Agreeing to Disagree: The Economics of Betting Exchanges

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Abstract

Betting exchanges match people to take opposite sides of a bet. We present a model of a betting exchange in which participants disagree about outcome probabilities but are, on average, correct. Traders maximize subjective expected profits and equilibrium emerges from a simple matching process. The model predicts those who post quotes (Makers) will earn higher returns than those who accept them (Takers) and that loss rates for Takers will rise as the probability of their accepted bet winning falls. Using a large sample of bets on soccer from Betfair Exchange, we implement a transaction-level empirical strategy that identifies Maker and Taker sides of each trade. We show that pre-match and early in-play behavior aligns closely with the model's predictions. However, as matches progress, behavior shifts: longshot bets generate large, systematic losses even for liquidity providers, and profits emerge for those who accept offers on favorites.

Keywords: Betting Exchanges, Bid-Ask Spreads, Betfair, Matching, Thick and Thin Markets

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1. Introduction

Over twenty years ago, Steve Levitt (2004) asked why gambling markets are organized so differently from financial markets. His puzzle was why betting revolved around bookmakers who set odds, rather than participants posting and matching offers the way traders do in stock or futures markets. As it happened, just as Levitt was posing this question, an alternative model was emerging: betting exchanges where every wager is matched between a backer and a "layer" (someone betting against an outcome) with participants able to either post or accept limit orders. In this paper, I develop the first formal model of such an exchange and evaluate its predictions using unique transaction-level data from Betfair, the world's largest platform of this kind.

Betting exchanges provide a natural laboratory for questions central to information economics and market microstructure. Their transparency and decentralized design allow analysis of order posting versus order acceptance, the role of heterogeneous beliefs and the formation of prices in real time. Yet despite having been around for more than twenty years, the rich data they generate have been underutilized by researchers. The Betfair data used here record the full order book at one-second intervals, letting us identify which side of each trade was taken by the Maker (the participant who acts like a market maker by posting a limit offer to be matched) and which by the Taker (the participant who accepts the limit offer). This clarity is rarely available in financial market datasets, and allows us to test sharp predictions from our model about the behavior of Makers and Takers.

Betfair shares many features with prediction markets such as the Iowa Electronic Markets which have long been studied by economists (Forsythe et al. 1992; Berg, Nelson and Rietz 2008). However, in addition to providing order book information, Betfair has a different market microstructure to IEM—trades happen via decentralized filling of limit orders—and also levies a commission on winnings. The theoretical contribution of the paper is to develop a model that incorporates these details and show how participants with heterogeneous subjective probabilities behave in this environment.

The central asymmetry on a betting exchange is between Makers and Takers. At any point in time, there are two sets of odds available for an outcome: one if you act as a Taker and accept an existing offer, and another if you act as a Maker and post your own offer, waiting to be matched. For example, you could back an outcome by clicking to accept the best available back odds already on the screen, or you could instead post your own offer and invite someone else to take the lay side of that bet. The first option guarantees execution but at a less favorable price; the second may yield better odds but carries the risk of not being matched.

Participants therefore face a menu of five possible actions: (i) accept an existing back offer, (ii) post a new back offer at better odds, (iii) abstain from betting, (iv) accept an existing lay offer, or (v) post a new lay offer at better odds. The logic of the model is that those with the strongest subjective beliefs are willing to act as Takers to ensure their bet is placed, while those with somewhat weaker beliefs prefer to post better odds as Makers and take the risk of not being matched. Participants with beliefs close to the population average may find no bet attractive and abstain altogether. In the model calibration, I follow the "wisdom of crowds" assumption that this average belief corresponds to the true probability, though that is not required for the structure of the model.

Equilibrium odds emerge from the interaction of these choices: the share of participants selecting each of the five actions must be consistent with the matching probabilities that make those choices worthwhile. Despite its parsimonious structure, the model generates a rich set of implications, including the possibility of multiple equilibria. In thick-market equilibria, match rates are high, bid–ask spreads are narrow and more offers are attractive enough for Takers to accept. In thin-market equilibria, low match rates lead Makers to demand wider spreads to justify posting, producing more execution risk and less liquidity.

As you would expect, Takers' willingness to accept lower odds means that they underperform Makers. But the model makes more specific predictions. The marginal Taker is more over-optimistic relative to their true win probability than the marginal Maker. This means Takers bet at odds that imply a proportional disadvantage. Because disagreement is specified as a fixed gap in beliefs, this proportional disadvantage implies Taker losses grow nonlinearly as their objective win probability falls, while Makers earn their highest returns when betting on longshots, where even a modest mispricing on the other side of the bet gives them a large proportional edge.

The empirical contribution of the paper is to exploit Betfair's unusually rich transaction-level data. Our dataset covers over 200,000 soccer matches between 2022 and 2024, recording the full order book at one-second intervals and the complete set of matched trades. By comparing trades to the offers available at the time, we can infer with high confidence which trades were made by Makers and which by Takers. This feature, not usually available in financial datasets, allows us to provide rare large-scale evidence on returns to Makers and Takers in financial transactions.

The data support the main predictions of the model. For bets placed before kick-off or early in the first half, Takers lose more heavily on longshots while Makers perform close to break-even for most bets, as predicted. The losses for Takers increase nonlinearly as the win probability falls, and we also find evidence of positive returns for Makers when laying favorites. During the matches, however, a striking anomaly emerges: Longshot bets perform ever more poorly as the match progresses, with Makers also recording losses. This pattern is consistent with the "Yogi Berra effect" described by Page (2012), whereby traders overestimate the likelihood of late-game comebacks.

The remainder of the paper is structured as follows. Section 2 reviews the relevant literature. Section 3 describes the institutional details of Betfair. Section 4 sets out the model, and Section 5 illustrates its predictions. Section 6 describes the data, while Section 7 presents the empirical results.

2. Previous Literature

This paper builds on previous work on prediction markets as well as a few papers that have analyzed the behavior of odds on Betfair Exchange.

2.1. Prediction Markets

Betting exchanges exist because of the Internet but their first forerunners, prediction markets that matched people up to take either side of wagers on events, predated the Internet. Forsythe et al. (1992) reported that the Iowa Presidential Stock Market, which operated on a computer network at the University of Iowa, yielded predictions of the vote shares in the 1988 US presidential election that outperformed polls. Subsequent research on what became known as the Iowa Electronic Markets (IEM), such as Berg, Nelson and Rietz (2008) and Berg and Rietz (2019), further reported impressive findings on its predictive accuracy. Wolfers and Zitzewitz (2004) reported similarly strong predictive power for several other prediction markets.

The theoretical literature on prediction markets has largely focused on whether it is reasonable to interpret a prediction market price as an unbiased estimate of the public's average belief about the probability of an event happening. Manski (2006) presented a simple model with risk-neutral investors who invested all their wealth in their preferred contract and showed that prediction market prices could deviate substantially from the average belief of the participants. Key contributions by Gjerstad (2004) and Wolfers and Zitzewitz (2006) showed that if agents set the size of their trading positions by maximizing log utility (thus investing according to the well-known Kelly criterion) and the distribution of beliefs about a probability in the population were symmetric, then the price that equated demand and supply on either side of a contract was the average belief of participants. He and Treich (2017) showed that this result does not apply for any other CRRA utility functions.

Modeled on the IEM, these papers all assumed there was a single price set to equate supply and demand from limit orders on both sides of a contract but this is not how prices are set on Betfair. In addition, they did not incorporate commission fees charged by the operator. Two more recent papers that introduce this element are Whelan (2023), which adapted the log utility model and showed the incidence of the commission tends to fall on those who decide to back longshot options and Bürgi, Deng and Whelan (2025), who present a model in which informed traders post offers and uninformed traders accept them to explain prices on the Kalshi prediction market which began trading in 2021. Our model differs from both of these—we keep bet size at one rather than endogenizing it via utility maximization and we treat Makers and Takers symmetrically. We will discuss later why these modeling decisions suit our goal of explaining how the Betfair Exchange works.

2.2. Betfair

Unlike bookmakers, Betfair has always been transparent about how its business operates and has made historical transaction-level data available, albeit users largely have to pay for access. Researchers have found Betfair's odds to be accurate predictors of outcomes. In an early contribution, Smith, Paton and Vaughan Williams (2006) compared bookmakers' odds for 799 races with odds available on Betfair and showed that, unlike the bookmakers' odds, the average odds on Betfair did not display a favorite-longshot bias. Franck, Verbeek and Nüesch (2010) examined 5,478 matches across Europe's highest-profile soccer leagues and found that the implicit forecasts of match outcomes in Betfair's odds were more accurate than those from bookmakers. One partial exception is Angelini, de Angelis and Singleton (2022) who use a sample of pre-play odds from 1,004 English Premier League matches. They conclude that pre-play odds are slightly biased, with the probability of favorites winning being somewhat over-stated. With a much larger sample of over 200,000 matches, we do not find evidence of bias, so this result may have stemmed from using a relatively small or unrepresentative sample.

Betfair provides comprehensive historical data files for each match, including a full recording of all trades that take place both before and during matches. Croxson and Reade (2013) used this data to study 160 matches that had goals just before half-time and analyzed the market's reaction. They found that the reaction of the Betfair odds to the goals was efficient, with no possibility of making profits by reacting to the new information during half-time. In contrast, Angelini, de Angelis and Singleton (2022) report that in-play Betfair odds appear to over-estimate the decisiveness of a first goal when it is scored by a favorite team and under-estimate it when it is scored by a longshot team. We will add to this literature by demonstrating some significant inefficiencies in real-time trading on soccer on Betfair during the second half.

3. How Betting Exchanges Work

Many readers will be familiar with betting websites where you can pick an event, select the outcome you are backing and decide how much to place on it. Relative to these sites, betting exchanges can seem a bit complicated. Here, we will explain how they work, illustrating via Betfair's web interface and the various decisions it allows. Figure 1 reproduces an example of data for a soccer match played between Germany and Italy in March 2025.

The simplest thing that you can do on the Betfair Exchange site is treat it as you would a regular bookmaker's site. You can click on the blue buttons and choose to back Germany at decimal odds of 1.93 (meaning you are risking £1 to potentially get back £1.93, so your profit would be £0.93). You can back Italy at odds of 4.3 and the draw at odds of 3.9. But another option is available that bookmakers generally do not offer. You can choose to place a "lay" bet that will pay off if the event does not happen. Betfair quotes the odds for lay bets from the perspective of the backer. So you can lay Italy at odds of 4.4. This means you will win £1 if Italy lose but will have to pay out £4.4 if Italy win (meaning a loss of £3.4 because the first pound they get back is the one they staked).[1]

Suppose Jim has made a £1 bet backing Italy at 4.3 by clicking on the blue button. And suppose Tom has accepted the offer to take the opposite side of a £1 back bet on Italy at 4.4 by clicking on the pink button. You might imagine that Betfair acts as an intermediary between Jim and Tom, taking £1 from Jim to back Italy and £3.4 from Tom (the exposure on his lay bet). Then, if Italy wins, Betfair could give £4.3 back to Jim, keeping £0.1 in profit.

But this is not how Betfair works. Betfair does not act as an intermediary between those who click on blue to back and those who click on pink to lay. These are separate transactions. Instead, every customer that backs an option is matched up with another customer that has chosen to take the lay bet.

So, to continue with names to be concrete, Betfair has matched Jim up with Alice, who offered the back bet at £4.3. It takes £1 from Jim and £3.3 from Alice and holds the combined funds awaiting the outcome of the game. If Italy wins, Jim gets paid the £4.3 in funds that Betfair has held, making a profit of £3.3. If Italy do not win, Alice gets paid the £4.3, making a profit of £1. Similarly, Betfair has matched up Tom with Mary, who offered the lay bet at £4.4. Betfair takes £1 from Mary and £3.4 from Tom, paying out £4.4 to Mary if Italy win and paying it to Tom if they don't.

