Advanced Macroeconomics
5. Rational Expectations and Asset Prices

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A New Topic

- We are now going to switch gear and leave the IS-MP-PC model behind us.
- One of the things we've focused on is how people formulate expectations about inflation. We put forward one model of how these expectations were formulated, an adaptive expectations model in which people formulated their expectations by looking at past values for a series.
- Over the next few weeks, we will look at an alternative approach that macroeconomists call "rational expectations".
- This approach is widely used in macroeconomics and we will cover its application to models of
  - Asset prices, particularly stock prices.
  - Household consumption and fiscal policy.
  - Exchange rates

Rational Expectations

- Almost all economic transactions rely crucially on the fact that the economy is not a "one-period game." Economic decisions have an intertemporal element to them.
- A key issue in macroeconomics is how people formulate expectations about the in the presence of uncertainty.
- Prior to the 1970s, this aspect of macro theory was largely *ad hoc* relying on approaches like adaptive expectations.
- This approach criticised in the 1970s by economists such as Robert Lucas and Thomas Sargent who instead promoted the use of an alternative approach which they called "rational expectations."
- In economics, saying people have "rational expectations" usually means two things:
    1. They use publicly available information in an efficient manner. Thus, they do not make systematic mistakes when formulating expectations.
    2. They understand the structure of the model economy and base their expectations of variables on this knowledge.

How We Will Describe Expectations

- We will write $E_t Z_{t+2}$ to mean the expected value the agents in the economy have at time t for what Z is going to be at time $t+2$.
- We assume people have a distribution of potential outcomes for Z_{t+2} and $E_t Z_{t+2}$ is mean of this distribution.
- So E_t is not a number that is multiplying Z_{t+2}. Instead, it is a qualifier explaining that we are dealing with people's prior expectations of a Z_{t+2} rather than the actual realised value of Z_{t+2} itself.
- We will use some basic properties of the expected value of distributions. Specifically, the fact that expected values of distributions is what is known as a linear operator meaning

$$E_t\left(\alpha X_{t+k} + \beta Y_{t+k}\right) = \alpha E_t X_{t+k} + \beta E_t Y_{t+k}$$

- For example,

$$E_t\left(5X_{t+k}\right) = 5E_t\left(X_{t+k}\right)$$

- And

$$E_t\left(X_{t+k} + Y_{t+k}\right) = E_t X_{t+k} + E_t Y_{t+k}$$

Example: Asset Prices

- Asset prices are an increasingly important topic in macroeconomics. The most recent two global recessions—the "dot com" recession of 2000/01 and the "great recession" of 2008/09—were triggered by big declines in asset prices following earlier large increases. A framework for discussing these movements is thus a necessary part of any training in macroeconomics.
- Consider an asset that can be purchased today for price P_t and which yields a dividend of D_t.
- The asset could be a share of equity in a firm with D_t being the dividend payment but it could also be a house and D_t could be the net return from renting this house out
- If this asset is sold tomorrow for price P_{t+1}, then it generates a rate of return on this investment of

$$r_{t+1} = \frac{D_t + \Delta P_{t+1}}{P_t}$$

- This rate of return has two components, the first reflects the dividend received during the period the asset was held, and the second reflects the *capital gain* (or loss) due to the price of the asset changing from period t to period $t+1$.

A Different Form for the Rate of Return Equation

- The *gross return* on the asset, i.e. one plus the rate of return, is

$$1 + r_{t+1} = \frac{D_t + P_{t+1}}{P_t}$$

- A useful re-arrangement of this equation that we will work with is:

$$P_t = \frac{D_t}{1 + r_{t+1}} + \frac{P_{t+1}}{1 + r_{t+1}}$$

- In this context, rational expectations means investors understand this equation and that all expectations of future variables must be consistent with it. This implies that

$$E_t P_t = E_t \left[\frac{D_t}{1 + r_{t+1}} + \frac{P_{t+1}}{1 + r_{t+1}} \right]$$

where E_t means the expectation of a variable formulated at time t.

- The stock price at time t is observable to the agent so $E_t P_t = P_t$, implying

$$P_t = E_t \left[\frac{D_t}{1 + r_{t+1}} + \frac{P_{t+1}}{1 + r_{t+1}} \right]$$

Constant Expected Returns

- Assume the expected return on assets is constant.

$$E_t r_{t+k} = r \qquad k = 1, 2, 3, ...$$

- Can think of this as a "required return", determined perhaps the rate of return available on some other asset.
- Last equation on the previous slide becomes

$$P_t = \frac{D_t}{1+r} + \frac{E_t P_{t+1}}{1+r}$$

- This is an example of a **first-order stochastic difference equation**. Stochastic means random or incorporating uncertainty.
- Because such equations occur commonly in macroeconomics, we will discuss a general approach to solving them.

