Advanced Macroeconomics
8. Growth Accounting

Karl Whelan

School of Economics, UCD

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Growth Accounting

- The final part of this course will focus on "growth theory."
- This branch of macroeconomics concerns itself with what happens over long periods of time.
- We will look at the following topics:
    1. What determines the growth rate of the economy over the long run and what can policy measures do to affect it?
    2. What makes some countries rich and others poor?
    3. How economies behaved prior to the modern era of economic growth.
    4. The tensions between economic growth and environmental sustainability.
- We will begin by covering "growth accounting" – a technique for explaining the factors that determine growth.

Production Functions

- We assume output is determined by an aggregate production function technology depending on the total amount of labour and capital.
- For example, consider the Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{\beta}$$

  where K_t is capital input and L_t is labour input.
- An increase in A_t results in higher output without having to raise inputs. Macroeconomists usually call increases in A_t "technological progress" and often refer to this as the "technology" term.
- A_t is simply a measure of productive efficiency and it may go up or down for all sorts of reasons, e.g. with the imposition or elimination of government regulations.
- Because an increase in A_t increases the productiveness of the other factors, it is also sometimes known as **Total Factor Productivity** (TFP).

Productivity Growth

- Output per worker is often labelled *productivity* by economists with increases in output per worker called *productivity growth*.
- Productivity with Cobb-Douglas is

$$\frac{Y_t}{L_t} = A_t K_t^{\alpha} L_t^{\beta-1} = A_t \left(\frac{K_t}{L_t}\right)^{\alpha} L_t^{\alpha+\beta-1}$$

- There are three potential ways to increase productivity:
  1. Technological progress: Improving the efficiency with which an economy uses its inputs, i.e. increases in A_t.
  2. Capital deepening (i.e. increases in capital per worker)
  3. Increases in the number of workers:
     - Only adds to growth if $\alpha + \beta > 1$, i.e. increasing returns to scale.
     - Most growth theories assume constant returns to scale: A doubling of inputs produces a doubling of outputs. Under CRS, $\alpha + \beta - 1 = 0$ and productivity is

$$\frac{Y_t}{L_t} = A_t \left(\frac{K_t}{L_t}\right)^{\alpha}$$

Determinants of Growth

- Let's consider what determines growth with a constant returns to scale Cobb-Douglas production function (so $\beta = 1 - \alpha$)

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

  ans assume that time is continuous: t evolves smoothly instead of just taking integer values like $t = 1$ and $t = 2$.

- Denote the growth rate of Y_t by G_t^Y. This can be defined as

$$G_t^Y = \frac{1}{Y_t} \frac{dY_t}{dt}$$

- Can characterise as a function of G_t^Y the growth rates of labour, capital and technology by differentiating production function with respect to time.
- Recall product rule of differentiation implies

$$\frac{dABC}{dx} = BC \frac{dA}{dx} + AC \frac{dB}{dx} + AB \frac{dC}{dx}$$

The Key Equation of Growth Accounting

- In our case, we have

$$\begin{aligned}\frac{dY_t}{dt} &= \frac{dA_t K_t^{\alpha} L_t^{1-\alpha}}{dt} \\ &= K_t^{\alpha} L_t^{1-\alpha} \frac{dA_t}{dt} + A_t L_t^{1-\alpha} \frac{dK_t^{\alpha}}{dt} + A_t K_t^{\alpha} \frac{dL_t^{1-\alpha}}{dt} \\ &= K_t^{\alpha} L_t^{1-\alpha} \frac{dA_t}{dt} + \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} \frac{dK_t}{dt} + (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha} \frac{dL_t}{dt}\end{aligned}$$

- Dividing across by $A_t K_t^{\alpha} L_t^{1-\alpha}$, this becomes

$$G_t^Y = G_t^A + \alpha G_t^K + (1-\alpha) G_t^L$$

- The growth rate of output equals the growth rate of the technology term plus a weighted average of capital growth and labour growth, where the weight is determined by the parameter α.
- This is the key equation in growth accounting studies. These studies provide estimates of how much GDP growth over a certain period comes from growth in the number of workers, how much comes from growth in the stock of capital and how much comes from improvements in TFP.

