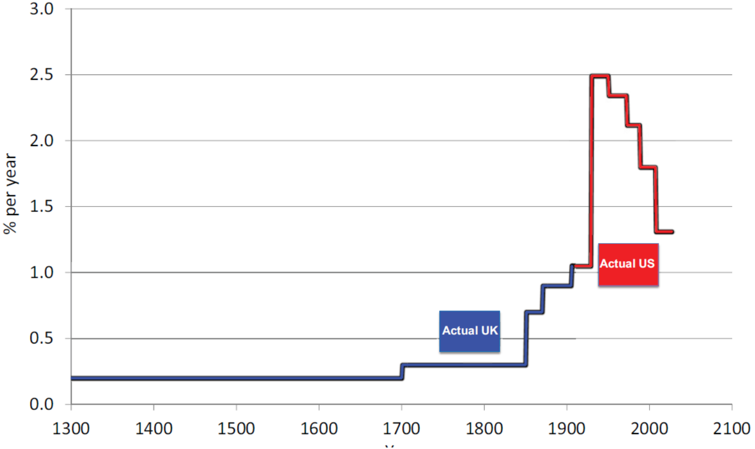
  "*Option B is that you get everything invented in the past decade right up to Facebook, Twitter, and the iPad, but you have to give up running water and indoor toilets. You have to haul the water into your dwelling and carry out the waste. Even at 3am on a rainy night, your only toilet option is a wet and perhaps muddy walk to the outhouse. Which option do you choose?*"

- You probably won't be surprised to find out that most people pick option B.
- As a fan of iPads and Twitter (not Facebook ...) I'm thankful we don't have to make the choice!

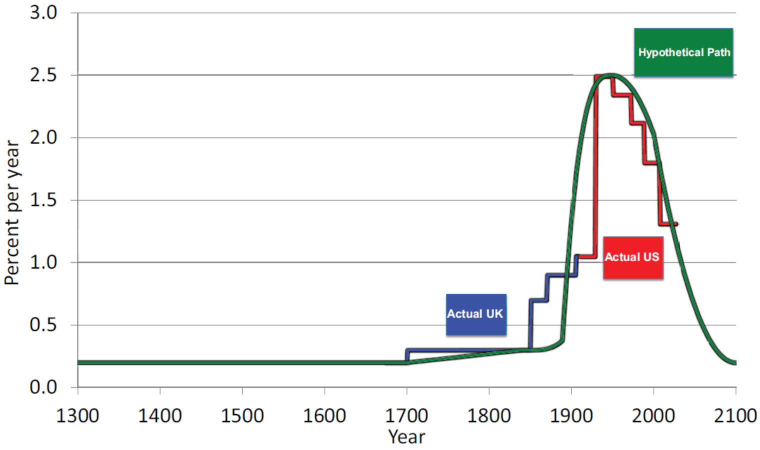
# Gordon on Future Growth

- Gordon believes that the technological innovations associated with computer technologies are far less important than those associated with the "second industrial revolution" and that growth may sputter out over time.
- The next slide repeats a chart from Gordon's paper showing the growth rate of per capita GDP for the world's leading economies (first the UK, then the US). It shows growth accelerating until 1950 and declining thereafter.
- The slide after shows a hypothetical chart in which Gordon projects a continuing fall-off in growth.
- Gordon also discusses other factors likely to holdback growth in leading countries - leveling off of educational achievement, an aging population and energy-related constraints.
- We should note, however, that economists are not very good at forecasting the invention of new technologies or their impact!
- Joel Mokyr's article "Is technological progress a thing of the past?" is a good counterpart to Gordon's scepticism.

# Gordon on the Growth Rate of Leading Economies



# Gordon's Hypothetical Path for Growth



# Things to Understand From This Topic

1. The Romer model's production function.
2. The model's assumptions about how the number of capital goods changes.
3. How to simplify the aggregate production function.
4. How to derive the steady-state growth rate.
5. The steady-state level of output per worker.
6. Why $A$ converges to its steady-state level.
7. The optimal level of R&D and why the observed level is probably below it.
8. Policy trade-offs suggested by the Romer model.
9. Robert Gordon on the history and future of technological innovation.