MA Macroeconomics
13. Cross-Country Technology Diffusion

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Cross-Country Differences in Output Per Worker

- The Romer model shows how the invention of new technologies promotes economic growth.
- However, only a very few countries in the world are "on the technological frontier".
- One way to illustrate this is to estimate the level of TFP for different countries.
- An important paper that did these calculations is by Hall and Jones (1999).
- The basis of the study is a "levels accounting" exercise starting from a production function

$$Y_i = K_i^{\alpha} \left(h_i A_i L_i\right)^{1-\alpha}$$

- Hall and Jones account for the effect of education on labour productivity.
- They construct measures of *human capital* based on estimates of the return to education—this is the h_i in the above equation.

Hall and Jones

- Hall and Jones show that their production function can be re-formulated as

$$\frac{Y_i}{L_i} = \left(\frac{K_i}{Y_i}\right)^{\frac{\alpha}{1-\alpha}} h_i A_i$$

- h_i estimated using evidence on educational levels and they set $\alpha = 1/3$.
- This allowed them to express all cross-country differences in output per worker in terms of three multiplicative terms: capital intensity (i.e. $\frac{K_i}{Y_i}$), human capital per worker, and technology or total factor productivity.
- They found that output per worker in the richest five countries was 31.7 times that in the poorest five countries.
- This was explained as follows:
  - Differences in capital intensity contributed a factor of 1.8.
  - Differences in human capital contributed a factor of 2.2
  - The remainder—a factor of 8.3—was due to differences in TFP.

Table from Hall-Jones Paper

TABLE I
PRODUCTIVITY CALCULATIONS: RATIOS TO U. S. VALUES

		Contribution from		
Country	Y/L	$(K/Y)^{\alpha/(1-\alpha)}$	H/L	A
United States	1.000	1.000	1.000	1.000
Canada	0.941	1.002	0.908	1.034
Italy	0.834	1.063	0.650	1.207
West Germany	0.818	1.118	0.802	0.912
France	0.818	1.091	0.666	1.126
United Kingdom	0.727	0.891	0.808	1.011
Hong Kong	0.608	0.741	0.735	1.115
Singapore	0.606	1.031	0.545	1.078
Japan	0.587	1.119	0.797	0.658
Mexico	0.433	0.868	0.538	0.926
Argentina	0.418	0.953	0.676	0.648
U.S.S.R.	0.417	1.231	0.724	0.468
India	0.086	0.709	0.454	0.267
China	0.060	0.891	0.632	0.106
Kenya	0.056	0.747	0.457	0.165
Zaire	0.033	0.499	0.408	0.160
Average, 127 countries:	0.296	0.853	0.565	0.516
Standard deviation:	0.268	0.234	0.168	0.325
Correlation with Y/L (logs)	1.000	0.624	0.798	0.889
Correlation with A (logs)	0.889	0.248	0.522	1.000

The elements of this table are the empirical counterparts to the components of equation (3), all measured as ratios to the U. S. values. That is, the first column of data is the product of the other three columns.

A Model with Leaders and Followers

- The Romer model should not be thought of as a model of growth in any one particular country.
- No country uses only technologies that were invented in that country; rather, products invented in one country end up being used all around the world.
- Thus, the model is best thought of as a very long-run model of the world economy.
- For individual countries, it suggests we need a model of how technology spreads or diffuses around the world.
- We will now describe such a model.

The Model

- There is a "lead" country with technology level, A_t that grows at rate g every period

$$\frac{\dot{A}_t}{A_t} = g$$

- All other countries in the world, indexed by j, have technology that whose growth rate is determined by

$$\frac{\dot{A}_{jt}}{A_{jt}} = \lambda_j + \sigma_j \frac{(A_t - A_{jt})}{A_{jt}}$$

- We assume $\sigma_j > 0$ because countries can learn from the superior technologies in the leader country.
- We also assume $\lambda_j < g$ so country j can't grow faster than the lead country without the learning that comes from having lower technology than the frontier.