First-Order Stochastic Difference Equations

- Lots of models in economics take the form

$$y_t = ax_t + bE_t y_{t+1}$$

- The equation just says that y today is determined by x and by tomorrow's expected value of y. But what determines this expected value? Rational expectations implies a very specific answer.
- Under rational expectations, the agents in the economy understand the equation and formulate their expectation in a way that is consistent with it:

$$E_t y_{t+1} = aE_t x_{t+1} + bE_t E_{t+1} y_{t+2}$$

  This last term can be simplified to

$$E_t y_{t+1} = aE_t x_{t+1} + bE_t y_{t+2}$$

  because $E_t E_{t+1} y_{t+2} = E_t y_{t+2}$.
- This is known as the Law of Iterated Expectations: It is not rational for me to expect to have a different expectation next period for y_{t+2} than the one that I have today.

Repeated Substitution

- Substituting this into the previous equation, we get

$$y_t = ax_t + abE_t x_{t+1} + b^2 E_t y_{t+2}$$

- Repeating this by substituting for $E_t y_{t+2}$, and then $E_t y_{t+3}$ and so on gives

$$y_t = ax_t + abE_t x_{t+1} + ab^2 E_t x_{t+2} + ... + ab^{N-1} E_t x_{t+N-1} + b^N E_t y_{t+N}$$

- Which can be written in more compact form as

$$y_t = a \sum_{k=0}^{N-1} b^k E_t x_{t+k} + b^N E_t y_{t+N}$$

- Usually, it is assumed that

$$\lim_{N \to \infty} b^N E_t y_{t+N} = 0$$

- So the solution is

$$y_t = a \sum_{k=0}^{\infty} b^k E_t x_{t+k}$$

This solution underlies the logic of a very large amount of modern macroeconomics.

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Summation Signs

- For those of you unfamiliar with the summation sign terminology, summation signs work like this

$$\sum_{k=0}^{2} z_k = z_0 + z_1 + z_2$$

$$\sum_{k=0}^{3} z_k = z_0 + z_1 + z_2 + z_3$$

  and so on.

- The term

$$\sum_{k=0}^{\infty} b^k E_t x_{t+k}$$

  is just a compact way of writing $x_t + bE_t x_{t+1} + b^2 E_t x_{t+2} + \ldots$

Applying Solution to Asset Prices

- Our asset price equation

$$P_t = \frac{D_t}{1+r} + \frac{E_t P_{t+1}}{1+r}$$

is a specific case of the first-order stochastic difference equation with

$$\begin{aligned} y_t &= P_t \\ x_t &= D_t \\ a &= \frac{1}{1+r} \\ b &= \frac{1}{1+r} \end{aligned}$$

- This implies that the asset price can be expressed as follows

$$P_t = \sum_{k=0}^{N-1} \left(\frac{1}{1+r}\right)^{k+1} E_t D_{t+k} + \left(\frac{1}{1+r}\right)^N E_t P_{t+N}$$

The Dividend Discount Model

- Solution is

$$P_t = \sum_{k=0}^{N-1} \left(\frac{1}{1+r}\right)^{k+1} E_t D_{t+k} + \left(\frac{1}{1+r}\right)^N E_t P_{t+N}$$

- Usually assume the final term tends to zero as N gets big:

$$\lim_{N\to\infty} \left(\frac{1}{1+r}\right)^N E_t P_{t+N} = 0$$

- What is the logic behind this assumption? One explanation is that if it did not hold then we could set all future values of D_t equal to zero, and the asset price would still be positive. But an asset that never pays out should be inherently worthless, so this condition rules this possibility out.
- With this imposed, our solution becomes

$$P_t = \sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^{k+1} E_t D_{t+k}$$

- This equation is known as the **dividend-discount model**.

Explaining the Solution Without Equations

- Suppose I told you that the right way to price a stock was as follows.

  Today's stock price should equal today's dividend plus half of tomorrow's expected stock price.

- Now suppose it's Monday. Then that means the right formula should be

  Monday's stock price should equal Monday's dividend plus half of Tuesday's expected stock price.

- It also means the following applies to Tuesday's stock price

  Tuesday's stock price should equal Tuesday's dividend plus half of Wednesday's expected stock price.

- If people had rational expectations, then Monday's stock prices would equal

  Monday's dividend plus half of Tuesday's expected dividend plus one-quarter of Wednesday's expected stock price

Explaining the Solution Without Equations

- Now, being consistent about it—factoring in what Wednesday's stock price should be—you'd get the price being equal to

  Monday's dividend plus half of Tuesday's expected dividend plus one-quarter of Wednesday's expected dividend plus one-eigth of Thursday's expected dividend and so on.

- This is the idea being captured when we write

$$P_t = \sum_{k=0}^{\infty} \left(\frac{1}{1+r} \right)^{k+1} E_t D_{t+k}$$

Constant Dividend Growth

- A useful special case that is to assume dividends are expected grow at a constant rate

$$E_t D_{t+k} = (1+g)^k D_t$$

- In this case, the dividend-discount model predicts that the stock price should be given by

$$P_t = \frac{D_t}{1+r} \sum_{k=0}^{\infty} \left(\frac{1+g}{1+r} \right)^k$$

- Now, remember the old multiplier formula, which states that as long as $0 < c < 1$, then

$$1 + c + c^2 + c^3 + ... = \sum_{k=0}^{\infty} c^k = \frac{1}{1-c}$$

- This geometric series formula gets used *a lot* in modern macroeconomics, not just in examples involving the multiplier. Here we can use it as long as $\frac{1+g}{1+r} < 1$, i.e. as long as r (the expected return on the stock market) is greater than g (the growth rate of dividends).

