# The Economics of Free Bets

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March 30, 2026

**Abstract**

Sportsbooks offer promotional "free bets" that return profits if the bets win, but not the stake provided. These bets can be hedged to produce guaranteed profits. This paper shows how to value the guaranteed profit from these stake-not-returned free bets under a simple constant-margin pricing rule in which sportsbooks, on average, keep a fraction $m$ of the money staked. The guaranteed hedged value is a nonlinear function of the free bet's probability of winning, reaching a maximum when this probability is $\sqrt{m(1-m)}$. Empirical evidence from odds on tennis bets confirms this prediction.

JEL Codes: D81, D84, L83

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# 1. Introduction

Sportsbooks frequently offer free bets as promotions to attract new customers and retain existing ones. If these bets worked the same as regular bets, whereby you receive your stake back as well as winnings, then customers could turn the free bets into almost risk-free money by placing them on short-odds favorites. For this reason, free bets are almost always offered in the "stake-not-returned" form, meaning you get to keep any winnings but not the stake.

This paper describes the optimal strategy for using stake-not-returned free bets when sportsbooks price each bet with a fixed profit margin. The expected profit is maximized by choosing the bets with lowest possible chance of success. But this strategy involves a lot of risk. An alternative strategy is to turn free bets into a *guaranteed* profit by placing a hedging bet against the outcome being backed with the free bet, a process sometimes called matched betting. Aboufadel (2025) provides an illustration, using American odds, of how to choose a bet that maximizes the guaranteed return under a specific numerical example of sportsbook pricing.

This paper provides a more general result, using decimal odds to show how the guaranteed return behaves when sportsbooks set odds with an expected profit margin of $m$, meaning each unit bet placed has an expected profit of $1-m$. We show that the guaranteed return is a nonlinear function of $p$, the free bet's probability of success and the profit margin $m$, and it is maximized by choosing a free bet with $p = \sqrt{m(1-m)}$. As the probability of winning falls, the potential win from the free bet rises but, after a certain point, the size of the hedging bet required to generate a guaranteed profit rises faster, so the optimal guaranteed profit bet is not an absolute longshot.

The paper concludes with evidence from thousands of quotes offered on tennis by two sportsbooks. The data show that average guaranteed profits follow the predictions of our model of sportsbook pricing.

# 2. Hedging a Stake-Not-Returned Free Bet

Here we show the expected uncertain value of a free bet, describe a hedging strategy to produce a guaranteed return and derive the probability of the free bet winning that delivers the highest guaranteed return as a function of the sportsbook's profit margin.

## 2.1. Expected Returns on Free Bets

Consider a two-outcome event. Let $p$ denote the true probability of outcome $A$ and $1-p$ the probability of outcome $B$. Decimal odds – representing the total amount of money returned to a winning bettor – of $D_A$ and $D_B$ are offered by a sportsbook on these two outcomes. Assume the sportsbook sets decimal odds according to the profit-margin rule

$$D_A = \frac{1-m}{p} \qquad D_B = \frac{1-m}{1-p} \tag{1}$$

where $m \in (0,1)$ denotes the sportsbook's margin. This means the expected payout on a $1 bet is the same for both bets

$$pD_A = (1-p)\, D_B = 1-m \tag{2}$$

so the odds satisfy what Thaler and Ziemba (1988) called *strong market efficiency*, meaning the expected returns on each bet in a contest are the same.

Assume the bettor is offered a stake-not-returned unit bet and places it on outcome $A$. If $A$ occurs, the profit from the free bet is $D_A - 1$ (because they don't get the stake back), otherwise the profit is zero. The expected profit from this bet is

$$E\left[\Pi_A^F\right] = p\left(\frac{1-m}{p} - 1\right) = 1-m-p \tag{3}$$

This implies you maximize the expected profit from your free bet by placing it on the lowest value of $p$, meaning the longest possible longshot. This result is well known in quantitatively-oriented betting circles. For example, sports betting industry experts Ed Miller and Matthew Davidow (2019, page 218) recommend using a free bet on "*the biggest longshot they will let you bet with it.*"

## 2.2. Hedging

Placing your free bet on a longshot is the best way to maximize its expected profit but it is also a very risky bet. You probably don't get many free bets of this sort and this strategy will result in you most likely not winning any money. An alternative is to hedge the free bet by betting on the alternative outcome.