Exponential Growth

- A very special number, $e = 2.71828...$, has the property that

$$\frac{de^x}{dx} = e^x$$

- Shows up a lot in theory of economic growth.

$$\frac{de^{gt}}{dt} = \frac{de^{gt}}{d(gt)} \frac{d(gt)}{dt} = ge^{gt}$$

- Now let's relate this back to our model. The fact that the lead country has growth such that

$$\frac{dA_t}{dt} = \dot{A}_t = gA_t$$

means that this country is characterised by what is known as exponential growth, i.e.

$$A_t = A_0 e^{gt}$$

A Differential Equation for Technology

- The equation for the dynamics of A_{jt} can be re-written as

$$\dot{A}_{jt} = \lambda_j A_{jt} + \sigma_j \left(A_t - A_{jt} \right)$$

- This is what is known as a first-order linear differential equation (differential equation because it involves a derivative; first-order because it only involves a first derivative; linear because it doesn't involve any terms taken to powers than are not one.) These equations can be solved to illustrate how A_j changes over time.
- Draw some terms together to re-write it as

$$\dot{A}_{jt} + \left(\sigma_j - \lambda_j \right) A_{jt} = \sigma_j A_t$$

- Remembering exponetial growth for leader country, this becomes

$$\dot{A}_{jt} + \left(\sigma_j - \lambda_j \right) A_{jt} = \sigma_j A_0 e^{gt}$$

One Possible Solution

- Looking at

$$\dot{A}_{jt} + (\sigma_j - \lambda_j) A_{jt} = \sigma_j A_0 e^{gt}$$

you might guess that one A_{jt} process that could satisfy this equation is something of the form $B_j e^{gt}$ where B_j is some unknown coefficient.

- Indeed, it turns out that this is the case. B_j must satisfy

$$g B_j e^{gt} + (\sigma_j - \lambda_j) B_j e^{gt} = \sigma_j A_0 e^{gt}$$

- Canceling the e^{gt} terms, we see that

$$B_j = \frac{\sigma_j A_0}{\sigma_j + g - \lambda_j}$$

- So, this solution takes the form

$$A^p_{jt} = B_j e^{gt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j} \right) A_0 e^{gt} = \left(\frac{\sigma_j}{\sigma_j + g - \lambda_j} \right) A_t$$

A More General Solution

- It turns out we can add on an additional term and still get a solution. Suppose there was a solution of the form

$$A_{jt} = B_j e^{gt} + D_{jt}$$

- If this satisfies

$$\dot{A}_{jt} + (\sigma_j - \lambda_j) A_{jt} = \sigma_j A_0 e^{gt}$$

- Then we must have

$$gBe^{gt} + \dot{D}_{jt} + (\sigma_j - \lambda_j) \left(Be^{gt} + D_{jt} \right) = \sigma_j A_0 e^{gt}$$

- The terms in e^{gt} cancel out by construction of B_j so

$$\dot{D}_{jt} + (\sigma_j - \lambda_j) D_{jt} = 0$$

- Again using the properties of the exponential function, this equation is satisfied by anything of the form

$$D_{jt} = D_{j0} e^{-(\sigma_j - \lambda_j)t}$$

Technology Convergence Over Time

- Express A_{jt} as a ratio of the frontier level of technology.

$$\frac{A_{jt}}{A_t} = \frac{\sigma_j}{\sigma_j + g - \lambda_j} + \frac{D_{j0}}{A_0} e^{-(\sigma_j + g - \lambda_j)t}$$

- Recall that $\lambda_j < g$, (without catch-up growth, the follower's technology grows slower than the leader) and also that $\sigma_j > 0$ (some learning takes place). This means

$$\sigma_j + g - \lambda_j > 0$$

- For this reason

$$e^{-(\sigma_j + g - \lambda_j)t} \to 0 \quad \text{as} \quad t \to \infty$$

- This means that the second term in the first equation above tends towards zero. Over time, as this term disappears, the country converges towards a level of technology that is a constant ratio, $\frac{\sigma_j}{\sigma_j + g - \lambda_j}$ of the frontier level, and its growth rate tends towards g.

Properties of the Steady-State Technology Level

- Because $g - \lambda_j > 0$ we know that

$$0 < \frac{\sigma_j}{\sigma_j + g - \lambda_j} < 1$$

  so each country never actually catches up to the leader but instead converges to some fraction of the lead country's technology level.