This structure means that Betfair does not earn money from the bid-ask spread between lay and back odds. Instead, they earn revenue from taking a percentage commission on winnings. In recent years, this commission rate has typically been 2%.

While not obvious on first viewing of the Betfair site, the way to act as a Maker via the web interface is to click on the side you want but rather than accepting the odds available, you toggle the

[1] Betfair allows you to scale your bet by the size of your liability, so in this case, you could risk £1 if Italy win, in order to win £$\frac{1}{3.4}$ =£ 0.294 if they do not win.

odds button to request a different number. For example, Figure 2 illustrates what happens when you click on the blue button to back Germany. If you want better odds for your back bet, you click on this button and toggle the 1.93 upwards to seek better odds.[2] If you have made the best offer available for Lay bettors, then your new offer will show up on the pink button.

Another difference from traditional betting sites is that bookmakers quote odds but don't indicate any limits on the quantities you can bet.[3] On Betfair, the numbers under the odds indicate how much money Makers are willing to accept from Takers at this price. From this, you can see how supply and demand can automatically drive odds. For example, if someone places a bet of £135 on Italy at 4.3 (a relatively small amount considering total volume thus far is £70,807) then those offers are gone and 4.2 will move up to be the best available back odds in the blue window. With these as the odds available for Takers on Italy, a Maker may now decide to seek 4.3 to back Italy by offering this as a lay bet that would show up in the pink window. The additional demand to back Italy has moved both back and lay odds downwards.

Betfair also displays the "overround" associated with the back and lay odds. This is the sum of the inverses of the decimal odds on each of the options. Hegarty and Whelan (2024) show that if odds were what Thaler and Ziemba (1988) termed strongly efficient, meaning each bet had the same expected payout, then this average payout equals the inverse of the overround. If Betfair's average odds are unbiased predictors of outcomes—and we show later that this is true—then the overround summarizes the expected profits for Makers and the corresponding expected losses for Takers.

In this example, the overround for accepting back offers is 100.7 percent, implying people who accept these offers will have an expected loss rate of 0.7%. The overround for accepting lay offers is 99%, implying the people who make lay offers will have an expected profit of 1% on average. These are typical values on the exchange, with high-profile games tending to have back overrounds of about 101% and lay overrounds typically being about 99%. These figures provide a clear suggestion that Takers who accept the offers in the blue and pink buttons will tend to lose money while Makers will tend to earn profits (at least before factoring in commission). Our model will confirm this intuition but it will also explain why some people choose to be Takers and some choose to be Makers.

[2]In practice, a huge amount of trading on Betfair occurs away from the web interface in the form of automated trading via bots that place orders and bets via Betfair's API.

[3]This is not to say that these limits don't exist. Retail bookmakers impose stake limits but they tend to be personalized based on betting history and may vary across different events.

Figure 1: How Betfair Presents Germany vs Italy

Match Odds						
Going In-Play						
Cash Out						Matched: **£70,807**
Rules						Refresh
Betfair SP						
3 selections	100.7%		**Back all**	**Lay all**		99.0%
Germany	**1.91** £3025	**1.92** £2563	**1.93** £1187	**1.95** £824	**1.96** £531	**1.97** £705
Italy	**4.1** £2154	**4.2** £1530	**4.3** £135	**4.4** £1167	**4.5** £752	**4.6** £2189
The Draw	**3.8** £1179	**3.85** £1232	**3.9** £361	**4** £1209	**4.1** £3475	**4.2** £3230

Figure 2: Betting on Germany

Current Odds Bet			
Back (Bet for)	Odds	Stake	Profit
x **Germany**	1.93 ▲▼		£0.00
			Liability: **£0.00**

4. Model

Here, we present our model and discuss its predictions of multiple equilibria.

4.1. Assumptions

There is an outcome of an event that will either happen or not happen. For clarity, let's say the outcome is Chelsea winning a soccer match. Participants differ in their beliefs about the probability of Chelsea winning, with each person having their own subjective value of this probability p and the cumulative distribution function of beliefs being $F(p)$. The average value of p among the population is q. In simulating the returns to participants later, we will follow Ali (1977) and Hegarty and Whelan (2025) in making the "wisdom of crowds" assumption that q is the true probability, so the public are on average correct in their beliefs, but this assumption is not needed to solve the model.

The agents in the model take actions to maximise their subjective expected profit. They are rational in the sense of Savage's (1954) formulation of expected utility theory in which "*probability measures the confidence that a particular individual has in the truth of a particular proposition.*" Savage noted his assumptions implied "*that the individual concerned is in some way "reasonable" but they do not deny the possibility that two reasonable individuals faced with the same evidence may have different degrees of confidence in the truth of the same proposition.*"

Those wishing to back Chelsea could choose to be a Taker and accept an available offer of D_B that has come from people betting against Chelsea. Similarly, those wishing to bet against Chelsea can accept the available offer from those who want to back Chelsea at odds of D_L. Picking either of these two options guarantees you have a matched bet. Alternatively, someone could choose to back Chelsea by being a Maker and thus seeking odds of D_L with this offer having a probability θ_B of being matched. Similarly, someone could choose to act as a Maker betting against Chelsea by offering odds of D_B with this offer having a probability θ_L of being matched.

Matching on the exchange is randomly assigned so the probability of being matched is the ratio of people willing to accept offers to those seeking to get them accepted (i.e. the ratio of Takers to Makers). The exchange charges a fee rate of τ on winnings. All back bet sizes are normalized to one.

The assumption of an exogenous bet size is common in theoretical models of sports betting (see, for example, Shin, 1991, Ottaviani and Sørensen, 2010 and Hegarty and Whelan, 2025). In contrast, the literature on prediction markets more commonly uses an endogenous bet size, generally set in accordance with the Kelly criterion implied by log utility. We don't use this approach here for a few reasons. The first is realism. The Kelly criterion generally recommends a more aggressive betting strategy than people are comfortable with in the real world, often recommending people stake large fractions of their wealth on bets in which they believe they have a modest edge. In practice, average bet sizes on Betfair are small. Based on the figures quoted earlier from their annual report, the average back bet size in 2024 was about £13.[4] So it seems unsatisfactory to apply a model in which people will bet ever-larger fractions of their wealth on bets as their confidence in their edge increases when there is little evidence that this fits with the market we are modeling. The second issue is technical. The model below, while simple to derive, is highly nonlinear. My attempts to extend it to include endogenous bet size produced a model that I could not get solutions for despite using a range of different nonlinear solver methods.

4.2. Solution

Let's first consider the possibility of placing a bet on Chelsea to win. In deciding whether to back Chelsea, people have three options: accept the odds D_B that have been offered to back Chelsea, seek better odds of D_L while knowing there is a probability $1 - \theta_B$ they will not get matched or choose not to back Chelsea at all. There will be two threshold levels of p that determine which of these three decisions they pick.

Consider first the decision to either accept odds of D_B or seek to back at D_L. The expected profit after commission from accepting will exceed that from seeking if

$$p\left(D_B - 1\right)\left(1 - \tau\right) - \left(1 - p\right) > \theta_B\left[p\left(D_L - 1\right)\left(1 - \tau\right) - \left(1 - p\right)\right] \tag{1}$$

This requires

$$p > \frac{1 - \theta_B}{\left[\left(1 - \tau\right)\left(\left(D_B - 1\right) - \theta_B\left(D_L - 1\right)\right) + 1 - \theta_B\right]} = p_t^B \tag{2}$$

Those with $p > p_t^B$ have strong enough beliefs in Chelsea winning that they are willing to accept the lower odds of D_B in order to definitely get their bet placed. Those with $p < p_t^B$ think the expected return from seeking the better odds of D_L is higher than accepting D_B, even if they will not necessarily be matched.

Those with $p < p_t^B$ also have the option of not seeking to back Chelsea. They will pick this option if they believe that the bet has a negative expected profit even at the higher odds of D_L. In other words, people will not believe backing Chelsea at D_L is profitable if

$$p\left(D_L - 1\right)\left(1 - \tau\right) - \left(1 - p\right) < 0 \tag{3}$$

This is satisfied if

$$p < \frac{1}{\tau + \left(1 - \tau\right)D_L} = p_n^B \tag{4}$$

For most of the parameter values we used in our calculations, these threshold rules generate three

[4] In a study of the *bwin* betting platform, Nelson et al. (2021) reported an average bet size of €18 and a median bet size of €6. This average is similar to the figure I have derived here from Betfair's report. The gap between mean and median also explains why £13 may seem higher than the typical bet we assume most punters place. Nelson et al. report that their average is driven upwards by the presence of a smaller number of bettors that have much larger average bet sizes.

clear regions among those with $p \geq q$. Those with $p \geq p_t^B$ accept the offer of odds at D_B; those with $p_n^B \leq p < p_t^B$ choose to seek to back Chelsea at odds of D_L; those with $p_n^B < p \leq q$ choose to not back Chelsea.

Similar considerations show that someone will prefer to accept the offer to lay Chelsea at D_L rather than to seek to lay them at D_B if

$$(1-p)(1-\tau) - p(D_L - 1) > \theta_L \left[(1-p)(1-\tau) - p(D_B - 1)\right] \tag{5}$$

This is satisfied if

$$p < \frac{(1-\tau)(1-\theta_L)}{D_L - \theta_L D_B + \tau(1-\theta_L)} = p_t^L \tag{6}$$

And the expected profit from seeking to lay Chelsea at D_B is negative if

$$(1-p)(1-\tau) - p(D_B - 1) < 0 \tag{7}$$

This is satisfied if

$$p > \frac{1-\tau}{D_B - \tau} = p_n^L \tag{8}$$

The typical outcome of this set of decisions is that people take one of five actions: Accept an offer to back Chelsea at D_B, seek to back Chelsea at D_L, do nothing, seek to lay Chelsea at D_B, accept an offer to lay Chelsea at D_L.

The model is completed by specifying the matching conditions that determine θ_B and θ_L. The mass of agents who choose to seek odds of D_L to back Chelsea is $F(p_t^B) - F(p_n^B)$ while the mass of agents on the lay side willing to take the other side of this bet is $F(p_t^L)$. Thus, the matching rate for those seeking to back at better odds is given by

$$\theta_B = \frac{F(p_t^B) - F(p_n^B)}{F(p_t^L)}. \tag{9}$$

And by the same logic, the matching condition for those seeking to lay at better odds is

$$\theta_L = \frac{F(p_n^L) - F(p_t^L)}{1 - F(p_t^B)} \tag{10}$$

Two special cases are worth noting. First, if the commission rate is high, it is possible that at some or all of the thresholds, the expected returns on each option are negative and so the optimal decision for people with these threshold beliefs is to do nothing. In that case, the formulas used here for numbers of seekers and acceptors would be wrong. In practice, none of the calibrations of the model shown in this paper have this feature. Second, it is possible for the thresholds to overlap so that $p_n^B > p_n^L$. In this case, there is a fraction of participants with beliefs close to q who act as "market makers" offering both back and lay odds because they believe they can earn money from the spread between the two sets of odds. This behavior does feature in some of our model calibrations.

4.3. Multiple Equilibria

The model can be summarized by six equations: four equations defining the various cut-off thresholds (equations 2, 4, 6, 8) and two equations defining the matching rates (equations 9 and 10). However, the model has eight endogenous variables: the four thresholds, the two odds, and the two matching rates. This implies the possibility of multiple equilibria.