To hedge the position, the bettor places a real-money bet $h$ on outcome $B$. If $A$ occurs, the free bet pays $D_A - 1$ while the hedge bet on $B$ loses its stake $h$. So the net profit is

$$\Pi_A = (D_A - 1) - h \tag{4}$$

If $B$ occurs, the free bet pays zero, while the hedge bet wins, so the net profit is

$$\Pi_B = h(D_B - 1) \tag{5}$$

To obtain a guaranteed return, the bettor chooses $h$ so that these two profits are equal. This implies a hedge stake of

$$h = \frac{D_A - 1}{D_B}. \tag{6}$$

Substituting this value back into either state-contingent profit gives the guaranteed return from hedging:

$$\Pi = (D_A - 1) - h = h(D_B - 1) = \frac{(D_A - 1)(D_B - 1)}{D_B}. \tag{7}$$

Using the margin-based odds formulas, these can also be written as

$$h = \left(\frac{1-m}{p} - 1\right)\frac{1-p}{1-m} \tag{8}$$

and

$$\Pi^h = \left(\frac{1-m-p}{1-m}\right)\left(1 - \frac{m}{p}\right) \tag{9}$$

This means the ratio of the guaranteed profit to the expected value is

$$\frac{\Pi^h}{E\left[\Pi_A^F\right]} = \left(\frac{1}{1-m}\right)\left(1 - \frac{m}{p}\right) \tag{10}$$

We can note several things about these formulae.

First, if $m = 0$, so there is no sportsbook profit margin, the ratio in equation 10 is one, so you can replicate the expected value of the free bet via hedging. Consider a coin-flip event so $p = 0.5$. Without any sportsbook margin, the decimal odds are $D_A = D_B = 2$. Use your free $1 bet on $A$ and bet $50c$ on $B$ and whatever happens you are 50c better off. Either $A$ happens, so you win a dollar but lose 50c on your bet on $B$. Or $B$ happens, so the free bet doesn't matter but you win 50c from the bet on $B$.

But suppose $m = 0.05$, so the odds on both options are now $D_A = D_B = 1.9$. If $A$ wins, your free bet gets you a 90c profit. The hedging formula now says to place 47c on $B$ and the guaranteed return falls to 43c. Both your free bet and your hedging bet are being placed at less than their fair value and this eats into the risk-free return.

Second, the guaranteed profit can be written as the product of the expected risky profit from the free bet $(1 - m - p)$ and a multiplicative factor that describes the discount due to the sportsbook's margin. The discount gets bigger as $m$ increases but it also depends negatively on $p$ because the size of hedging bets get bigger as $p$ falls, with the discount falling particularly fast as $p$ reaches low levels. In fact, with $p = m$, the guaranteed return is zero and it is negative for $p < m$. For these low values of $p$, the size of the hedging bet gets so large, it wipes out the guaranteed profits.

Third, differentiating equation 9 with respect to $p$, we find that the guaranteed profit is maximized when the probability of the free-bet outcome satisfies

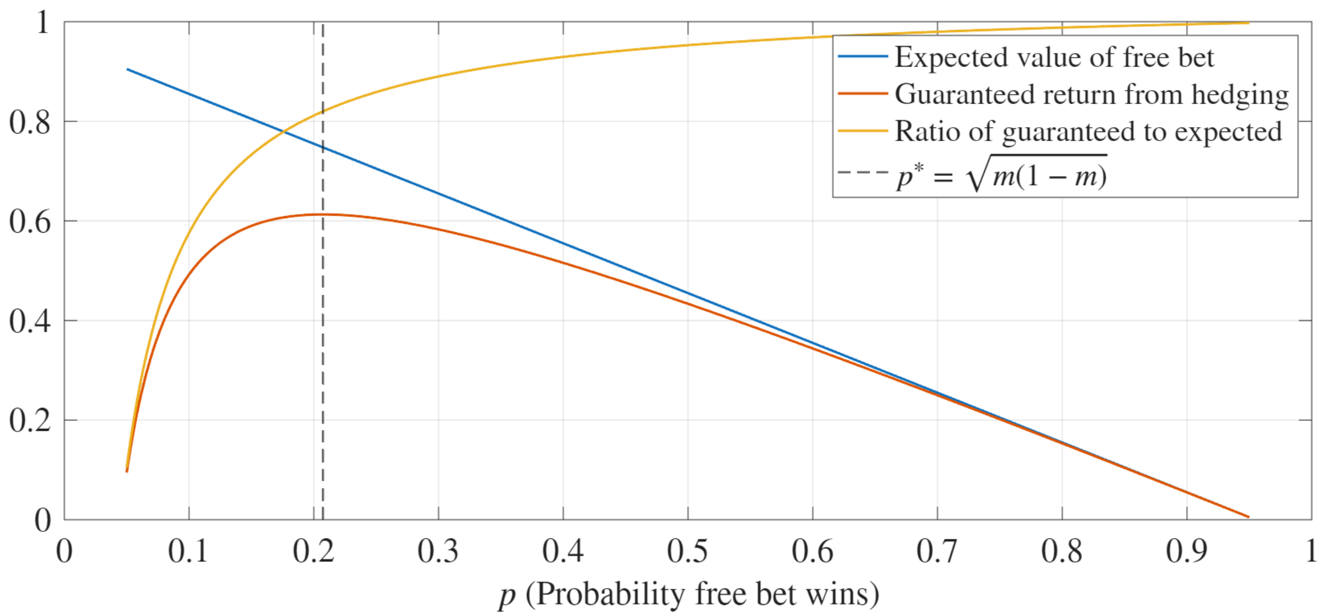
$$p^* = \sqrt{m(1-m)} \tag{11}$$

The optimal probability reflects a trade-off.[1] Lower-probability outcomes offer larger expected winnings from the free bet, but they also require bigger hedge stakes to equalize profits across states.[2]