- Also, $g - \lambda_j > 0$ means that

$$\frac{d}{d\sigma_j}\left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) > 0$$

  so the equilibrium ratio of the country's technology to the leader's depends positively on the "learning parameter" σ_j.

- It's also true that

$$\frac{d}{d\lambda_j}\left(\frac{\sigma_j}{\sigma_j + g - \lambda_j}\right) > 0$$

  so the more growth the country can generate each period independent of learning from the leader, the higher will be its equilibrium ratio of technology relative to the leader.

Transition Paths

- Remember equation for A_{jt} as a ratio of the frontier level of technology.

$$\frac{A_{jt}}{A_t} = \frac{\sigma_j}{\sigma_j + g - \lambda_j} + \frac{D_{j0}}{A_t} e^{-(\sigma_j + g - \lambda_j)t}$$

- Just because the second term tends to disappear to zero over time doesn't mean it's unimportant. How a country behaves along its "transition path" depends on the value of the initial parameter D_{j0}.
- If $D_{j0} < 0$, then the term that is disappearing over time is a negative term that is a drag on the level of technology. This means that the country starts out below its equilibrium technology ratio and grows faster than the leader for some period of time.
- If $D_{j0} > 0$, then the term that is disappearing over time is a positive term that is boosting the level of technology. This means that the country starts out above its equilibrium technology ratio and grows slower than the leader for some period of time.

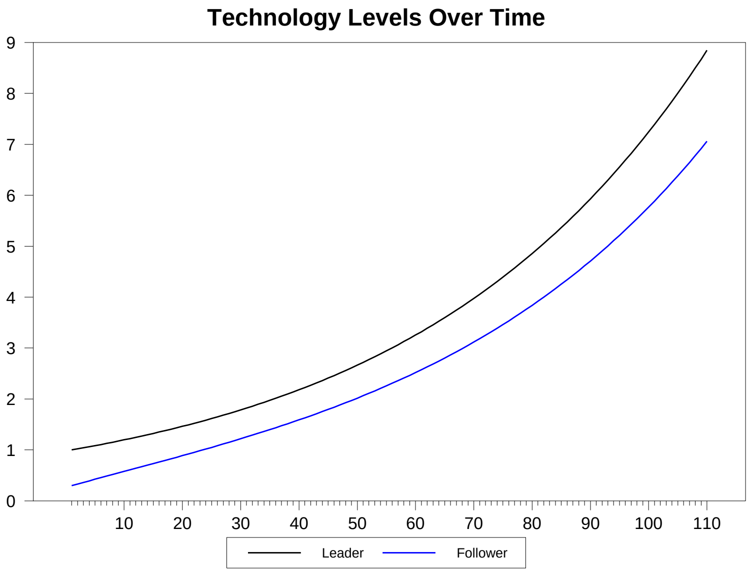
Illustrating Transition Dynamics

- The charts on the next six pages illustrate how these dynamics work.
- They charts show model simulations for a leader economy with $g = 0.02$ and a follower economy with $\lambda_j = 0.01$ and $\sigma_j = 0.04$. These values mean

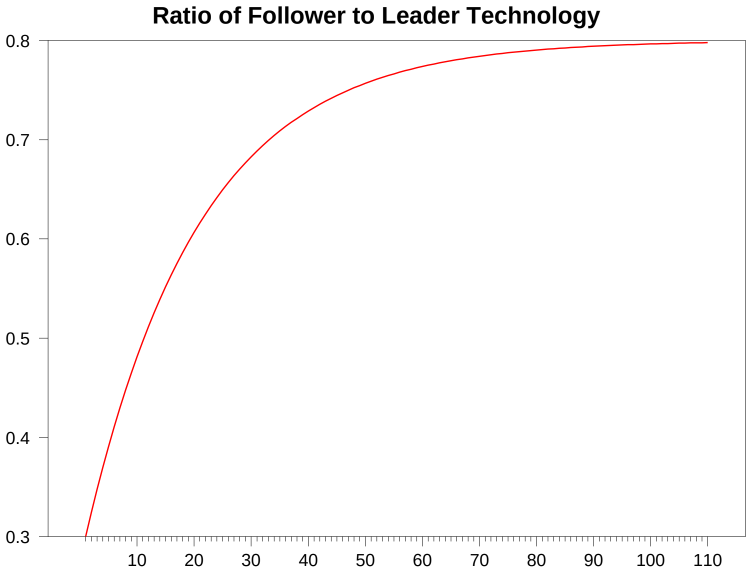
$$\frac{\sigma_j}{\sigma_j + g - \lambda_j} = \frac{0.04}{0.04 + 0.02 - 0.01} = 0.8$$