In particular, the model contains both thick and thin market equilibria. One way to frame this is in terms of the "spread" in odds relative to the breakeven odds for betting on Chelsea to win, which is $\frac{1}{q}$:

$$D_B = \frac{1}{q} - x_B \tag{11}$$

$$D_L = \frac{1}{q} + x_L \tag{12}$$

We refer to equilibria as thick when the matching probabilities (θ_B and θ_L) are high, and thin when they are low. In thick equilibria, Makers are more confident that their offers will be matched, so the gap in odds required to justify making an offer is smaller. This allows for narrower spreads. By contrast, in thin market equilibria, the lower likelihood of being matched needs to be offset by more favorable odds for Makers, resulting in wider spreads.

Importantly, thickness in this model is not defined by the number of people willing to participate ex ante, but by the *realized* volume of matched bets. As matching rates increase and bid-ask spreads fall, the range of beliefs for which it is optimal to act as a Maker narrows, leading fewer participants with middling beliefs to submit offers. So, as matching rates rise, the number of people choosing to participate actually falls. However, the lower bid-ask spread associated with the higher matching rate means that some people with beliefs further away from q switch from being Makers to Takers. Since it is the total amount of Takers that determines how many bets are matched, this means that higher matching rates are associated with greater trading volume. In this way, the thicker market equilibria do not reflect greater willingness to participate but rather the market's overall ability to facilitate matched trades.

The coexistence of thick and thin equilibria is a common feature in models involving search and matching, with Diamond (1982) providing a foundational example. Similar forms of multiplicity arise in contexts such as urban economics (Gautier and Teulings, 2009) and marriage markets (Burdett and Coles, 1997). A common feature our model shares with these other examples is that the attractiveness of the decision to search depends on how many others are making similar decisions.

What is unusual here, though, is that the markets with the highest volumes of transactions feature fewer people participating (via choosing to be either Makers or Takers).

In what follows, we treat the matching probabilities θ_B and θ_L as exogenous, and choose values that generate bid-ask spreads similar to those observed on the Betfair exchange. An equilibrium in the model is defined as a set of odds and thresholds such that individual decisions defined by the thresholds are consistent with both the odds and the exogenously-specified match rates.

One clear example of how multiple equilibria may operate in practice comes from the current structure of betting exchange markets. Betfair has about 90% of the volume, with the rest divided among competitors such as Betdaq, Smarkets, and Matchbook. These competitors offer the same kind of service but have much lower volumes, fewer markets, and typically wider bid-ask spreads. Anecdotal evidence suggests that many of the larger providers of liquidity to Betfair value its narrow spreads, broader market selection, high match rates, and the ability to place large bets without materially shifting prices — even though better odds might occasionally be available elsewhere.[5] Betfair also levies a significant charge on highly profitable users — 20% of net profits above £25,000 in a year, rising to 40% above £100,000 — whereas its competitors do not have such charges. That many successful gamblers continue to use Betfair despite these charges underscores the value they place on its higher liquidity and lower execution frictions.

5. Model Solutions

We will now discuss how the model was solved and illustrate the multiple equilibria, showing the properties of equilibria with high and low match rates and outlining how the typical bid-ask spreads on Betfair Exchange suggest it is a market with relatively high match rates.

5.1. Solution Methods

Due to the extreme nonlinearity of the model, even with six equations in six unknowns once we specify exogenous values for θ_B and θ_L, we are not guaranteed either that there is an equilibrium or that any calculated equilibrium is unique. To solve the model, I used two different nonlinear solution techniques in Matlab, `fsolve` and `lsqnonlin`. The details of these methods are described in an appendix. Despite their differing numerical approaches—`fsolve` is a root-finding algorithm while `lsqnonlin` is a norm minimization technique—both methods converged to the same solutions for this model, which increases confidence that the equilibria we are calculating (conditional on specified values for θ_B and θ_L) are unique.

[5] See, for example, the discussion here on a betting discussion board: https://arbusers.com/betfair-vs-smarkets-a-comprehensive-comparison-of-betting-exchanges-t10136

5.2. Illustrating the Multiple Equilibria

We start by illustrating the model's multiple equilibria and the properties of thick versus thin markets. We formulate disagreement among the participants so that the cross-sectional distribution of their beliefs is a Normal distribution $N\left(q, \sigma^2\right)$ centred on the true probability. We set the standard deviation as $\sigma = 0.03$, which implies a relatively small amount of disagreement: 90% of participants have beliefs within 0.05 of q. We choose this value for σ because it combines with the Betfair commission rate of $\tau = 0.02$ to give predictions that most closely match the data. In particular, other values of σ produces unrealistically narrow or wide spreads.

Figure 3 shows the bid-ask spread, $D_L - D_B$ for different values of $\theta = \theta_Y = \theta_N$ in a market with no commission ($\tau = 0$), looking at true probabilities ranging from $q = 0.05$ to $q = 0.95$.[6] As expected, spreads rise as the matching rates fall and the market moves from thick to thin. For each value of the matching rate, spreads rise nonlinearly as q falls. While the model generates values for D_B that are lower than $\frac{1}{q}$ and values for D_L that are higher than $\frac{1}{q}$, the average of these are generally extremely close to $\frac{1}{q}$.

Other differences between the thick and thin market equilibria are illustrated in Figures 4 and 5. Figure 4 shows the fraction of participants that choose to neither accept offers or make them. As noted above, slightly paradoxically, as θ rises so the market becomes "thicker," the fraction of people who decide not to participate increases. In the thin market case of $\theta = 0.6$, everybody decides to participate and, as described above, those with beliefs close to q seek to act like traditional dual-sided market makers, making both back and lay offers, hoping to get matched and make profits off the high bid-ask spread. In contrast, when $\theta = 0.8$, we get larger numbers choosing not to participate, particularly if the true value of q is close to 0.5. Figure 5 shows the total fraction of participants who participate in a matched trade for different values of θ.[7] The lower spreads associated with the higher match rates push additional people into being Takers, so the equilibria with higher match rates generate more trades.

[6]For some of the more extreme values of q available, this will mean some fraction of participants would have subjective probabilities below zero or above one. This doesn't cause us any problems. We use these probabilities to assign decisions to people and it is clear which decisions those with these extreme beliefs will make. They will act the same as those with beliefs close to zero or close to one.

[7]The figures in this chart make an adjustment to not double count those who make both Back and Lay offers.

Figure 3: Bid-ask spreads ($D_L - D_B$) for $\tau = 0$, $\sigma = 0.03$ and for various values of q and of $\theta = \theta_B = \theta_L$.

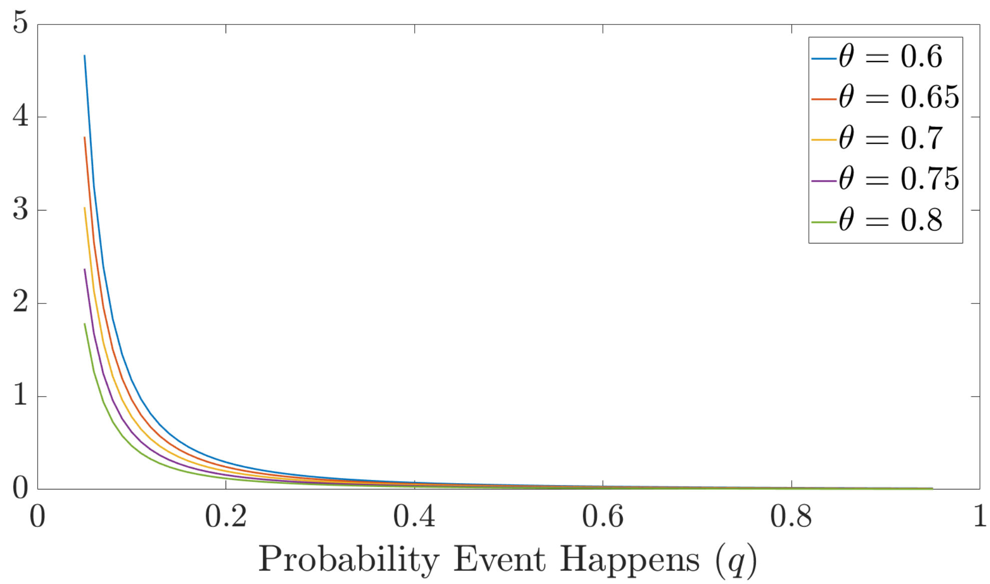


Figure 4: Share who choose to be neither seek or accept offers for $\tau = 0$, $\sigma = 0.03$ and for various values of q and of $\theta = \theta_B = \theta_L$.

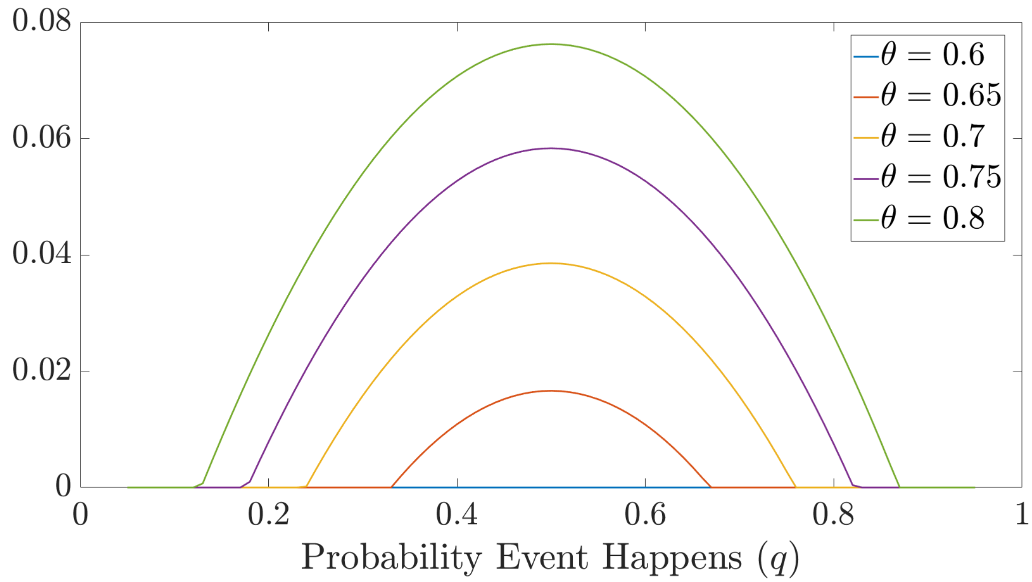
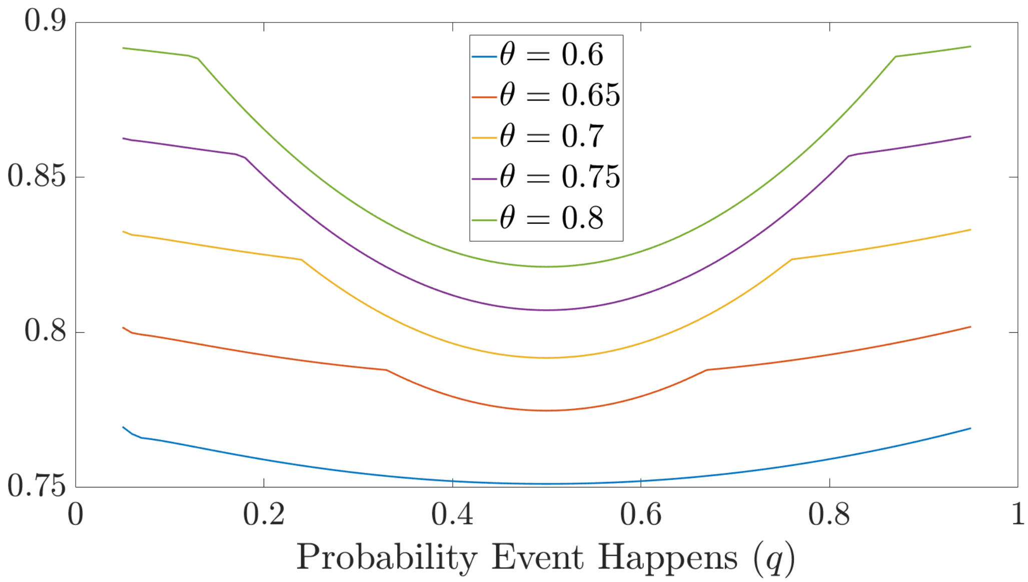


Figure 5: Share of participants who are matched for $\tau = 0$, $\sigma = 0.03$ and for various values of q and of $\theta = \theta_B = \theta_L$.