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The Gordon Growth Formula

- Assuming $r > g$, then we have

$$\begin{aligned} P_t &= \frac{D_t}{1+r}\frac{1}{1-\frac{1+g}{1+r}} \\ &= \frac{D_t}{1+r}\frac{1+r}{1+r-(1+g)} \\ &= \frac{D_t}{r-g} \end{aligned}$$

- Prices are a multiple of current dividend payments, where that multiple depends positively on the expected future growth rate of dividends and negatively on the expected future rate of return on stocks.
- This means that the dividend-price ratios is

$$\frac{D_t}{P_t} = r - g$$

- This is often called the **Gordon growth model**, after the economist that popularized it. It is often used as a benchmark for assessing whether an asset is above or below the "fair" value implied by rational expectations.

Trends and Cycles

- An alternative assumption: Suppose dividends fluctuate around a steady-growth trend. An example this is

$$
\begin{aligned}
D_t &= c(1+g)^t + u_t \\
u_t &= \rho u_{t-1} + \epsilon_t
\end{aligned}
$$

- These equations state that dividends are the sum of two processes:
  - The first grows at rate g each period.
  - The second, u_t, measures a cyclical component of dividends, and this follows an AR(1) process. Here ϵ_t is a zero-mean random "shock" term. Over large samples, we would expect u_t to have an average value of zero, but deviations from zero will be more persistent the higher is the value of the parameter ρ.
- We will now derive the dividend-discount model's predictions for stock prices when dividends follow this process.

Trend Component

- The model predicts that

$$P_t = \sum_{k=0}^{\infty} \left(\frac{1}{1+r} \right)^{k+1} E_t \left(c(1+g)^{t+k} + u_{t+k} \right)$$

- Let's split this sum into two. First the trend component,

$$\begin{aligned} \sum_{k=0}^{\infty} \left(\frac{1}{1+r} \right)^{k+1} E_t \left(c(1+g)^{t+k} \right) &= \frac{c(1+g)^t}{1+r} \sum_{k=0}^{\infty} \left(\frac{1+g}{1+r} \right)^k \\ &= \frac{c(1+g)^t}{1+r} \frac{1}{1-\frac{1+g}{1+r}} \\ &= \frac{c(1+g)^t}{1+r} \frac{1+r}{1+r-(1+g)} \\ &= \frac{c(1+g)^t}{r-g} \end{aligned}$$

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&= \frac{c(1+g)^t}{1+r} \frac{1}{1-\frac{1+g}{1+r}} \\
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&= \frac{c(1+g)^t}{1+r} \frac{1}{1-\frac{1+g}{1+r}} \\
&= \frac{c(1+g)^t}{1+r} \frac{1+r}{1+r-(1+g)} \\
&= \frac{c(1+g)^t}{r-g}
\end{aligned}$$

Cyclical Component

- Next, the cyclical component. Because $E(\epsilon_{t+k}) = 0$, we have

$$\begin{aligned} E_t u_{t+1} &= E_t(\rho u_t + \epsilon_{t+1}) = \rho u_t \\ E_t u_{t+2} &= E_t(\rho u_{t+1} + \epsilon_{t+2}) = \rho^2 u_t \\ E_t u_{t+k} &= E_t(\rho u_{t+k-1} + \epsilon_{t+k}) = \rho^k u_t \end{aligned}$$

So, this second sum can be written as

$$\begin{aligned} \sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^{k+1} E_t u_{t+k} &= \frac{u_t}{1+r} \sum_{k=0}^{\infty} \left(\frac{\rho}{1+r}\right)^k \\ &= \frac{u_t}{1+r} \frac{1}{1-\frac{\rho}{1+r}} \\ &= \frac{u_t}{1+r} \frac{1+r}{1+r-\rho} \\ &= \frac{u_t}{1+r-\rho} \end{aligned}$$

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The Full Solution

- Putting these two sums together, the stock price at time t is

$$P_t = \frac{c(1+g)^t}{r-g} + \frac{u_t}{1+r-\rho}$$

- Stock prices don't just grow at a constant rate. Instead they depend positively on the cyclical component of dividends, u_t, and the more persistent are these cyclical deviations (the higher ρ is), the larger is their effect on stock prices.
- Concrete examples, suppose $r = 0.1$. When $\rho = 0.9$ the coefficient on u_t is $\frac{1}{1+r-\rho} = \frac{1}{1.1-0.9} = 5$ but if $\rho = 0.6$, then the coefficient falls to $\frac{1}{1+r-\rho} = \frac{1}{1.1-0.6} = 2$
- When taking averages over long periods of time, the u components of dividends and prices will average to zero. This is why the Gordon formula is normally seen as a guide to long-run average valuations rather than a prediction as to what the market should be right now.