  so the follower economy converges to a level of technology that is 20 percent below that of the leader.
- The first three charts show what happens when this economy has a value of $D_{j0} = -0.5$, so that it starts out with a technology level only 30 percent that of the leader.
- The second three charts show what happens when this economy has a value of $D_{j0} = 0.5$, so that it starts out with a technology level 30 percent above that of the leader, even though the equilibrium value is 20 percent below.

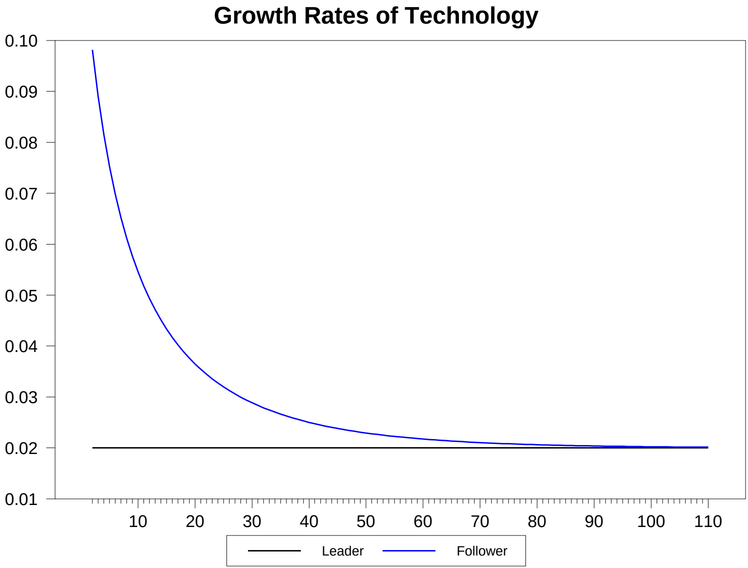
Follower Starts Out Below Equilibrium Technology Ratio



Follower Starts Out Below Equilibrium Technology Ratio



Follower Starts Out Below Equilibrium Technology Ratio



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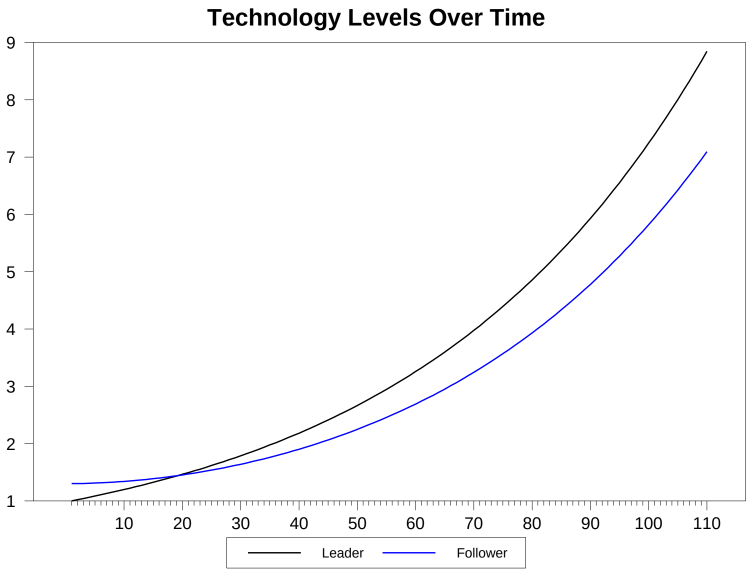