5.3. Match Rates and Returns

An important feature of the model is that bid-ask spreads rise faster than $1/q$ as q falls. This has an impact on the average rates of return. For example, a Taker who places a one unit back bet on Chelsea at odds of D_B will have an average rate of return of

$$r_{TB} = q\left(D_B - 1\right)\left(1 - \tau\right) - \left(1 - q\right) = q\left[D_B\left(1 - \tau\right) + \tau\right] - 1 \tag{13}$$

so returns are driven by the product qD_B.

The model output shows that as q falls, D_B rises but by not as much. This means Takers' returns on back bets get worse as their bet becomes more a longshot. The same result applies for Takers accepting lay bets on Chelsea at odds of D_L, with returns getting worse as $1 - q$ falls, so laying Chelsea becomes the longshot bet. The opposite pattern applies to Makers. They make money from bid-ask spreads, so Makers will earn higher profits from getting their longshot bets accepted because of the wider spreads associated with these bets.

What are the economic forces underlying these patterns? That Makers have a better average financial outcome than Takers is, of course, true by design. When they get matched, they do so at superior odds to Takers. But the specific patterns of returns relative to the probability of a bet's success are driven by the subtleties of our disagreement model. To see this, define a *breakeven win rate* for a bet as the inverse of its decimal odds. If I take a back bet at odds of D_B, then I need to win a fraction $p_B = \frac{1}{D_B}$ of the time to break even. The term that drives returns for this bet can thus be re-expressed as

$$qD_B = \frac{q}{p_B} \tag{14}$$

so returns are driven by the ratio of the win rate to the breakeven win rate.

The equilibrium odds require just the right fraction of Makers and Takers to all view their position as profitable. All of these participants are too optimistic about their bets but this applies more so to the marginal Taker than it does to the marginal Maker. Thus, Makers' breakeven rates exceed their true win probabilities. And because we have specified the magnitude of disagreement to be independent of the value of q, the gap between breakeven rates and true probabilities is approximately fixed. For example, the breakeven rate for Takers accepting odds of D_B can be accurately approximated as

$$p_B \approx q + \mu \tag{15}$$

where μ is the same for all q. Returns for Takers of back bets are now driven by

$$qD_B \approx \frac{q}{q + \mu} \tag{16}$$

This proportional disadvantage for Takers rises sharply as q falls.

To give a numerical example, suppose $q = 0.1$ and $p_B = 0.11$, so the market generates odds for a back bet that requires an 11% winning rate to break even. Before factoring in commission, this leads to Takers losing about 9% (driven by the ratio of 10 to 11) on average because they only win 10% of the time. In contrast, the Maker on the other side of this contract earns only a modest positive return. Their breakeven rate is 89% and they actually win 90% but this only implies a modest profit of about 1% (driven by the ratio of 90 to 89).

The same logic explains why Makers earn a high return when the Taker's bet is highly likely to succeed. One can easily see how the numbers flip if we suppose $q = 0.9$ and $p_B = 0.91$. The Taker loses about 1% while the Maker has a breakeven rate of 9% but wins 10% of the time, generating a return of about 11%.

Figure 6 illustrates the average rates of return on money placed at risk for Takers (i.e. the average returns across both accepting Back offers and accepting Lay offers) sorted by the probability that the Taker's bet will win. Figure 7 shows the same chart for Makers, also sorted by the probability that the Taker's bet will win. Returns for Takers are negative while returns for Makers are positive. For most probabilities, the deviations in these returns from zero are relatively small but there is a nonlinear pattern: As bets move further into longshot territory, Takers' losses accelerate while Makers' profits rise disproportionately. The figures also show that as the market gets thinner, with falling match rates, loss rates for Takers and profit rates for Makers get bigger.

Figure 6: Post-commission average returns for Takers with $\tau = 0$, $\sigma = 0.03$ and various values of $\theta = \theta_B = \theta_L$.

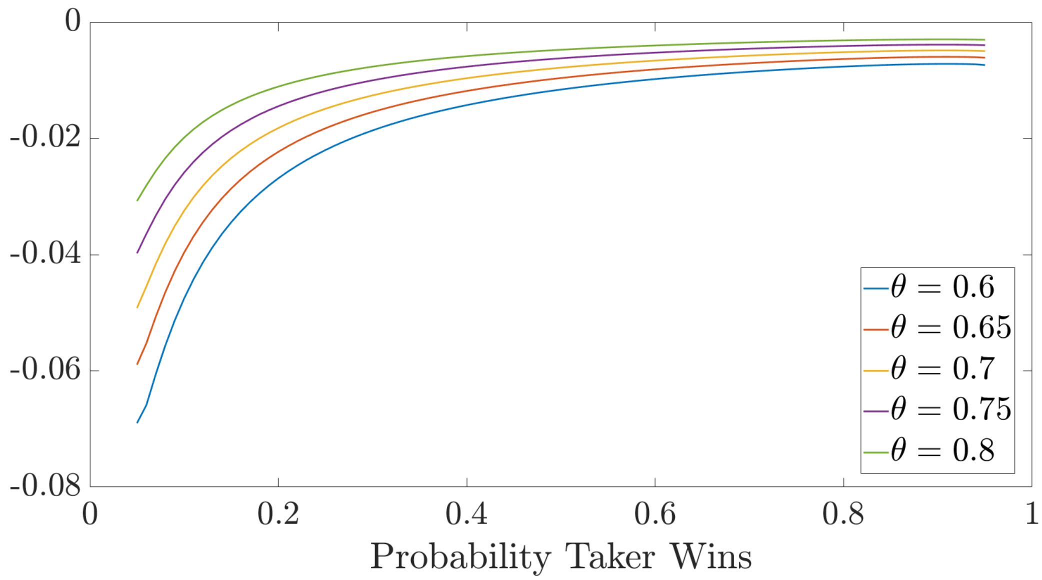
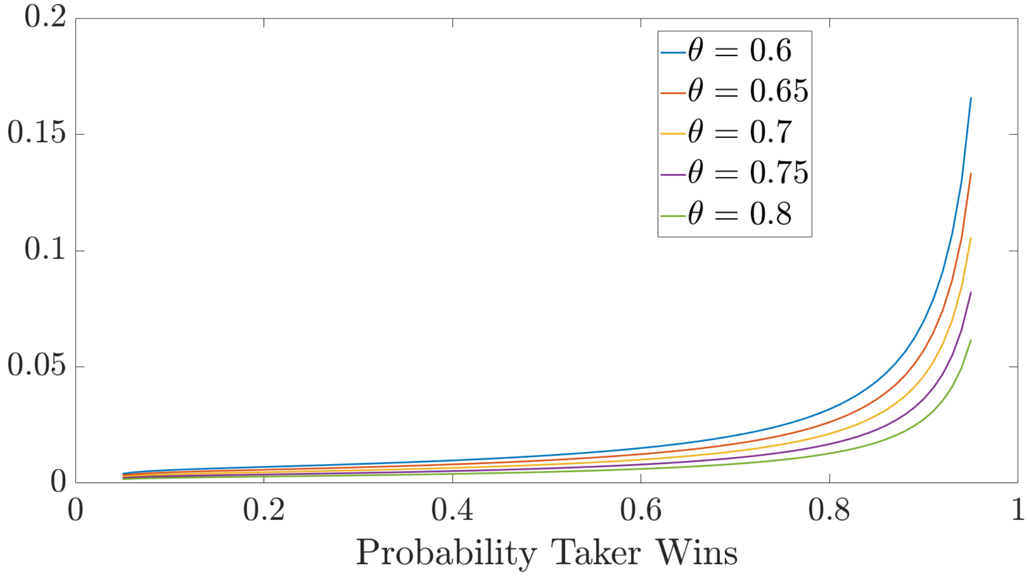


Figure 7: Post-commission average returns for Makers with $\tau = 0$, $\sigma = 0.03$ and various values of $\theta = \theta_B = \theta_L$.



5.4. Bid-Ask Spreads on Betfair

We have so far assumed that the odds can take any value. However, this is not the case in reality. Betfair sets a minimum "tick size", meaning there are minimum increments in the odds, with this tick size varying with the value of the decimal odds. Table 1 shows these minimum tick sizes. These restrictions imply, for example, going back to the illustration in Figure 2 of seeking better odds on Italy when the best available back odds on offer were 4.3, it is not possible to ask for 4.35. If you want better odds, you must ask for at least 4.4. These restrictions promote liquidity and matching rates in the market. By reducing the number of different possible odds that can be sought or accepted, there are more participants at each set of feasible odds.

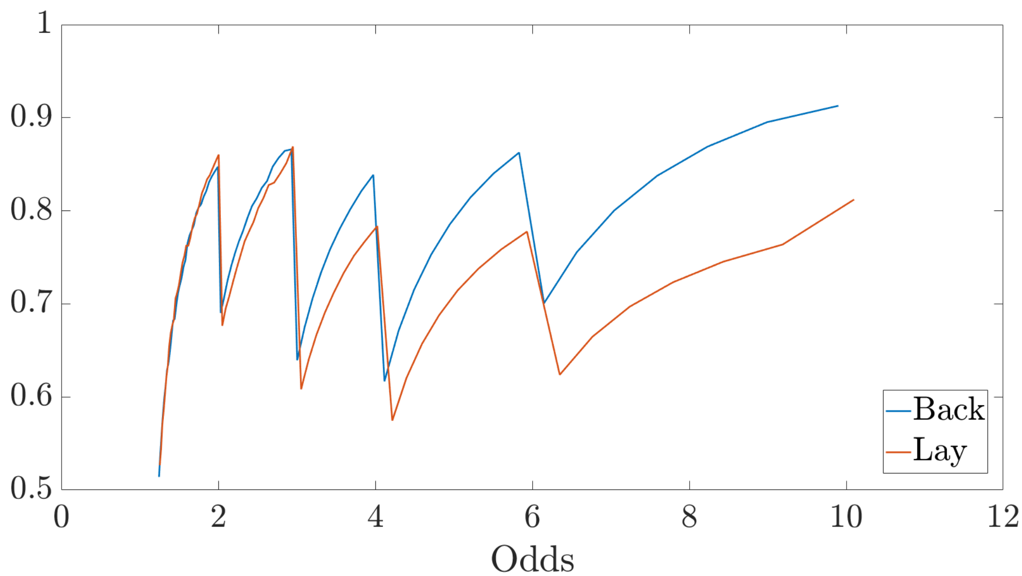
Table 1: Betfair Minimum Tick Sizes by Price Range

Decimal Odds	**Tick Size**
0 to 2.0	0.01
2.0 to 3.0	0.02
3.0 to 4.0	0.05
4.0 to 6.0	0.10
6.0 to 10	0.20
10 to 20	0.50
20 to 30	1.00
30 to 50	2.00
50 to 100	5.00
100+	10.00

In practice, in most active Betfair markets, the gaps between back and lay odds are either equal to the minimum tick size or slightly higher: Note this pattern for the Germany v Italy match in Figure 1. These empirical spread patterns can help us figure out the match rates that would be consistent with our model. Specifically, we can solve the model with the spreads x_B and x_L set equal to half of the minimum tick size, so the odds are exogenous and the match rates become the endogenous variables.