Changes in Stock Prices

- The model has important predictions for changes in stock prices. It predicts

$$
\begin{aligned}
P_{t+1} - P_t \quad = \quad & \left[\left(\frac{1}{1+r} \right) D_{t+1} + \left(\frac{1}{1+r} \right)^2 E_{t+1} D_{t+2} + \left(\frac{1}{1+r} \right)^3 E_{t+1} D_{t+3} + ... \right] \\
& - \left[\left(\frac{1}{1+r} \right) D_t + \left(\frac{1}{1+r} \right)^2 E_t D_{t+1} + \left(\frac{1}{1+r} \right)^3 E_t D_{t+2} + ... \right]
\end{aligned}
$$

- This can be re-written as

$$
\begin{aligned}
P_{t+1} - P_t \quad = \quad & - \left(\frac{1}{1+r} \right) D_t + \left[\left(\frac{1}{1+r} \right) D_{t+1} - \left(\frac{1}{1+r} \right)^2 E_t D_{t+1} \right] \\
& + \left[\left(\frac{1}{1+r} \right)^2 E_{t+1} D_{t+2} - \left(\frac{1}{1+r} \right)^3 E_t D_{t+2} \right] + \\
& + \left[\left(\frac{1}{1+r} \right)^3 E_{t+1} D_{t+3} - \left(\frac{1}{1+r} \right)^4 E_t D_{t+3} \right] + ...
\end{aligned}
$$

What Drives Changes in Stock Prices?

Change in stock prices is

$$
\begin{aligned}
P_{t+1} - P_t &= -\left(\frac{1}{1+r}\right) D_t + \left[\left(\frac{1}{1+r}\right) D_{t+1} - \left(\frac{1}{1+r}\right)^2 E_t D_{t+1}\right] \\
&+ \left[\left(\frac{1}{1+r}\right)^2 E_{t+1} D_{t+2} - \left(\frac{1}{1+r}\right)^3 E_t D_{t+2}\right] + \\
&+ \left[\left(\frac{1}{1+r}\right)^3 E_{t+1} D_{t+3} - \left(\frac{1}{1+r}\right)^4 E_t D_{t+3}\right] + ...
\end{aligned}
$$

Three reasons why prices change from period P_t to period P_{t+1}.

- P_{t+1} differs from P_t because it does not take into account D_t – this dividend has been paid now and has no influence any longer on the price at time $t+1$.
- P_{t+1} applies a smaller discount rate to future dividends because have moved forward one period in time, e.g. it discounts D_{t+1} by $\left(\frac{1}{1+r}\right)$ instead of $\left(\frac{1}{1+r}\right)^2$.
- People formulate new expectations for the future path of dividends e.g. $E_t D_{t+2}$ is gone and has been replaced by $E_{t+1} D_{t+2}$

Unpredictability of Stock Returns

- In the notes, we shows that the equation on the previous slide can be re-written as

$$r_{t+1} = \frac{D_t + \Delta P_{t+1}}{P_t} = r + \frac{\sum_{k=1}^{\infty} \left(\frac{1}{1+r}\right)^k (E_{t+1} D_{t+k} - E_t D_{t+k})}{P_t}$$

- The rate of return on stocks depends on how people change their minds about what they expect to happen to dividends in the future.
- If we assume that people formulate rational expectations, then the return on stocks should be unpredictable.
- Research in the 1960s and 1970s by Eugene Fama and co-authors found that stock returns did seem to be essentially unpredictable.
- This was considered a victory for the rational expectations approach and many people believed that financial markets were "efficient" meaning they priced on the basis of all relevant information.

A Problem: Excess Volatility

- In an important 1981 paper, Robert Shiller argued that the dividend-discount model cannot explain the *volatility* of stock prices.
- Shiller's argument began by observing that the *ex post* outcome for a variable can be expressed as:

$$X_t = E_{t-1}X_t + \epsilon_t$$

- This means that the variance of X_t can be described by

$$\text{Var}\left(X_t\right) = \text{Var}\left(E_{t-1}X_t\right) + \text{Var}\left(\epsilon_t\right) + 2\text{Cov}\left(E_{t-1}X_t, \epsilon_t\right)$$

- This covariance term—between the "surprise" element ϵ_t and the ex-ante expectation $E_{t-1}X_t$—should equal zero if expectations are rational.
- If there was a correlation—for instance, so that a low value of the expectation tended to imply a high value for the error—and then you could systematically construct a better forecast.

Variance Bounds for Stock Prices

- So, if expectations are rational, then we have

$$\text{Var}\left(X_t\right) = \text{Var}\left(E_{t-1}X_t\right) + \text{Var}\left(\epsilon_t\right)$$

- Provided there is some unpredictability, then the variance of the *ex post* outcome should be higher than the variance of *ex ante* rational expectation

$$\text{Var}\left(X_t\right) > \text{Var}\left(E_{t-1}X_t\right)$$

- Stock prices are an *ex ante* expectation of a discount sum of future dividends. Shiller's observation was that rational expectations should imply

$$\text{Var}(P_t) < \text{Var}\left[\sum_{k=0}^{\infty}\left(\frac{1}{1+r}\right)^{k+1} D_{t+k}\right]$$

- A check on this calculation, using a wide range of possible values for r, reveals that this inequality does not hold: Stocks are actually much more volatile than suggested by realized movements in dividends.