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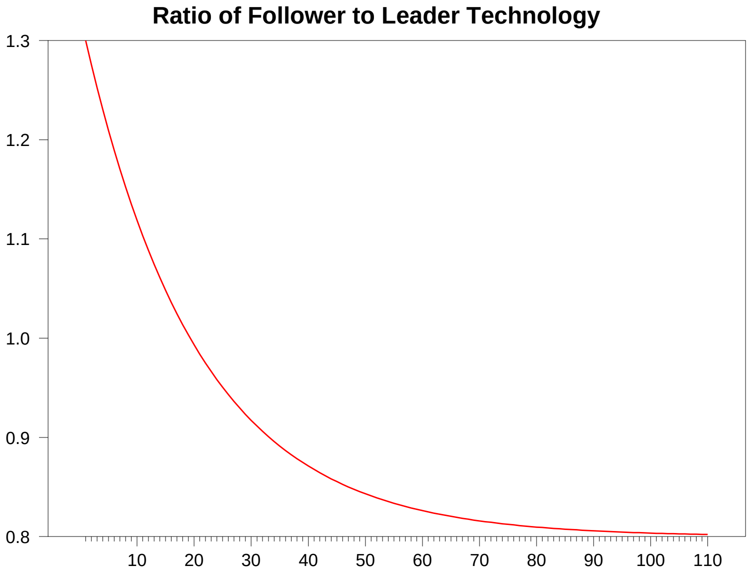

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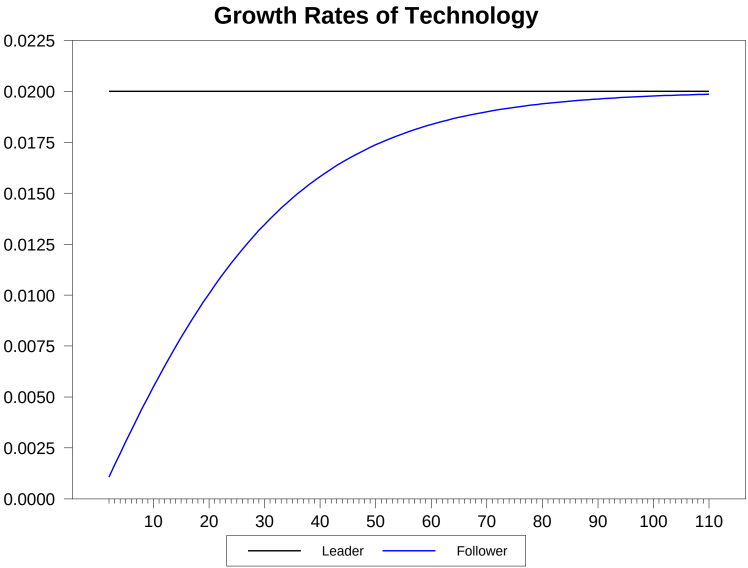
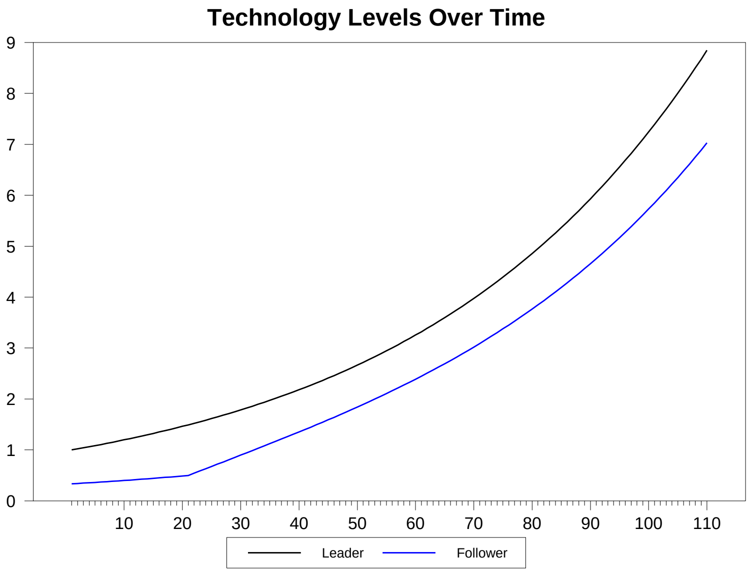