Figure 8 shows the resulting match rates for back and lay offers plotted against the odds with the commission rate set equal to Betfair's standard rate in recent years of $\tau = 0.02$. The match rates show a pattern of rising as the odds increase (because the spreads get larger relative to the true probabilities) and then ratcheting downwards when the minimum tick sizes increase. Overall, the results show that Betfair has set minimum tick sizes consistent with match rates being relatively high, with the average for both back and lay match rates being about 75%.[8] We will use this value now to illustrate the impact of the commission rates and to generate predictions that can be checked with data.

Figure 8: Equilibrium match rates with $\tau = 0.02$, $\sigma = 0.03$ and spreads set by Betfair's minimum tick size



[8] I have in mind that this is a realistic matching rate for those offers displayed as best available to back and best available to lay, such as the 4.3 back odds and 4.4 lay odds shown for Italy in Figure 1. Match rates for offers to the left and right of these are presumably lower than this. These offers are made by people who hope the market odds will move in their favor and they can be matched later at a better price for them than the current ones.

5.5. Introducing Fees with a High Matching Rate

We now set the matching rates equal to $\theta_B = \theta_L = 0.75$ and look at various values for the commission rate. The first thing to note is that the introduction of a commission fee has almost no effect on bid-ask spreads and does not change the pattern of the average of the back and lay odds approximately equaling $1/q$. In the model of Bürgi, Deng and Whelan (2025), in which Makers set odds to seek a post-commission rate of return and Takers simply accept these offers, higher commission rates increase the bid-ask spread because Makers make less attractive offers to compensate for the higher fees. However, in this model, both sides of each bet take the commission fee into account. Fees reduce the perceived profits available but the equilibrium odds are barely affected by the commission rate being higher.

While the commission rate has little impact on odds, it does affect the number of participants who choose to do nothing. Figure 9 shows the fraction of participants who decide to be neither a Maker nor a Taker. Interestingly, at this matching rate, the introduction of a 1 percent commission does not induce anyone to take this option. As described above, in this case, those with beliefs close to the true value of q believe that the bid-ask spread is wide enough for them to make profits. They choose to act as "stockbrokers" and make both Back and Lay offers intending to profit from the bid-ask spread. For higher levels of commission, we see an inverse-U shape for non-participation rates, with lower participation for events with middling levels of q and higher participation when q is closer to the extremes.

As the commission rate rises, more people choose to participate but this effect is fairly modest. Within the range modeled here, total expected commission revenue rises as the commission rate increases, with the higher fee offsetting the smaller volume. This suggests that the exchange operator could earn more profits by charging a higher commission rate. However, these calculations do not take into account the fact that there is competition in the betting exchange market. Betfair used to have a complex fee structure, with a base rate of 5% but lower commissions for those who placed more bets. However, its competitors, Betdaq and Matchbook, had a simple 2% fee structure and Betfair has also moved to this rate in recent years.

Figures 10 and 11 show the post-commission returns on money placed at risk for various commission rates on winnings. Higher commission rates imply worse rates of return but do not change the overall shape of the returns for Makers and Takers. Because the exchange we have data for has a 2% commission, Figure 12 shows average returns for Makers and Takers with a 2% commission. For most values of q, average returns are negative for both sides of contracts. For Makers, loss rates increase slightly as the Taker's chance of winning rises from low levels but then we see Makers earning sharply rising profits when the probability of the Taker winning goes above 90%. Losses for Takers increase nonlinearly as their probability of winning falls with loss rates of about 5% for contracts with $q = 0.05$.

We also examined the implications of variations in the extent of disagreement as measured by σ. Higher values of σ raise bid-ask spreads and increase loss rates for Takers and profit rates for Makers on contracts. As noted, we have chosen $\sigma = 0.03$ because the patterns it produces for returns in Figure 12 match the data well.

Figure 9: Fraction of participants who do nothing for $\theta_B = \theta_L = 0.75$, $\sigma = 0.03$ and various values of τ.

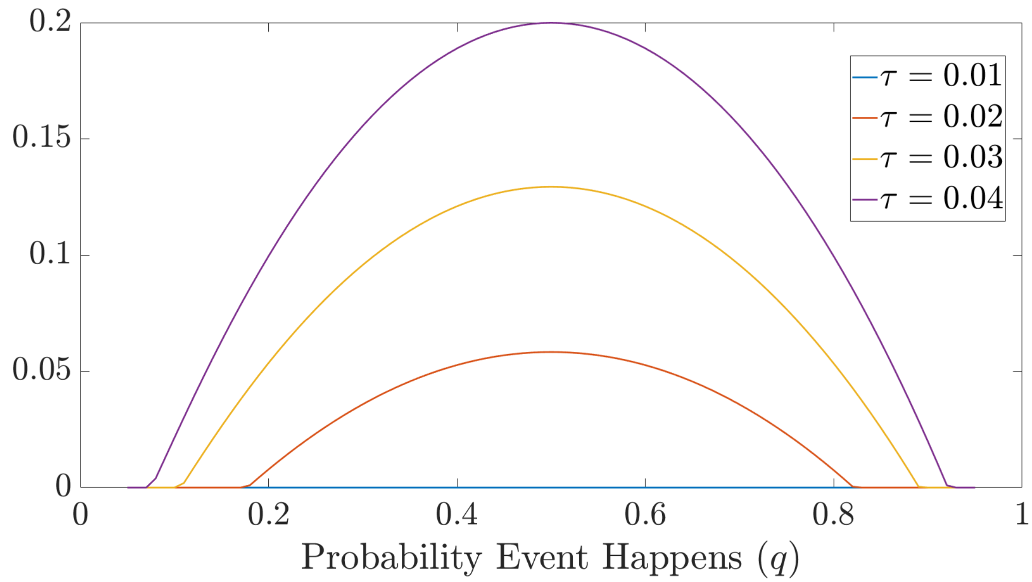


Figure 10: Post-commission average returns for Takers with for $\theta_B = \theta_L = 0.75$, $\sigma = 0.03$ and various values of τ.

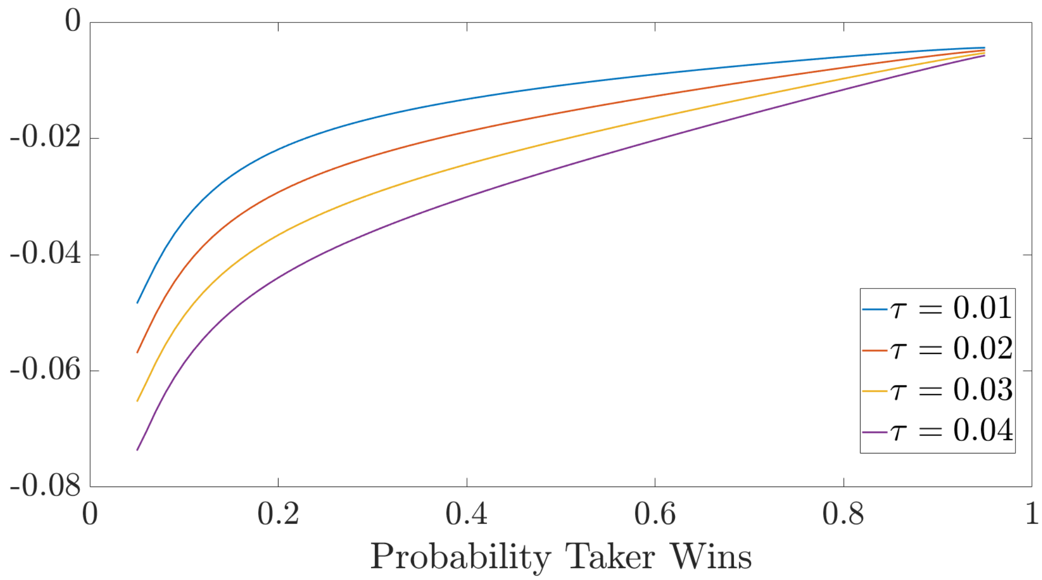


Figure 11: Post-commission average returns for Makers with for $\theta_B = \theta_L = 0.75$, $\sigma = 0.03$ and various values of τ.

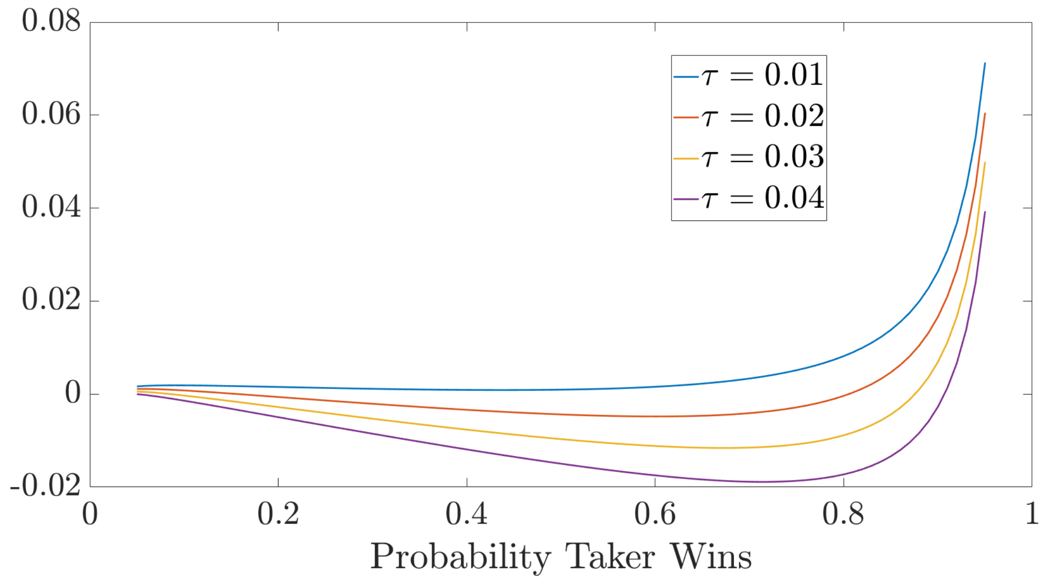
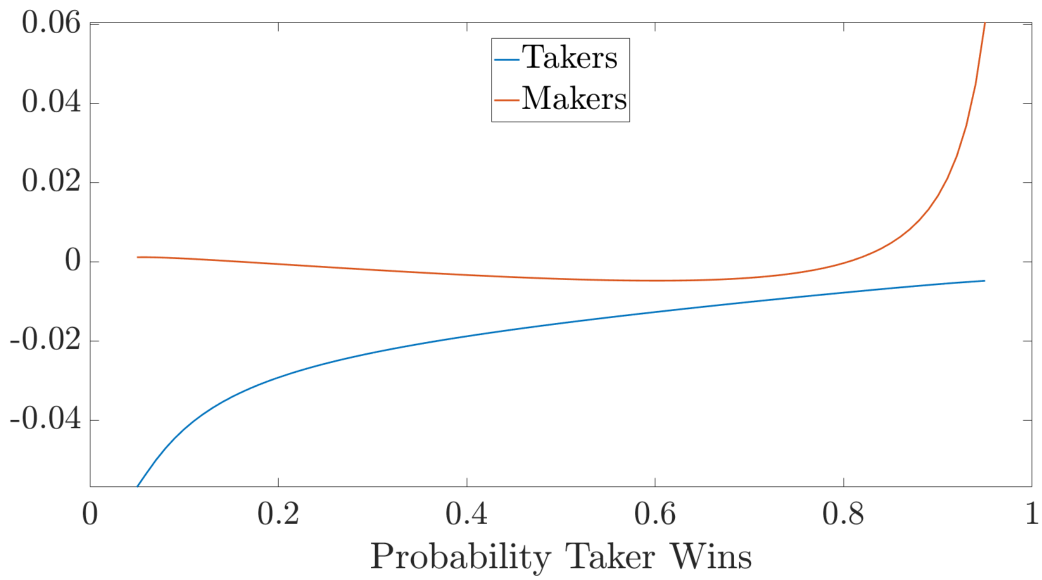


Figure 12: Post-commission average returns for Makers and Takers with for $\theta_B = \theta_L = 0.75$, $\sigma = 0.03$ and $\tau = 0.02$.



