Advanced Macroeconomics
0. Some Preliminaries on Equations

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How to Read the Equations in this Course

- We will use both graphs and equations to describe the models in this class.
- I know many students don't like equations and believe they are best studiously avoided but it isn't as hard is it might look to start with.
- The equations in this class will often look a bit like this.

$$y_t = \alpha + \beta x_t$$

There are two types of objects in this equation.

1. The **variables**, y_t and x_t. These will correspond to economic variables that we are interested in (inflation for example). We interpret y_t as meaning "the value that the variable y takes during the time period t").
2. There are the **parameters** or **coefficients**. In this example, these are given by α and β. These are assumed to stay fixed over time. There are usually two types of coefficients: Intercept terms like α that describe the value that series like y_t will take when other variables all equal zero and coefficients like β that describe the impact that one variable has on another.

Squiggly Letters

- Some of you are probably asking what those squiggly shapes — α and β — are. They are Greek letters.
- While it's not strictly necessary to use these shapes to represent model parameters, it's pretty common in economics.
- So let me introduce them:
  1. α is alpha (Al-Fa)
  2. β is beta (Bay-ta)
  3. γ is gamma
  4. δ is delta
  5. θ is theta (Thay-ta)
  6. π naturally enough is pi.

Why Not Just Use Numbers?

- Consider again the equation

$$y_t = \alpha + \beta x_t$$

- One question you might ask: If α and β are fixed numbers, then why don't you just write down numbers? For example if $\alpha = 1$ and $\beta = 2$, then why don't you just right

$$y_t = 1 + 2x_t$$

- The answer is that we don't usually know exactly what the coefficient numbers are in macroeconomic relationships.
- For example, we may know that β is positive, meaning y_t goes up when x_t goes up, but we don't want to pretend that we know precisely that $\beta = 2$.
- So we want to be able to focus on the things that will generally emerge from the model as being true, rather than results that only apply specifically when $\beta = 2$, which would mean that y_t quadruples when x_t doubles.
- In some cases, however, we will put specific values of coefficients and use them to give specific examples of how the variables in our models behave.

Subscripts and Superscripts

- When we write y_t, we mean the value that the variable y takes at time t.
- Note that the t here is a **subscript** – it goes at the bottom of the y.
- Some students don't realise this is a subscript and will just write yt but this is incorrect (it reads as though the value t is multiplying y which is not what's going on).
- We will also sometimes put indicators above certain variables to indicate that they are special variables.
- For example, in the model we present now, you will see a variable written as π_t^e which will represent the public's expectation of inflation.
- In the model, π_t is inflation at time t and the e above the π in π_t^e is there to signify that this is not inflation itself but rather it is the public's expectation of it.

Dynamic Equations

- One of the things we will be interested in is how the variables we are looking at will change over time. We will characterise these changes with **dynamic equations** like

$$y_t = \beta y_{t-1} + \gamma x_t$$

- Reading this equation, it says that the value of y at time t will depend on the value of x at time t and also on the value that y took in the previous period i.e. $t-1$.
- By this, we mean that this equation holds in every period. In other words, in period 2, y depends on the value that x takes in period 2 and also on the value that y took in period 1.
- Similarly, in period 3, y depends on the value that x takes in period 3 and also on the value that y took in period 2.
- And so on.

Dynamics Generated by Difference Equations

- A difference equation is a formula that generates a sequence of numbers. In economics, these sequences can be understood as a pattern over time for a variable of interest.
- After supplying some starting values, the difference equation provides a sequence explaining how the variable changes over time.
- For example, consider a case in which the first value for a series is $z_1 = 1$ and then z_t follows a difference equation

$$z_t = z_{t-1} + 2$$

  This will give $z_2 = 3$, $z_3 = 5$, $z_4 = 7$ and so on.
- So the sequence of numbers generated is $1, 3, 5, 7, ...$

A More Relevant Example

- More relevant to this module is the multiplicative model

$$z_t = bz_{t-1}$$

- For a starting value of $z_1 = x$, this difference equation delivers a sequence of values x, xb, xb^2, xb^3, xb^4..... If b is between zero and one, the sequence converges to zero but if $b > 1$ it explodes to either plus or minus infinity depending on whether x is positive or negative.
- For example, for a starting value of $z_1 = 5$ and $b = 2$, this difference equation delivers the following sequence of values $5, 10, 20, 40, 80, 160$.... and so on.
- The same logic prevails if we add a constant term

$$z_t = a + bz_{t-1}$$

  If b is between zero and one, the sequence converges over time to $\frac{a}{1-b}$ but if $b > 1$, the sequence explodes towards infinity.
- You can use spreadsheet packages like Excel to get sequences of values generated by difference equations.

A Model with Random Shocks

- The difference equations we have just looked at our termed **deterministic** models. Once you know what happens at the start, everything that happens after that point is pre-determined and perfectly predictable.
- But macroeconomic variables like GDP and inflation don't behave this way. At best, we can make an imperfect forecast about what future values they may take.
- For this reason, in this course, we will sometime assume that variables are partly determined by random factors or "shocks". In other words, they are what statisticians call **stochastic** variables.
- For example, we can alter the multiplicative model to add random shocks

$$z_t = a + bz_{t-1} + \epsilon_t$$

where ϵ_t is a series of zero-mean random shocks. This is called a first-order autoregressive or AR(1) model. Then if $0 < b < 1$ the series tends to oscillate above and below the average value of $\frac{a}{1-b}$ while if $b > 1$ the series will tend to explode over time.

Simulating Stochastic Difference Equations

- How would you generate examples of how the following variable would behave over time?

$$z_t = a + bz_{t-1} + \epsilon_t$$

- See the next page for a sample time path for this model with $a = 0$, $b = 0.9$, $z_0 = 1$ and ϵ_t a set of random numbers drawn from a mean-zero uniform distribution (so that all numbers between -0.5 and 0.5 were equally likely.)
- This chart was generated using Excel. You should look at the video that has been made available showing how to implement deterministic and stochastic difference equations using Excel.

Sample Output From an AR(1) Stochastic Difference Equation