Some Preliminaries on Equations

I want to start with some discussion of equations. We will use both graphs and equations to describe the models in this class. Now I know many students don't like equations and believe they are best studiously avoided. However, that won't be a good strategy for doing well in this course, so I strongly encourage you to engage with the technical material in this class. It isn't as hard is it might look to start with.

Variables and Coefficients

The equations in this class will generally have a certain format. They will often look a bit like this.

$$y_t = \alpha + \beta x_t \tag{1}$$

There are two types of objects in this equation. First, there are the **variables**, y_t and x_t. These will correspond to economic variables that we are interested in (inflation or GDP for example). We interpret y_t as meaning "the value that the variable y takes during the time period t". For most models in this course, we will treat time as marching forward in discrete intervals, i.e. period 1 is followed by period 2, period t is followed by period $t+1$ and so on.

Second, there are the **parameters** or **coefficients**. In this example, these are given by α and β. These are assumed to stay fixed over time. There are usually two types of coefficients: Intercept terms like α that describe the value that series like y_t will take when other variables all equal zero and coefficients like β that describe the impact that one variable has on another. In this case, if β is a big number, then a change in the variable x_t has a big impact on y_t while if β is small, it will have a small impact.

One question you might ask: If α and β are fixed numbers, then why don't you just write down numbers? For example if $\alpha = 1$ and $\beta = 2$, then why don't you just right

$$y_t = 1 + 2x_t \tag{2}$$

The answer is that we don't usually know exactly what the coefficient numbers are in macroeconomic relationships. For example, we may know that β is positive, meaning y_t goes up when x_t goes up, but we don't want to pretend that we know precisely that $\beta = 2$. So we want to be able to focus on the things that will generally emerge from the model as being true, rather than results that only apply specifically when $\beta = 2$, which would mean that y_t quadruples when x_t doubles.

In some cases, however, we will put specific values of coefficients and use them to give specific examples of how the variables in our models behave.

Some of you are probably asking what those squiggly shapes — α and β — are. They are Greek letters. While it's not strictly necessary to use these shapes to represent model parameters, it's pretty common in economics. So let me introduce them: α is alpha (Al-Fa), β is beta (Bay-ta), γ is gamma, δ is delta, θ is theta (Thay-ta) and π naturally enough is pi.

Subscripts and Superscripts

When we write y_t, we mean the value that the variable y takes at time t. Note that the t here is a *subscript* – it goes at the bottom of the y. Some students don't realise this is a subscript and will just write yt but this is incorrect (it reads as though the value t is multiplying y which is not what's going on).

We will also sometimes put indicators above certain variables to indicate that they are special variables. For example, in the model we present now, you will see a variable written as π_t^e which will represent the public's expectation of inflation. In the model, π_t is inflation at time t and the e above the π in π_t^e is there to signify that this is not inflation itself but rather it is the public's expectation of it.

Dynamic Equations

One of the things we will be interested in is how the variables we are looking at will change over time. We will characterise these changes with **dynamic equations** like

$$y_t = \beta y_{t-1} + \gamma x_t \tag{3}$$

Reading this equation, it says that the value of y at time t will depend on the value of x at time t and also on the value that y took in the previous period i.e. $t-1$. By this, we mean that this equation holds in every period. In other words, in period 2, y depends on the value that x takes in period 2 and also on the value that y took in period 1. Similarly, in period 3, y depends on the value that x takes in period 3 and also on the value that y took in period 2. And so on.

Dynamics Generated by Difference Equations

A difference equation is a formula that generates a sequence of numbers. In economics, these sequences can be understood as a pattern over time for a variable of interest. After supplying some starting values, the difference equation provides a sequence explaining how the variable changes over time. For example, consider a case in which the first value for a series is $z_1 = 1$ and then z_t follows a difference equation

$$z_t = z_{t-1} + 2 \tag{4}$$

This will give $z_2 = 3$, $z_3 = 5$, $z_4 = 7$ and so on.

More relevant to economics is the multiplicative model

$$z_t = bz_{t-1} \tag{5}$$

Note that to actually obtain the series of numbers, you need to have a starting value and a specific value for b. For a starting value of $z_1 = 5$ and $b = 2$, this difference equation delivers a sequence of values that looks like this: $5, 5b, 5b^2, 5b^3, 5b^4....$ which is $5, 10, 20, 40, 80....$ and so on.

Note that if b is between zero and one, then this sequence converges to zero over time no matter what value x takes but if $b > 1$, the sequence will explode off towards either plus or minus infinity depending on whether the initial value was positive or negative. The same logic prevails if we add a constant term to the difference equation. Consider this equation:

$$z_t = a + bz_{t-1} \tag{6}$$

If b is between zero and one, then no matter what the starting value is, the sequence converges over time to $\frac{a}{1-b}$ but if $b > 1$, the sequence explodes towards infinity.

Difference Equations with Random Shocks

The difference equations we have just looked at our termed **deterministic** models. Once you know what happens at the start, everything that happens after that point is pre-determined and perfectly predictable. But macroeconomic variables like GDP and inflation don't behave this way. At best, we can make an imperfect forecast about what future values they may take. For this reason, in this course, we will sometime assume that variables are partly determined by random factors or "shocks". In other words, they are what statisticians call **stochastic** variables.

If we add random shocks to the model—making it what is known as a first-order autoregressive or AR(1) model—the key thing remains the value of b. If the model is

$$z_t = a + bz_{t-1} + \epsilon_t \tag{7}$$

where ϵ_t is a series of independently drawn zero-mean random shocks, then the presence of the shocks will mean the series won't simply converge to a constant or steadily explode. But as long as we have $0 < b < 1$ then the series will tend to oscillate above and below the average value of $\frac{a}{1-b}$ while if $b > 1$ the series will tend to explode to infinity over time.

See the next page for a sample time path for this model with $a = 0$, $b = 0.9$, $z_0 = 1$ and ϵ_t a set of random numbers drawn from a mean-zero uniform distribution (so that all numbers between -0.5 and 0.5 were equally likely.) This chart was generated using Excel. You should look at the video that has been made available showing how to implement deterministic and stochastic difference equations using Excel.

Sample Output From a Stochastic Difference Equation