6. The Betfair Dataset

Here we describe the dataset from Betfair and briefly discuss the accuracy of the closing odds.

6.1. Data format and trade identification

Our data are the "Advanced" historical data from Betfair for all soccer matches from 2022 to 2024. They are a commercial product sold by Betfair but they were made available to me free upon request for use for research purposes.

Each match has a JSON (JavaScript Object Notation) file that tells the story of the betting on the exchange for that match. The match's meta-data is at the start of the file (team names, IDs for the different possible outcomes, market opening time). Then there is a set of "market change messages" (mcm) at 1 second intervals which update the best available back offers, the best available lay offers, the last traded prices, the prices and volumes of specific trades and total volume. There is an indicator to note when the match goes in play and, at the end of the file, it records which bets won and lost.

Figure 13 shows a stylized version of what a short sequence of market change messages looks like, in this case just for one outcome (styled here to be like backing Italy in our earlier example). I used Python code to process these files and save data in a spreadsheet. This produced 214,992 matches which had last traded prices before kick-off for bets on home wins, away wins and draws and a similar number for each of the six total goals markets that we looked at (0.5 goals up to 5.5 goals).

We use the information in the market change messages prior to particular moments (at first kick-off but also various points throughout the game) to identify trades that were offered by Makers and accepted by Takers. Going back to the Germany-Italy example in Figure 1, if you saw a trade on Italy win at 4.3, you can tell it was someone accepting an offer to back Italy. In the same way, Figure 13 shows a trade happening at 4.3 and you can go back and see that the best available to back (batb) odds just prior to this trade was 4.3 and hence someone accepted an offer to back Italy. The cumulative volume available for each price is shown next to the odds. In this case, we can see how a £5 trade reduced the available liquidity at 4.3, how it was the first trade at this price and how total traded volume (*tv*) increased by £5.

This method did not produce an identified back and lay trade for every option available for each match (e.g. home, away and draw) because sometimes trades are reported at odds that were not previously quoted in the JSON files, most likely because offers appeared and were taken up in between the one-second intervals recorded in this data. However, as discussed below, the process was able to produce very large datasets of identified trades in which we know which side was the Maker and which was the Taker.[9]

[9] This method mirrors the logic of the widely used Lee and Ready (1991) algorithm in finance, which infers trade direction from quote and transaction data. However, unlike most financial market datasets, where the trade initiator must

Figure 13: Example snippet of Betfair JSON file


```
{"op":"mcm","clk":"5488001000","pt":1648387200000,
"mc":[{"id":"1.196074655","rc":[
  {
    "batb":[[0,4.3,12.34],[1,4.2,14.00],[2,4.1,16.75]],
    "batl":[[0,4.4,7.25],[1,4.5,9.00],[2,4.6,12.00]],
    "tv":340.00,
    "id":44517
  }
]}]}
{"op":"mcm","clk":"5488001001","pt":1648387205000,
"mc":[{"id":"1.196074655","rc":[
  {
    "batb":[[0,4.3,7.34],[1,4.2,14.00],[2,4.1,16.75]],
    "batl":[[0,4.4,7.25],[1,4.5,9.00],[2,4.6,12.00]],
    "trd":[[4.3,5.00]],
    "tv":345.00,
    "id":44517
  }
]}]}
```


6.2. Predictive power of pre-play odds

The model predicts that average odds from a betting exchange should be good predictors of outcomes. Here, we demonstrate that Betfair's odds produce unbiased forecasts for home wins, away wins and draws. Our model predicts that relative to the true probability of a bet winning q, the inverse of the back odds ($\frac{1}{D_B}$) will be too high and the inverse of the lay odds ($\frac{1}{D_L}$) will be too low but that the average of these inverses will approximately equal q. Here, we use the last traded price before the match's kick-off. About half the time, this traded price will be for a back bet and the other half it will be for a lay bet, so on average we expect these odds to be accurate.

One technical issue with interpreting the inverse of the odds as a probability is that the sum of the inverse odds for each possible outcome generally does not equal one. The size of this overround is very small for the Betfair last traded prices when compared with bookmakers: The average deviation from 1 was only 0.003. However, just to be able to strictly interpret the measures we use as probabilities, we constructed "normalized" probabilities by dividing the inverse of the odds by their sum, so the adjusted probability estimates sum to one. This gave us a sample of 200,622 matches where we have normalized probabilities for each final pre-play bet, as well as match outcomes

be inferred with error – often misclassified 10–20% of the time (Odders-White, 2000; Theissen, 2001) – Betfair's real-time reporting of both sides of the order book enables us to infer trade direction with high confidence.

We tested whether the Betfair implied probabilities for match outcomes are unbiased as follows. Let Y_{ij} equal one if match i ends in outcome j. For each match i, we have three possible outcomes (home win, away win, draw) and for each of those outcomes we have a Betfair-derived probability P_{ij}. A simple way to test whether these probabilities are unbiased forecasts of the outcome is the classic Mincer-Zarnowitz (1969) regression

$$Y_{ij} - P_{ij} = \alpha + \psi P_{ij} + \epsilon_{ij} \tag{17}$$

where the dependent variable is the realized "error". The null hypothesis of the price being an unbiased predictor can be tested via the standard F-test of $\alpha = \psi = 0$.

Table 2 shows the results from this regression. We have a sample of 601,866 bets, with 3 observations for each match corresponding to the three possible outcomes. Two technical issues are worth noting. First, dummy variables that equal one with a probability P have a variance of $P(1-P)$, implying the error terms will feature heteroscedasticity. We follow Pope and Peel (1989) in using Weighted Least Squares (WLS) for estimation with the variances approximated by the $P_{ij}(1-P_{ij})$. Second, because each match shows up as multiple observations in the regressions, there are negative correlations between the errors for outcomes of individual matches (i.e. the ϵ_{ij} terms are correlated for each specific value of i). To address this, we cluster standard errors at the match level following Angelini and De Angelis (2019), Elaad, Reade and Singleton (2020) and others.

The key result is that the F-statistic cannot reject the null hypothesis that the probabilities based on the last traded prices prior to kick-off are unbiased forecasts of the outcome of matches, thus confirming one of the model's predictions.

Table 2: Weighted OLS regression of forecasting errors on probability estimates with clustered standard errors

	Coefficient	Std. Error	t-statistic
Betfair-implied probabilities	-0.0041	0.0033	-1.25
Constant	0.0020	0.0011	1.78
Number of observations: 601,866			
Number of clusters: 200,622			
F-statistic: 1.56 (df = 1, 200621)			
p-value: 0.2110			

7. Evidence on Returns

We now turn to assessing whether the predicted patterns for post-commission rates of return on bets can be seen in the data.

7.1. Pre-Play Betting

By matching trades with recently offered odds, we produced a dataset of 152,102 matches in which we could identify the side taken by the Maker and Taker on all six possible options: backing and laying either a home win, away win, or a draw. This produced 902,568 bets from which we could use back and lay bets on all options to calculate win probabilities and ex post outcomes. For these 902,568 bets, the average return for Takers was -2.5% while the average return for Makers was 0.6%. The t-statistic for the difference between these means is 11.8 which is highly statistically significant.

Figure 14 shows the average post-fee returns for these bets placed just prior to kick-off, with the data organized by decile of the estimated probability of the Taker bet's success—estimated as the average of the inverse back and inverse lay odds. In other words, the data are organized in the same way as the model output in Figure 12. The shaded areas in the graphs are 95% confidence intervals.

The results conform pretty well with the predictions of the model. For Takers, there is a clear pattern of increasing loss rates as the probability of their bet winning falls, with a steep drop-off for the lowest-probability bets—just as predicted by the model calibrated in Figure 12 with the commission rate set to Betfair's 2% and the disagreement parameter set to $\sigma = 0.03$. Losses for Takers are statistically significant, and the magnitudes of the losses closely match the model's predictions.

For Makers, post-commission returns are close to zero and statistically insignificant. This also fits with the predictions in Figure 12. The top decile shows the uptick in returns predicted by the model but the size of the pattern is not as large. Worth noting, however, is that the model predicts this pattern will be most evident for bets where the Taker's win probability exceeds 90%. In practice, very few home, away or draw bets have win probabilities this high.

For the total goals markets, we identified 2,088,032 bets across the different markets (from 0.5 goals up to 5.5 goals) here we know the side taken by the Makers and Takers. The average return on bets placed by Takers was -2.2% while the average return for Makers was 2%.[10]

Figure 15 shows the results for the total goals market sorted again by decile of the estimated probability of the Taker bet's success. The patterns are similar: returns for Takers again show increasing losses as their win probability falls. For Makers, returns are generally flat at about zero up until the Taker's probability of winning reaches 0.6, at which point it becomes positive. This sample includes

[10]This positive return for Makers of 0.6% may look attractive given that it can be repeated across many different bets. But the standard deviation of this return is 198% so the prospect of ruin would be very high if you devoted much of your fortune to it. The mean-variance approximation to the Kelly criterion would recommend staking only 0.5% of your wealth on this bet.

more cases with win probabilities above 90%, and here we do see a big uptick in returns for Makers in that range.

Overall, the returns for Makers and Takers on bets placed prior to kick-off conform well with the model's predictions.

7.2. High and Low Bid-Ask Spreads

The model predicts a more marked difference between returns for Makers and Takers in thin markets with low matching rates and high spreads. To check this prediction, Figure 16 shows the pre-play returns for home/away/draw bets and Figure 17 for the returns in the total goals market separately for the bottom and top halves of the bid-ask spread distribution, as measured by total traded volume just before kickoff. The patterns from the full distribution are largely reproduced but there is evidence of bigger losses for Takers and bigger losses for Makers for those bets with larger bid-ask spreads, particularly for longshot bets.