How to Calculate the Sources of Growth: Solow (1957)

- For most economies, we can calculate GDP, number of workers and get some estimate of the stock of capital. We don't directly observe the value of the Total Factor Productivity term, A_t.
- However, if we knew the value of the parameter α, we could figure out the growth rate of TFP:

$$G_t^A = G_t^Y - \alpha G_t^K - (1-\alpha) G_t^L$$

- In a famous 1957 paper, Robert Solow pointed out that we could arrive at an estimate of α by looking at the shares of GDP paid to workers and to capital.

Solow (1957) Continued

- Consider the case of a perfectly competitive firm that is seeking to maximise profits.
- Suppose the firm sells its product for a price P_t, pays wages of W_t and rents its capital for a rate of R_t.
- This firm's profits are given by

$$
\begin{aligned}
\Pi_t &= P_t Y_t - R_t K_t - W_t L_t \\
&= P_t A_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - W_t L_t
\end{aligned}
$$

- Now consider how the firm chooses how much capital and labour to use. It will maximise profits by differentiating the profit function with respect to capital and labour and setting the resulting derivatives equal to zero. This gives two conditions

$$
\begin{aligned}
\frac{\partial \Pi_t}{\partial K_t} &= \alpha P_t A_t K_t^{\alpha-1} L_t^{1-\alpha} - R_t = 0 \\
\frac{\partial \Pi_t}{\partial L_t} &= (1-\alpha) P_t A_t K_t^{\alpha} L_t^{-\alpha} - W_t = 0
\end{aligned}
$$

Estimating α

- These can be simplified to read

$$\begin{aligned} \frac{\partial \Pi_t}{\partial K_t} &= \alpha \frac{P_t Y_t}{K_t} - R_t = 0 \\ \frac{\partial \Pi_t}{\partial L_t} &= (1-\alpha) \frac{P_t Y_t}{L_t} - W_t = 0 \end{aligned}$$

- Solving these we get

$$\begin{aligned} \alpha &= \frac{R_t K_t}{P_t Y_t} \\ 1-\alpha &= \frac{W_t L_t}{P_t Y_t} \end{aligned}$$

- $P_t Y_t$ is total nominal GDP.
- $W_t L_t$ is the total amount of income paid out as wages.
- $R_t K_t$ is the total amount of income paid to capital.
- These equations tell us that we can calculate $1-\alpha$ as the fraction of income paid to workers rather than to compensate capital.

Solow's Findings

- In most countries, national income accounts show that wage income accounts for most of GDP, meaning $\alpha < 0.5$.
- A standard value that gets used in many studies, based on US estimates, is $\alpha = \frac{1}{3}$.
- However, note that some studies do this calculation assuming firms are imperfectly competitive – if this is the case, then the shares of income earned by labour and capital depend on the degree of monopoly power.
- Solow's 1957 paper concluded that capital deepening had not been that important for U.S. growth.
- In fact, he calculated that TFP growth accounted for 87.5% of growth in output per worker over that period.
- TFP is sometimes called "the Solow residual" because it is a "backed out" calculation that makes things add up.

BLS Multifactor Productivity Figures

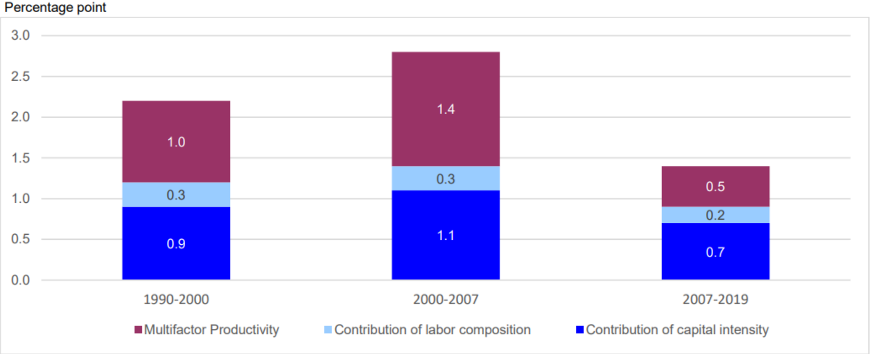
- Most growth accounting calculations are done as part of academic studies. However, in some countries the official statistical agencies produce growth accounting calculations.
- In the U.S. the Bureau of Labor Statistics (BLS) produces them under the name "multifactor productivity" calculations.
- The BLS add some additional factors, for example account for improvements in the "quality" of the labour force (educational qualifications and work experience of employees). In other words, they view the production function as being of the form