Shiller's 1981 Chart Illustrating Excess Volatility

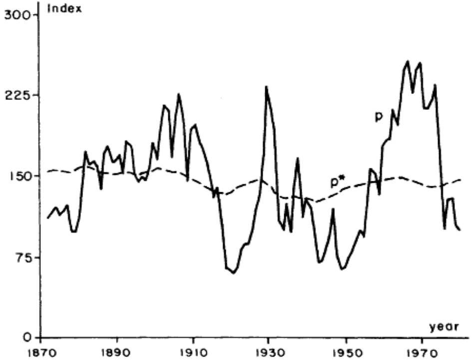
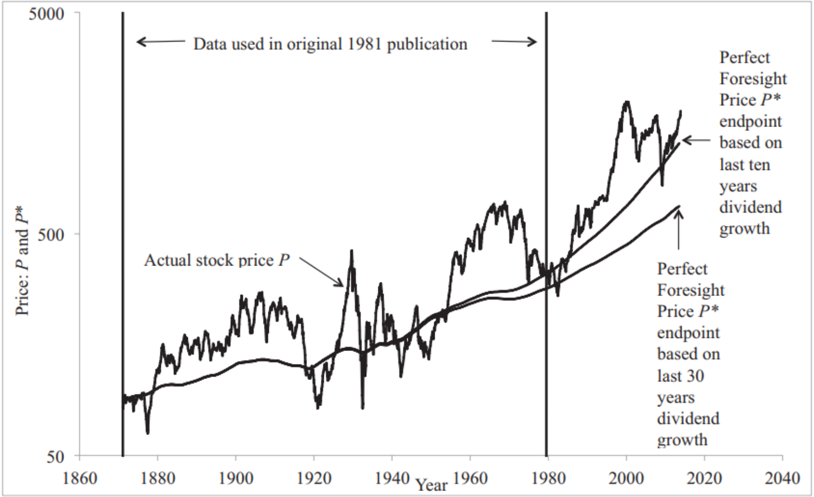


FIGURE 1

Note: Real Standard and Poor's Composite Stock Price Index (solid line p) and *ex post* rational price (dotted line p^*), 1871–1979, both detrended by dividing a long-run exponential growth factor. The variable p^* is the present value of actual subsequent real detrended dividends, subject to an assumption about the present value in 1979 of dividends thereafter. Data are from Data Set 1, Appendix.

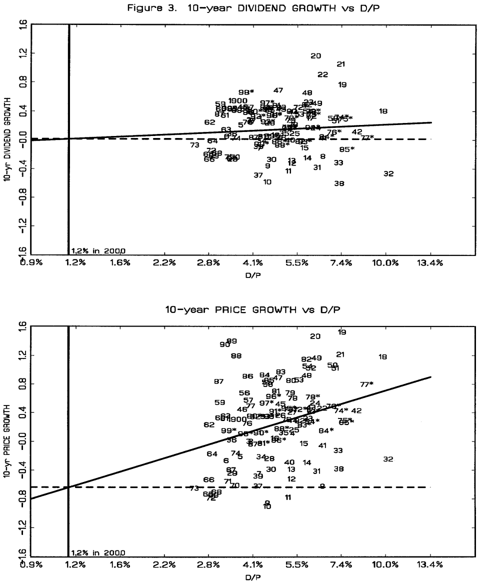
A 2014 Update of Shiller's Chart



Long-Horizon Predictability

- The model predicts that when the ratio of dividends to prices is low, investors are confident about future dividend growth. So a low dividend-price ratio should help to predict higher future dividend growth.
- Shiller's volatility research pointed out, however, that there appears to be a lot of movements in stock prices that never turn out to be fully justified by later changes dividends.
- Later research went a good bit further. For example, Campbell and Shiller (2001) show that over longer periods, dividend-price ratios are of essentially no use at all in forecasting future dividend growth.
- In fact, a high ratio of prices to dividends, instead of forecasting high growth in dividends, tends to forecast lower future returns on the stock market albeit with a relatively low R-squared.

Campbell and Shiller's 2001 Chart



Reconciling the Various Findings

- This last finding seems to contradict Fama's earlier conclusions that it was difficult to forecast stock returns but these results turn out to be compatible with both those findings and the volatility results.
- Fama's classic results on predictability focused on explaining short-run stock returns e.g. can we use data from this year to forecast next month's stock returns?
- But the form of predictability found by Campbell and Shiller (and other studies) related to predicting average returns over multiple years.
- It turns out an inability to find short-run predictability is not the same thing as an inability to find longer-run predictability.
- To understand this, we need to develop some ideas about forecasting time series.