Growth Miracles

- Finally, we show how the model may also be able to account for the sort of "growth miracles" that are occasionally observed when countries suddenly start experiencing rapid growth.
- If a country can increase its value of σ_j via education or science-related policies, its position in the steady-state distribution of income may move upwards substantially, with the economy then going through a phase of rapid growth.
- The next three charts show what happens when, in period 21, an economy changes from having $\sigma_j = 0.005$ to $\sigma_j = 0.04$. The equilbrium technology ratio changes from one-third to 0.8 and the economy experiences a long transitional period of rapid growth.
- An important message from this model is that for most countries, it is not their ability to invent new capital goods that is key to high living standards, but rather their ability to learn from those countries that are more technologically advanced.

An Increasing in the Rate of Learning



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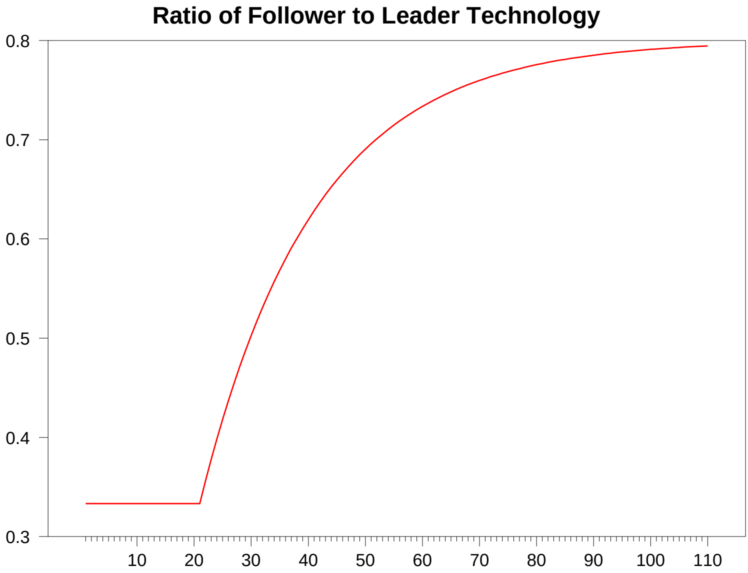

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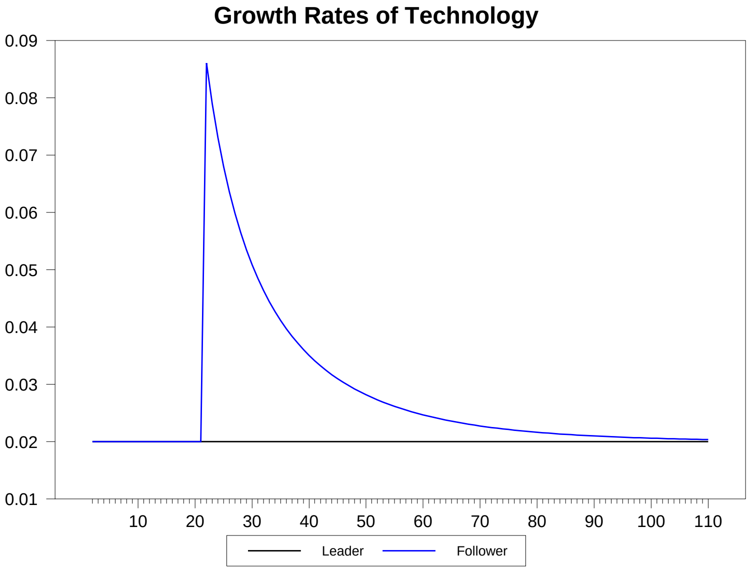

Things to Understand From This Topic

1. Evidence on the sources of cross-country differences in output per worker.
2. The model's assumptions and the meaning of its parameters.
3. Exponential growth: The properties of the function e^{gt}.
4. The model's differential equation and its two-part solution method.
5. Properties of the solution: How dynamics depend on σ_j, λ_j and A_0^g.
6. How the model can explain long periods of rapid growth or protacted slumps.
7. "What if" scenarios: What happens if a parameter changes?