For realistic margins, this formula says the guaranteed profit from hedging is maximized by placing the free bet on outcomes that are unlikely but not huge longshots. For example, if $m = 0.045$ (the typical sportsbook margin on traditional -110 on both sides bets) the optimal probability for the free bet is $p^* = 0.207$ and the guaranteed profit for a unit free bet is $\Pi^* = 0.613$.

Figure 1 shows how the rising expected value of the free bet combines with the declining efficiency of turning that expected value into guaranteed profits to produce an optimal probability at low, but not very low, probabilities for the free bet.

Figure 1: The factors determining the guaranteed value of the hedged bet with $m = 0.045$




[1] The second derivative is always negative, confirming that this is a strict global maximum.

[2] A natural question is whether, if a bettor has free bets available at two sportsbooks, they should use them on opposite sides of the same event. This is not optimal. A higher guaranteed return is obtained by using the free bets separately and hedging each one with a real-money bet.

### 2.3. Comparison with Aboufadel (2025)

Aboufadel (2025) also examines the question of how best to use a free bet. Rather than a general pricing framework, he uses a specific assumption applied to American odds. When the winning profit is greater than the stake, American odds have a positive sign and show the profit you can earn from a \$100 stake (so +200 means you will make a \$200 profit on a \$100 bet). When the winning profit is less than the stake, American odds have a negative sign and the magnitude tells you how big your stake needs to be to earn \$100 profit (so -110 means you need to bet \$110 to win \$100).

Aboufadel assumes American odds on an underdog and favorite of

$$U = \frac{100}{1.02 - p_F} - 100 \qquad F = -\frac{100}{0.98 - p_F} + 100 \tag{12}$$

where $U$ denotes the underdog odds, $F$ the favourite odds and $p_F$ is the probability of the favorite winning. These can be converted to decimal odds of

$$D_F = \frac{1}{p_F + 0.02} \qquad D_U = \frac{1}{1.02 - p_F} \tag{13}$$

A common estimate of the bookmaker's margin (see, for example, Hegarty and Whelan, 2025) is the sum of the inverses of the decimal odds. From equation 1, you can see that these inverse odds can be interpreted as probabilities if there was no sportsbook margin and their sum being greater than one is an indicator of the size of the margin. In this case, the inverse decimal odds are

$$\pi_F = \frac{1}{D_F} = p_F + 0.02 \qquad \pi_U = \frac{1}{D_U} = 1.02 - p_F \tag{14}$$

and their sum is 1.04, so Aboufadel's pricing model approximates a 4% profit margin for sportsbooks. However, the model does not allocate this margin equally across the two options because the expected profit on the favorite is greater than on the underdog ($p_F D_F > (1-p_F) D_U$). For example, if $p_F = 0.7$, the expected loss rates are 2.8% on the favorite and 6.2% on the underdog, while if $p_F = 0.8$, the expected loss rates are 2.4% on the favorite and 9.1% on the underdog.

While this is quite a different pricing model to the simple constant-margin one that we have assumed, Aboufadel concludes the optimal probability for the free bet in this case is 0.18, which lines up pretty well with you get if you use the 4% proxy for the margin in our formula $\sqrt{(0.04)(0.96)}$ = 0.196.

## 3. Evidence from Tennis Bets

From www.the-odds-api.com, I collected data on 6,259 tennis matches on the ATP and WTA tours between February 2021 and November 2025, for which odds on both players were offered by both FanDuel and DraftKings, the two most popular US sportsbooks. The odds were collected about 30 minutes before the match. In theory, you could place your free bet and the hedge with the same firm, but sportsbooks disapprove of this kind of activity, viewing it as an "abuse of promotions" and you are likely to get banned and perhaps have your remaining free bets revoked if you do this. So the dataset consists of 25,036 quotes which you could take with one book, while hedging with the other.

Tennis was selected because it fits our framework of having only two possible outcomes and because, with realistic margins, our formula suggests the optimal placement of a free bet is on an outcome with about a one-in-five chance of winning. These isn't a common situation for moneyline bets (placed on a competitor to win) in most sports but it is in tennis, and there are plenty of tennis matches to bet on most days of the year. So tennis offers a good opportunity for maximizing the guaranteed value of free bets.