Forecasting Short-Term Changes in Time Series

- Consider a series that follows the following $AR(1)$ time series process:

$$y_t = \rho y_{t-1} + \epsilon_t$$

  where ϵ_t is a random and unpredictable "noise" process with a zero mean.

- If $\rho = 1$ then the change in the series is

$$y_t - y_{t-1} = \epsilon_t$$

  so the changes cannot be predicted.

- Suppose, however, $\rho = 0.99$. The change in the series is now

$$y_t - y_{t-1} = -0.01 y_{t-1} + \epsilon_t$$

- Now suppose you wanted to assess whether you could forecast the change in the series based on last period's value of the series.
- The true coefficient in this relationship is -0.01 with the ϵ_t being the random error. This is so close to zero that you will probably be unable to reject that the true coefficient is zero.

Forecasting Long-Term Changes in Time Series

- What if you were looking at forecasting longer-term changes?
- Another repeated substitution trick.

$$\begin{aligned} y_t &= \rho y_{t-1} + \epsilon_t \\ &= \rho^2 y_{t-2} + \epsilon_t + \rho \epsilon_{t-1} \\ &= \rho^3 y_{t-3} + \epsilon_t + \rho \epsilon_{t-1} + \rho^2 \epsilon_{t-2} \\ &= \rho^N y_{t-N} + \sum_{k=0}^{N-1} \rho^k \epsilon_{t-k} \end{aligned}$$

- Now try to forecast change in y_t over N periods:

$$y_t - y_{t-N} = \left(\rho^N - 1\right) y_{t-N} + \sum_{k=0}^{N-1} \rho^k \epsilon_{t-k}$$

- If $\rho = 0.99$ and $N = 50$ the coefficient is $\left(0.99^{50} - 1\right) = -0.4$. So regressions predicting returns over longer periods find statistically significant evidence even though this evidence cannot be found for predicting returns over shorter periods.

Results from a Simple Simulation

```
Statistics on Series TSTATS_1LAG
Observations              10000
Sample Mean           -1.244998      Variance             0.463179
Standard Error         0.680573      of Sample Mean       0.006806
t-Statistic (Mean=0) -182.933896     Signif Level         0.000000
Skewness              -0.123331      Signif Level (Sk=0)  0.000000
Kurtosis (excess)      0.512791      Signif Level (Ku=0)  0.000000
Jarque-Bera          134.915363      Signif Level (JB=0)  0.000000

Statistics on Series TSTATS_20LAG
Observations              10000
Sample Mean           -5.693510      Variance            11.267177
Standard Error         3.356662      of Sample Mean       0.033567
t-Statistic (Mean=0) -169.618235     Signif Level         0.000000
Skewness              -0.277797      Signif Level (Sk=0)  0.000000
Kurtosis (excess)      0.778124      Signif Level (Ku=0)  0.000000
Jarque-Bera          380.901249      Signif Level (JB=0)  0.000000

Statistics on Series TSTATS_50LAG
Observations              10000
Sample Mean           -8.951905      Variance            31.554735
Standard Error         5.617360      of Sample Mean       0.056174
t-Statistic (Mean=0) -159.361419     Signif Level         0.000000
Skewness              -0.338830      Signif Level (Sk=0)  0.000000
Kurtosis (excess)      0.926286      Signif Level (Ku=0)  0.000000
Jarque-Bera          548.845975      Signif Level (JB=0)  0.000000
```

A Model to Explain the Findings?

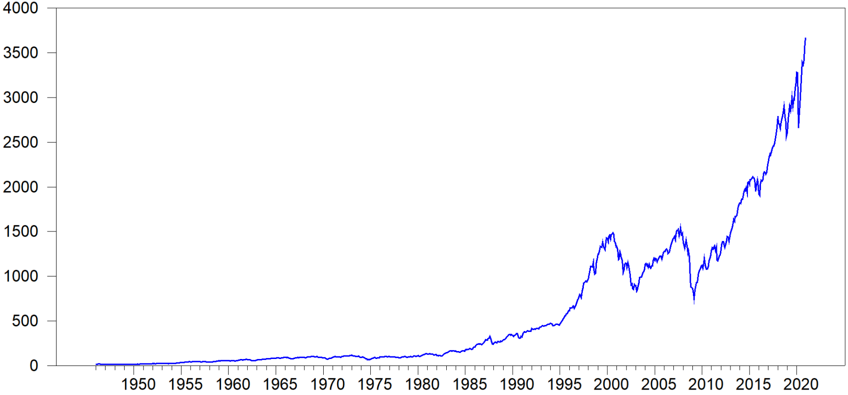
- Pulling these ideas together to explain the various stock price results, suppose prices were given by

$$P_t = \sum_{k=0}^{\infty} \left[\left(\frac{1}{1+r} \right)^{k+1} E_t D_{t+k} \right] + u_t$$