$$Y_t = A_t K_t^{\alpha} \left(q_t L_t\right)^{1-\alpha}$$

  where q_t is a measure of the "quality" of the labor input.
- The next slide shows a summary of the BLS's calculations of the sources of growth in the US from 1987 to 2018.

Growth Accounting Calculations for the U.S.

Chart 3. Contributions to labor productivity growth, private nonfarm business sector, selected time periods

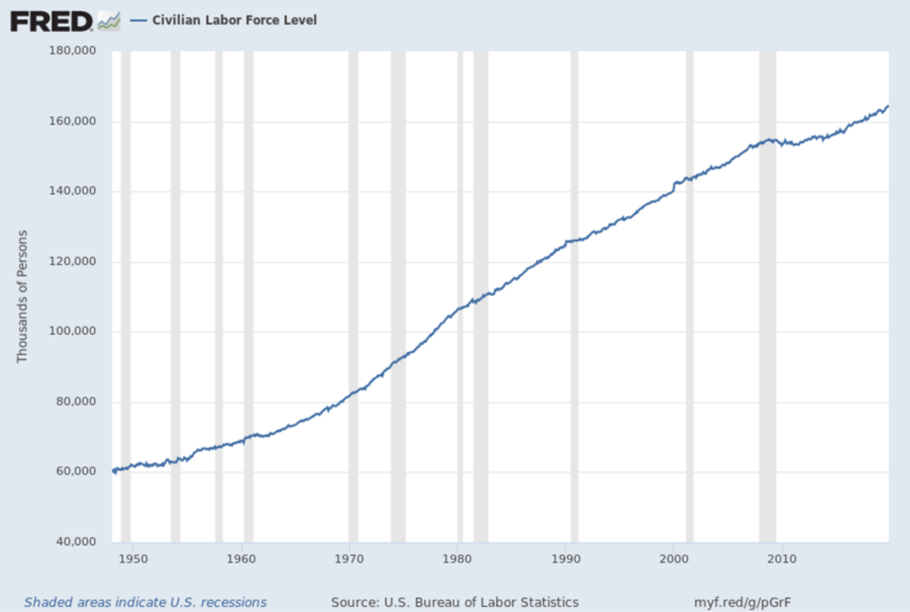


Note: Multifactor productivity plus the contributions of capital intensity and labor composition may not sum to labor productivity due to independent rounding.

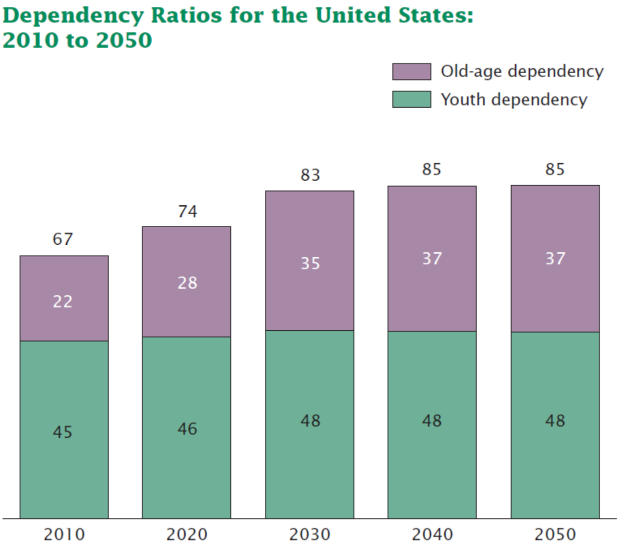
Weakening Prospects for Long-Run Growth?