Figure 14: Average post-fee returns by ex-ante probability for Makers and Takers:
Final pre-play trades on **home, away or draw** in soccer (2022-24)
Shaded areas are 95% confidence intervals

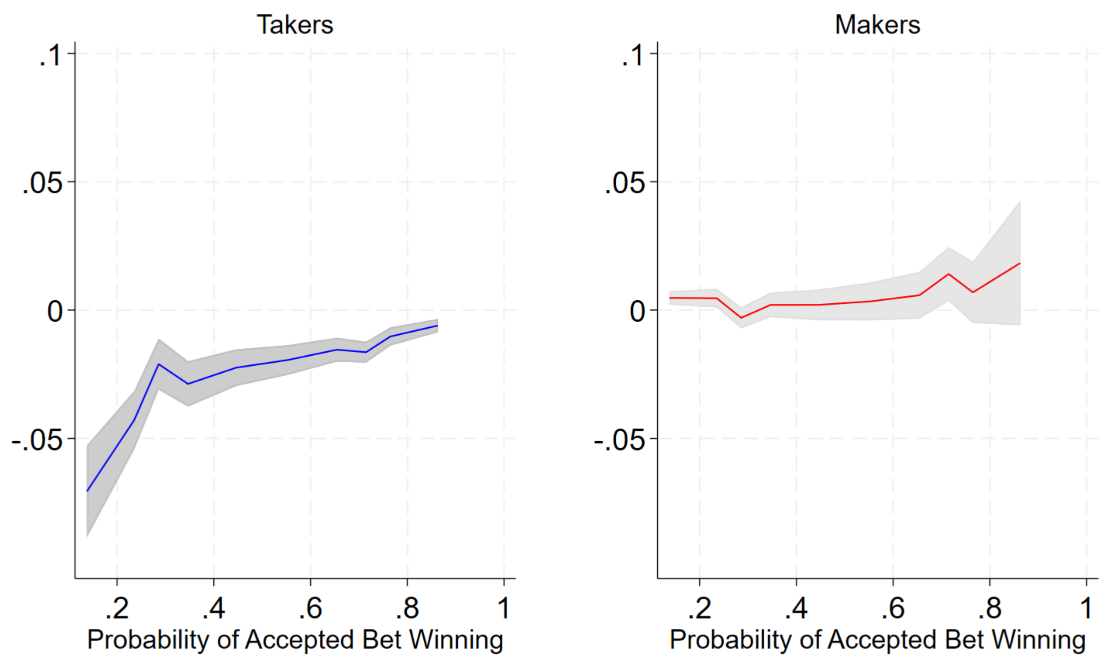


Figure 15: Average post-fee returns by deciles of ex-ante probability for Makers and Takers. Final pre-play trades for **total goals** (over-under) bets in soccer (2022-24). Shaded areas are 95% confidence intervals

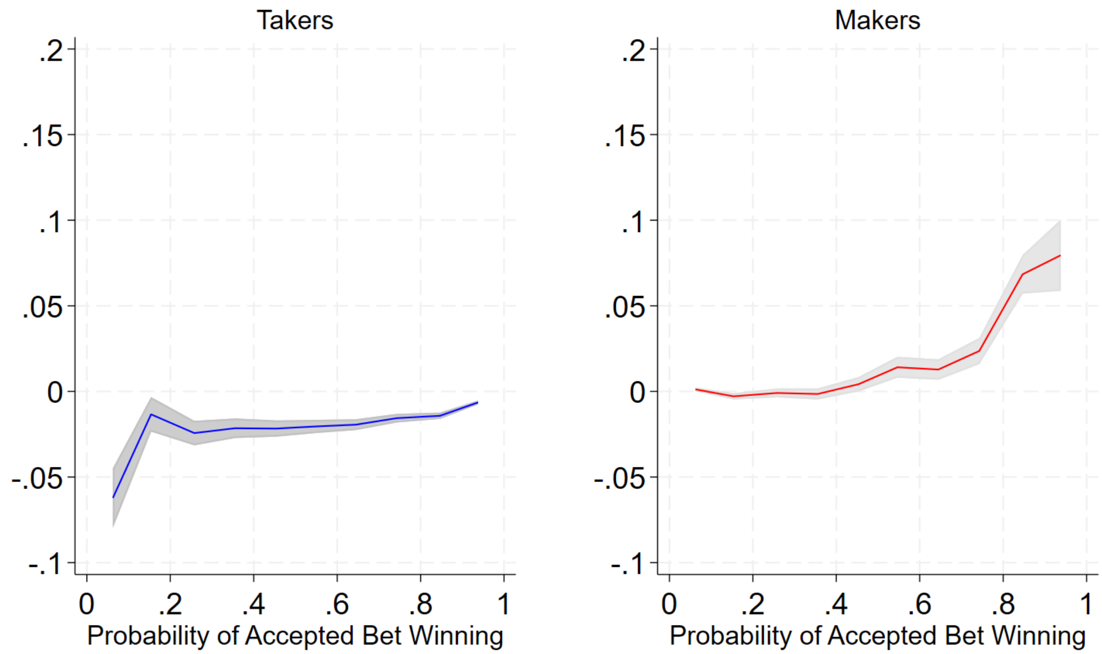
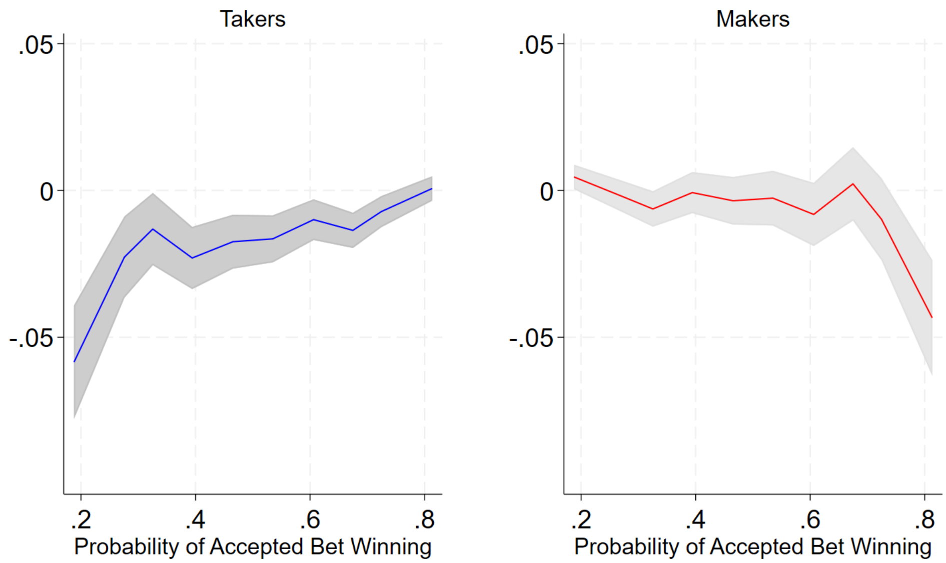
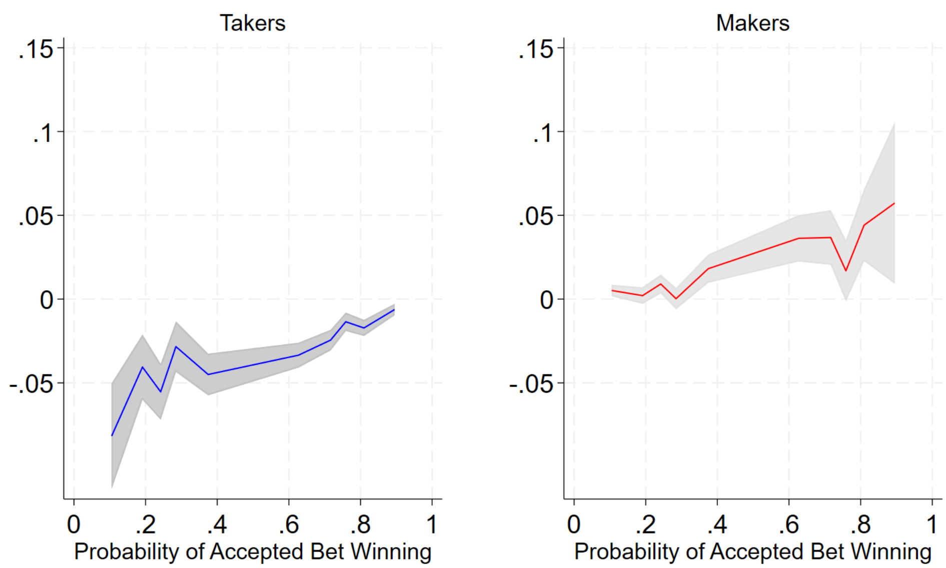


Figure 16: Split by average bid-ask spread size: Average post-fee returns by deciles of ex-ante probability for Makers and Takers: Final pre-play trades on **home, away or draw** in soccer at various timestamps (2022-24). Shaded areas are 95% confidence intervals

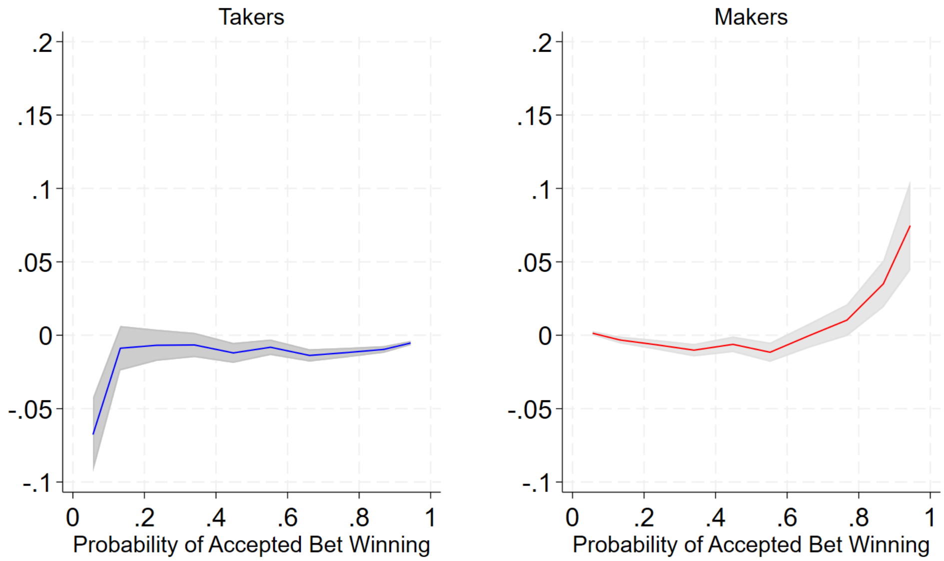


(a) Bottom Half for Bid-Ask Spreads

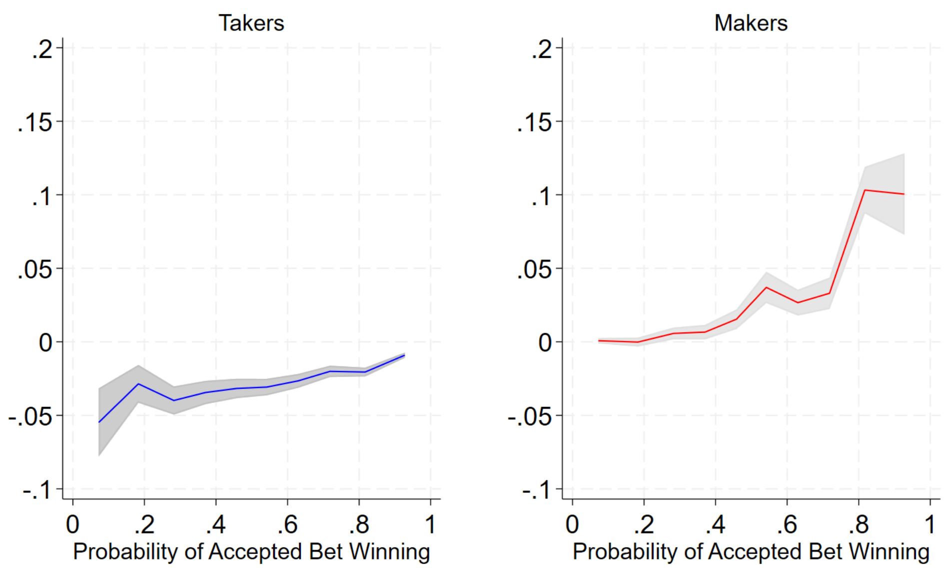


(b) Top Half for Bid-Ask Spreads

Figure 17: Split by average bid-ask spread size: Average post-fee returns by deciles of ex-ante probability for Makers and Takers. Final pre-play trades for **total goals** (over-under) bets in soccer (2022-24). Shaded areas are 95% confidence intervals



(a) Bottom Half for Bid-Ask Spreads



(b) Top Half for Bid-Ask Spreads

7.3. In-Play Betting

Betfair's data records all bets made right up to the end of a match. One advantage from our perspective is that as matches go on and goals are scored, some outcomes become very likely or very unlikely. This results in a much larger number of bets where Takers are either backing outcomes that are highly likely to happen or laying outcomes that have a very low probability of happening. Figure 18 shows post-commission returns from applying our procedure to bets placed just prior to six time-stamps in the match: 15 minutes, 30 minutes, 45 minutes, 75 minutes, 90 minutes and 105 minutes. Allowing 3 minutes for the average amount of added time at the end of the first half and 15 minutes for the half-time break, the final three timestamps would correspond to roughly 57 minutes, 72 minutes and 87 minutes on the match clock.[11]

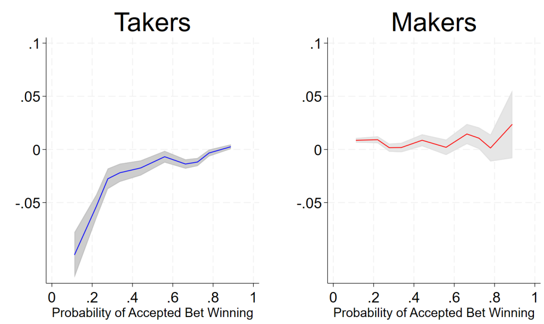
Several interesting results emerge. The results for bets placed 15 minutes in closely match those for pre-play bets. However, as matches progress, losses for Takers get larger, particularly for longshot bets. Returns for Makers placing longshot bets (bets where the Taker's probability of winning is high) also start to show higher average loss rates as matches progress. By 105 minutes, loss rates for Takers in the bottom decile of probability reach about 70% and loss rates for Makers taking longshot bets are almost as bad. Small but statistically significant profits for Takers emerge during the second half for bets with a reasonably high chance of success. These data suggest it may be possible to earn money on Betfair just by accepting offers on strong favorites in the second half!