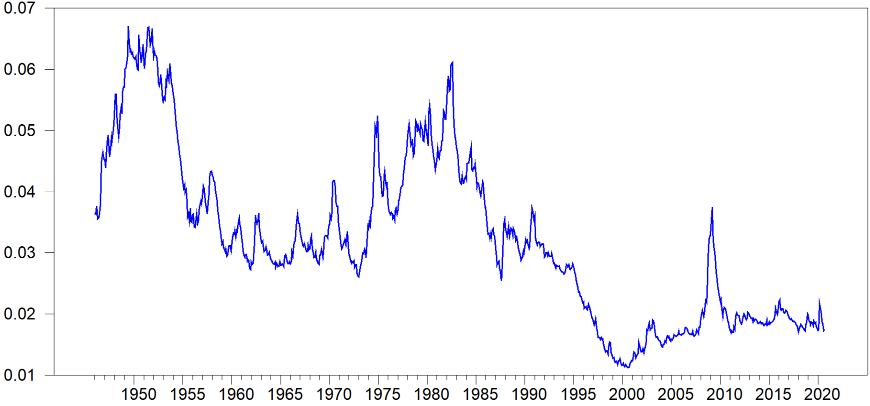
  where $u_t = \rho u_{t-1} + \epsilon_t$ with ρ being close to one and ϵ_t being an unpredictable noise series.

- In this case, statistical research would generate three results:
  1. Short-term stock returns would be very hard to forecast. This is partly because of the rational dividend-discount element but also because changes in the non-fundamental element are hard to forecast over short-horizons.
  2. Longer-term stock returns would have a statistically significant forecastable element, though with a relatively low R-squared.
  3. Stock prices would be more volatile than predicted by the dividend-discount model, perhaps significantly.

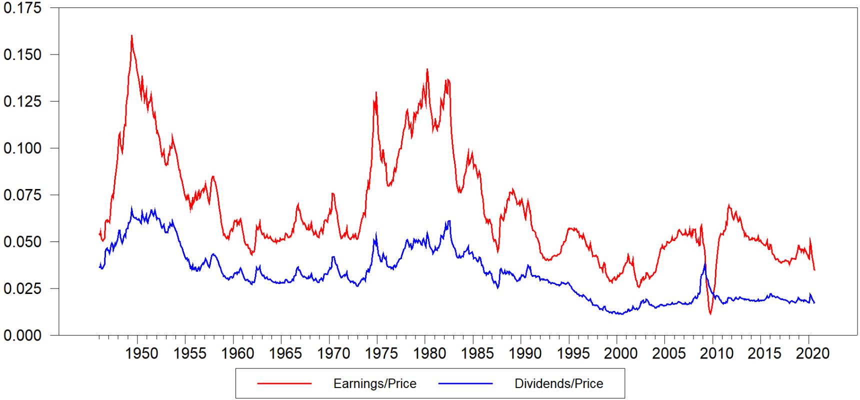
Example: Are U.S. Stock Prices Over-Valued? The S&P 500 Index



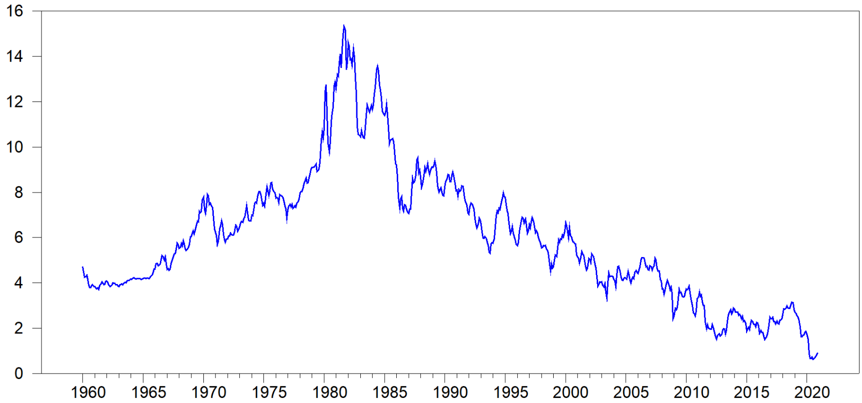
S&P 500 Dividend-Price Ratio



Dividend-Price and Earnings-Price Ratios



Real 10-Year Treasury Bond Rate



Incorporating Time-Varying Expected Returns

- Perhaps the problem with the model is its assumption that expected returns are constant. Can re-formulate the model with expected returns varying over time. Define

$$R_t = 1 + r_t$$

- Start again from the first-order difference equation for stock prices

$$P_t = \frac{D_t}{R_{t+1}} + \frac{P_{t+1}}{R_{t+1}}$$

- This implies

$$P_{t+1} = \frac{D_{t+1}}{R_{t+2}} + \frac{P_{t+2}}{R_{t+2}}$$

- Substitute this into the original price equation to get

$$\begin{aligned} P_t &= \frac{D_t}{R_{t+1}} + \frac{1}{R_{t+1}} \left(\frac{D_{t+1}}{R_{t+2}} + \frac{P_{t+2}}{R_{t+2}} \right) \\ &= \frac{D_t}{R_{t+1}} + \frac{D_{t+1}}{R_{t+1} R_{t+2}} + \frac{P_{t+2}}{R_{t+1} R_{t+2}} \end{aligned}$$