- The BLS calculations show US productivity growth is weakening.
- Another factor that is weighing on the potential for output growth is a slow growth rate of the labour force.
- After years of increasing numbers of people available for work due to normal population growth, immigration and increased female labour participation, the US labour force has grown slower over the past decade.
- This is being driven by long-run demographic trends as the large "baby boom" generation starts to retire.
- This trend is set to continue over the next few decades.
- The dependency ratio (the ratio of non-working to working people) is projected to increase significantly as the populations grows older on average.
- Demographic and productivity patterns are even worse in Europe. See the chart and table from my paper with Kieran McQuinn.

The U.S. Labour Force



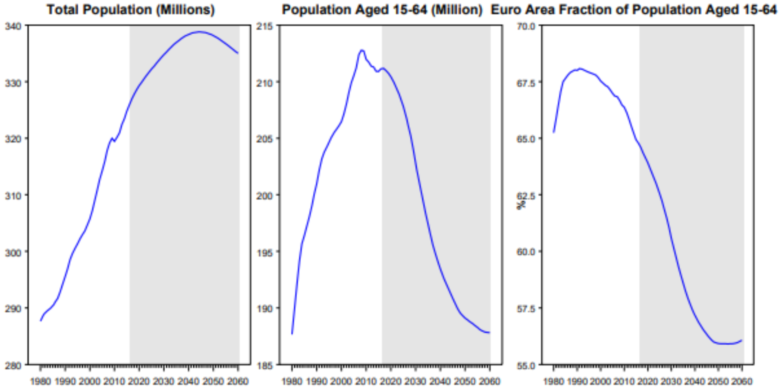
The Ratio of Non-Working to Working People in U.S.



Growth Accounting for the Euro Area

Period	Euro Area			
	$\triangle y$	$\triangle a$	$\triangle k$	$\triangle l$
1970-1976	3.6	2.7	1.5	-0.6
1977-1986	2.1	1.6	0.8	-0.4
1987-1996	2.3	1.5	0.8	0.0
1997-2006	2.2	0.7	0.8	0.7
2007-2016	0.3	0.1	0.5	-0.2

Demographic Projections for the Euro Area



Example: A Tale of Two Cities

- Alwyn Young's 1992 paper "A Tale of Two Cities: Factor Accumulation and Technical Change in Hong Kong and Singapore" is an interesting example of a growth accounting study.
- Both economies were successful: Hong Kong had total growth of 147% between the early 1970s and 1990 and Singapore had growth of 154%.
- But Young was interested in exploring the extent to which TFP contributed to growth in these two economies.
- He found that Singapore's approach (capital deepening and forced saving) did not produce any TFP growth while Hong Kong's more free market approach lead to strong TFP growth.
- Hong Kong achieved the growth without having to divert a huge part of national income towards investment rather than consumption.
- As we will see in the next lecture, TFP-based growth has another advantage over growth based on capital accumulation because it is more sustainable.

Table from Alwyn Young's 1992 Paper

Table 5 CRUDE ESTIMATE OF TOTAL FACTOR PRODUCTIVITY GROWTH

Time period	*Growth of*			*Average capital share*	*Percentage contribution of*		
	Output	*Labor*	*Capital*		*Labor*	*Capital*	*TFP Δ*
Hong Kong							
71–76	0.406	0.165	0.447	0.330	0.27	0.36	0.36
76–81	0.512	0.253	0.527	0.386	0.30	0.40	0.30
81–86	0.294	0.095	0.388	0.421	0.19	0.55	0.26
86–90	0.260	0.036	0.237	0.414	0.08	0.38	0.54
71–90	1.472	0.549	1.599	0.384	0.23	0.42	0.35
Singapore							
70–75	0.454	0.247	1.005	0.553	0.24	1.22	−0.47
75–80	0.408	0.256	0.503	0.548	0.28	0.68	0.04
80–85	0.300	0.069	0.620	0.491	0.12	1.01	−0.13
85–90	0.383	0.252	0.273	0.468	0.35	0.33	0.31
70–90	1.545	0.825	2.402	0.533	0.25	0.83	−0.08

Things to Understand from this Topic

- The sources of growth in output per worker.
- How to derive the growth rate of output under constant returns as a function of the growth rates of capital, labour and TFP.
- Solow's method for calculating TFP growth.
- Evidence from the BLS on US productivity growth.
- Evidence on growth in Europe.
- Young's Tale of Two Cities.