These results echo findings presented by Lionel Page (2012) on what he termed the "Yogi Berra effect" (it ain't over till it's over). Page analyzed trades on sports from 1,100 different US sports markets on the InTrade prediction market. He found that large losses on longshot bets emerged as the games went into their final 15 minutes.

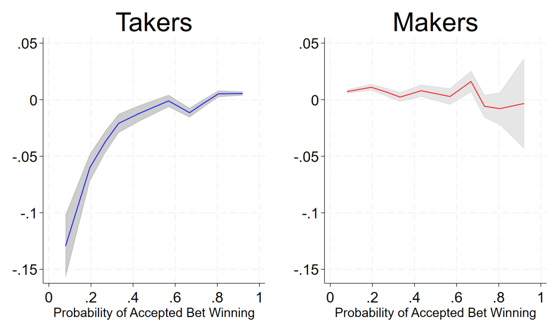
Poor returns for longshot bets is, of course, a common theme in the economics of betting with many possible explanations, as discussed in the well-known papers such as Thaler and Ziemba (1988), Ottaviani and Sørensen (2008) and Snowberg and Wolfers (2008). But those explanations normally focus on behavioral patterns like risk-seeking preferences or failure to assess small probabilities accurately. If these were innate traits of Betfair's participants, we would expect to see low returns to longshot bets prior to kick-off and during the first half. Page notes that the "higher emotional tension" near the end of games may distort the traders' ability to judge probabilities. Also, unlike pre-play betting where you have lots of time to research the various probabilities, late-game bets will also often be placed by people making rapid judgments while watching the match live, knowing there isn't much time left. In Kahneman's (2011) terminology, perhaps bettors' thinking moves from being "slow" prior to and early in the game to "fast" as the game progresses.

[11] The data do not record the minutes the first half ended or second half began.

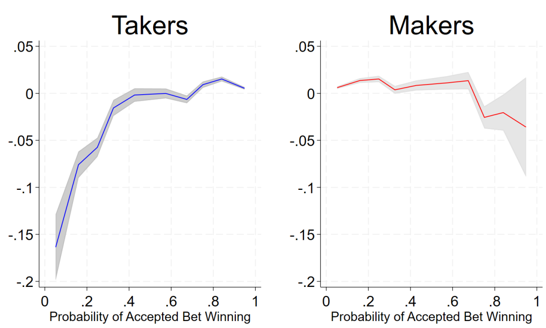
Figure 18: Average post-fee returns by deciles of ex-ante probability for Makers and Takers: Last in-play trades on **home, away or draw** in soccer at various timestamps (2022-24)
Shaded areas are 95% confidence intervals



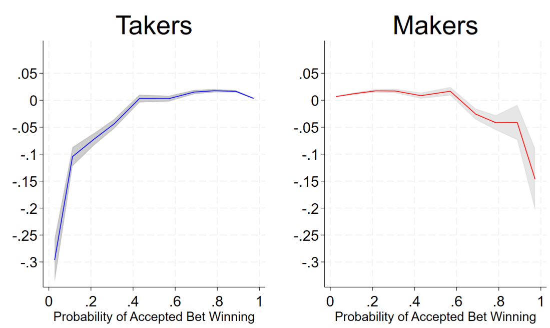
(a) 15 Minutes after kick-off



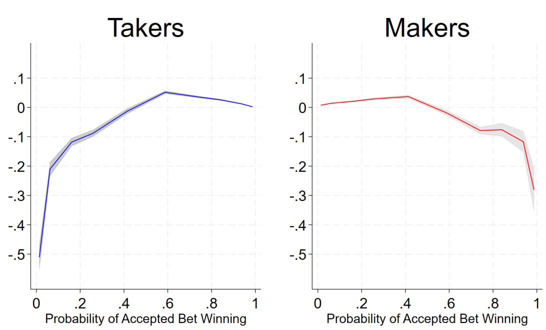
(b) 30 Minutes after kick-off



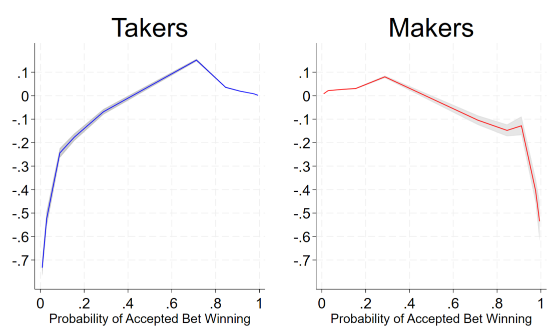
(c) 45 Minutes after kick-off



(d) 75 Minutes after kick-off



(e) 90 Minutes after kick-off



(f) 105 Minutes after kick-off

8. Conclusions

I hope this paper has shown that betting exchanges offer a valuable setting for testing models featuring decentralized trading, belief dispersion, and price formation under uncertainty, elements that are core to many financial markets. The combination of their market microstructure and Betfair's transaction-level data provides a rare opportunity to observe behaviors and outcomes that are difficult to measure in traditional financial market settings.

The paper introduces the first formal model of a betting exchange, in which decentralized trade arises from rational agents with heterogeneous but (on average) unbiased beliefs. Using Betfair's uniquely transparent data, we test the model and provide the first large-scale empirical comparison of returns to order-makers and order-takers, a contrast that standard financial datasets do not allow. Despite its relative simplicity, the model generates striking predictions, including the possibility of multiple equilibria. Some equilibria are characterized by high trade volumes and narrow spreads, while others feature low match rates and wide spreads. The contrast between Betfair's high-volume, high match-rate business and that of its lower-volume competitors is perhaps an example of these multiple equilibria in practice.

The model has been solved under a specific treatment of disagreement: the cross-sectional standard deviation of beliefs about an event's probability is constant across all true probabilities. This strong assumption could certainly be challenged. But an advantage of making specific assumptions is they generate specific predictions and the observed patterns of post-commission returns on bets for Makers and Takers fit well with the model's predictions when examining bets made prior to the match or during the first half.

By contrast, we document a pronounced favorite–longshot bias in second-half betting, with longshot wagers yielding large negative returns for both Makers and Takers. These losses are substantial enough to generate small but statistically significant profits for their counterparties. It remains to be seen whether this "Yogi Berra" effect on Betfair survives now that it has been publicly documented, or whether it will follow the pattern of other financial market anomalies in tending to disappear once they are publicized (see Zaremba, Umutlu and Maydybura, 2020, Shanaev and Ghimire, 2021).

One important caveat to the positive returns to Makers reported here is that acting as a Maker does not guarantee getting offers accepted. The bets that generated the positive returns reported here constitute the full sample of matched bets. These bets are unlikely to be equally good for those who place them and those seeking to act as Makers could face an "adverse selection" problem in which their less favorable offers are more likely to be accepted, while many of their potentially more profitable ones remain unmatched behind others in the queue. Whether this approach can actually be turned into a practical trading strategy is left as an optional exercise for the reader.

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A Numerical Algorithms

We used two different numerical algorithms to solve the model and they produced the same output. I would note that to reduce the nonlinearity in the equations used by the models, we used the four indifference conditions (equations 1, 3, 5 and 7) rather than the closed-form solutions presented above for the thresholds.

The function `fsolve` is designed to compute solutions to systems of nonlinear equations of the form $F(x) = 0$ where $F : \mathbb{R}^n \to \mathbb{R}^n$ is a vector-valued function. Numerically, `fsolve` employs a trust-region method. At each iteration k, the algorithm constructs a linear model of the residual vector using the Jacobian matrix:

$$F(x_k + \Delta x) \approx F(x_k) + J(x_k)\Delta x,$$

where $J(x_k) \in \mathbb{R}^{n\times n}$ is the Jacobian of F evaluated at the current point x_k. The algorithm then seeks a step $\Delta x \in \mathbb{R}^n$ that (approximately) solves the following subproblem:

$$\min_{\Delta x} \|F(x_k) + J(x_k)\Delta x\| \quad \text{subject to} \quad \|\Delta x\| \leq \Delta_k,$$

where $\Delta_k > 0$ defines the radius of the trust region around x_k.

This subproblem is not solved exactly. Instead, `fsolve` uses efficient numerical strategies, such as dogleg or subspace methods, to compute an approximate minimizer that yields sufficient decrease in the norm of the residual. Based on the agreement between the predicted and actual reduction in $\|F(x)\|$, the algorithm adaptively updates the trust-region radius Δ_k. The iteration terminates when the norm of the residual $\|F(x_k)\|$ falls below a user-defined threshold, the step $\|\Delta x_k\|$ becomes sufficiently small, or the gradient $J(x_k)^\top F(x_k)$ is nearly zero, indicating satisfaction of first-order optimality conditions. This trust-region framework enhances robustness relative to standard Newton-type methods, especially when the system is highly nonlinear.[12]

Alternatively, `lsqnonlin` solves the equilibrium system by formulating it as a nonlinear least-squares problem:

$$\min_{x\in\mathbb{R}^n} \|F(x)\|^2 = \sum_{i=1}^{m} F_i(x)^2.$$

In this approach, the residuals $F_i(x)$ need not vanish individually; the solver instead minimizes their collective squared magnitude. Because we impose various constraints on the solution (e.g. thresholds must be between zero and one), `lsqnonlin` employs the trust-region-reflective (TRR) algorithm, which is specifically designed to handle restrictions on the allowable range of each variable. The

[12] See Dennis and Schnabel (1996) for a discussion of these methods.

update step is computed by solving a constrained subproblem:

$$\min_{\Delta x} \; \|F(x_k) + J(x_k)\Delta x\|^2 \quad \text{subject to} \quad x_k + \Delta x \in [\text{lb}, \text{ub}], \quad \|\Delta x\| \le \Delta_k,$$

where $J(x_k)$ is the Jacobian of F at the current iterate x_k, and $\Delta_k > 0$ defines the trust-region radius. When a step would take a variable outside its allowed range, the algorithm adjusts it to stay within bounds and continues the search along the edge of the feasible region. Iterations terminate when the objective $\|F(x_k)\|^2$ is sufficiently small, the step size $\|\Delta x_k\|$ becomes negligible, or the gradient norm $\|J(x_k)^\top F(x_k)\|$ is close to zero, indicating that further progress is numerically insignificant.[13]

[13]See Coleman and Li (1996) for a discussion of these methods.