Solution with Time-Varying Expected Returns

- Applying the same trick to substitute for P_{t+2} we get

$$P_t = \frac{D_t}{R_{t+1}} + \frac{D_{t+1}}{R_{t+1}R_{t+2}} + \frac{D_{t+2}}{R_{t+1}R_{t+2}R_{t+3}} + \frac{P_{t+3}}{R_{t+1}R_{t+2}R_{t+3}}$$

- The general formula is

$$P_t = \sum_{k=0}^{N-1} \left(\frac{D_{t+k}}{\prod_{m=1}^{k+1} R_{t+m}} \right) + \frac{P_{t+N}}{\prod_{m=1}^{N} R_{t+m}}$$

where $\prod_{n=1}^{h} x_i$ means the product of x_1, x_2 .... x_h.

- Setting final term to term to zero as $N \rightarrow \infty$ and taking expectations

$$P_t = \sum_{k=0}^{\infty} E_t \left(\frac{D_{t+k}}{\prod_{m=1}^{k+1} R_{t+m}} \right)$$

Solution with Time-Varying Expected Returns

- Applying the same trick to substitute for P_{t+2} we get

$$P_t = \frac{D_t}{R_{t+1}} + \frac{D_{t+1}}{R_{t+1}R_{t+2}} + \frac{D_{t+2}}{R_{t+1}R_{t+2}R_{t+3}} + \frac{P_{t+3}}{R_{t+1}R_{t+2}R_{t+3}}$$

- The general formula is

$$P_t = \sum_{k=0}^{N-1} \left(\frac{D_{t+k}}{\prod_{m=1}^{k+1} R_{t+m}} \right) + \frac{P_{t+N}}{\prod_{m=1}^{N} R_{t+m}}$$

where $\prod_{n=1}^{h} x_i$ means the product of x_1, x_2 .... x_h.

- Setting final term to term to zero as $N \to \infty$ and taking expectations

$$P_t = \sum_{k=0}^{\infty} E_t \left(\frac{D_{t+k}}{\prod_{m=1}^{k+1} R_{t+m}} \right)$$

Time-Varying Expected Returns as an Explanation?

- This approach gives one potential explanation for the failure of news about dividends to explain stock price fluctuations—perhaps it is news about future stock returns that explains movements in stock prices.
- One way to think about expected returns on stocks is to break them into the return on low risk bonds and the premium for holding risky assets

$$E_t r_{t+1} = E_t i_{t+1} + \pi$$

  In other words, next period's expected return on the market needs to equal next period's expected interest rate on bonds, i_{t+1}, plus a risk premium, π, which we will assume is constant.
- Perhaps surprisingly, research has generally found that accounting for fluctuations in interest rates does little to explain movements in stock prices or resolve the various puzzles that have been documented.

Time-Varying Risk Premia?

- A final possibility is that that changes in expected returns do account for the bulk of stock market movements, but that the principal source of these changes comes, not from interest rates, but from changes in the risk premium that determines the excess return that stocks must generate relative to bonds (the π_t above) .
- In favour of this conclusion: Market commentary often discusses fluctuations in prices of stocks and other financial products in terms of investors having a "risk on" or "risk off" attitude.
- A problem with this conclusion is that it implies that, most of the time, when stocks are increasing it is because investors are anticipating lower stock returns at a later date.
- However, the evidence points in the other direction. For example, surveys have shown that even at the peak of the most recent bull market, average investors still anticipate high future returns on the market.

Behavioural Finance

- The last possibility is that people to act in a manner inconsistent with pure rational expectations.
- Indeed, inability to explain aggregate stock prices is not the only failure of modern financial economics, e.g. failure to explain why the average return on stocks exceeds that on bonds by so much, or discrepancies in the long-run performance of small- and large-capitalisation stocks.
- For many, the answers to these questions lie in abandoning the pure rational expectations, optimising approach.
- **Behavioural finance** is booming, with various researchers proposing all sorts of different non-optimising models of what determines asset prices.
- But there is no clear front-runner "alternative" behavioural-finance model of the determination of aggregate stock prices.
- And don't underestimate the rational expectations model as a benchmark. Asset prices probably eventually return towards the "fundamental" level implied by rational expectations.

Things to Understand From This Topic

1. The meaning of rational expectations, as used by economists.
2. The repeated substitution method for solving first-order stochastic difference equations.
3. How to derive the dividend-discount formula.
4. How to derive the Gordon growth formula and the variant with cyclical dividends.
5. The dividend-discount model's predictions on predictability.
6. Fama's evidence and the meaning of efficient markerts.
7. The logic behind Robert Shiller's test of the dividend-discount model and his findings.
8. How the dividend-price ratio forecasts longer-horizon stock returns.
9. Why stock returns may be predictable over long horizons but not a short.
10. How to incorporate time-varying expected returns, interest rates and risk premia.
11. The state of debate about rational expectations and asset pricing.