Under the assumption of strong market efficiency, described by equation 1, the probability of outcome $A$ can be calculated as

$$p = \frac{\frac{1}{D_A}}{\frac{1}{D_A} + \frac{1}{D_B}} \tag{15}$$

and the bookmaker's margin can be calculated as

$$m = 1 - \frac{1}{\frac{1}{D_A} + \frac{1}{D_B}} \tag{16}$$

Figure 2 shows the data. Every dot in the chart corresponds to a guaranteed profit obtained by backing with one tennis player with one book and hedging with the other. The pink line shows the best fit between realized hedge value and normalized probability, using a LOWESS smoother with a bandwidth of 0.2.[3]

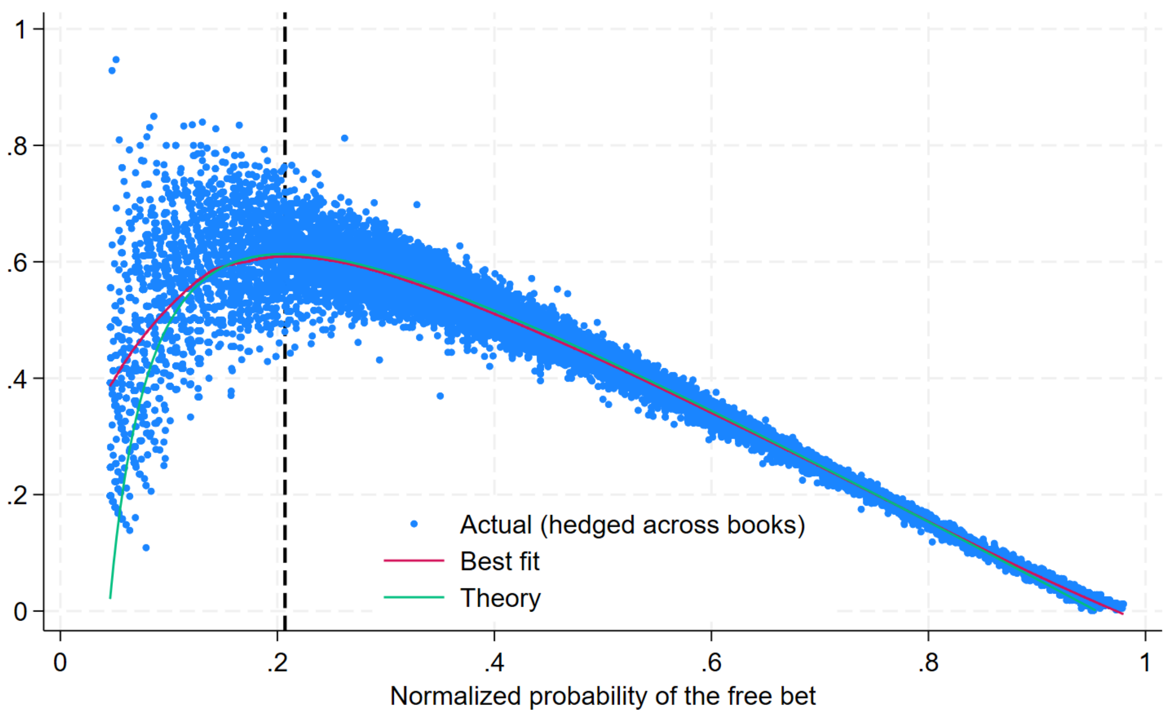
The green line shows the predicted guaranteed profit from our model of sportsbook pricing assuming $m = 0.045$, which is the average value of the margin implied by applying equation 16 to the data. This implies the best guaranteed return comes from placing a free bet with a probability $p = \sqrt{(0.045)\,(1 - 0.045)} = 0.207$ of winning (shown as the dashed black vertical line), which implies decimal odds of about 4.6.

The theory matches the data very well as we go from high probabilities for the free bet down to the optimal hedge probability. 21% does indeed appear to be the best normalized probability for your free bet and the chart shows that bets of roughly this type are often available. While the optimal guaranteed profit is 61c per dollar of free bet, about one-fifth of the bets in the sample gave

[3] More complex kernel regression methods produced essentially the same results.

guaranteed returns above 58c. As the probability of the free bet falls below this optimal level, the guaranteed returns in the data do not fall as much as the theory suggests. This is due to favourite-longshot bias, which has been documented for tennis previously (e.g. Hegarty and Whelan, 2025). Odds on more extreme favourites are a bit better than predicted by the normalized probability of the underdog. But this does not change the conclusions about the optimal use of a free bet.

Figure 2: Actual and predicted profit rate from hedged free bets on tennis



## 4. Conclusion

Sportsbooks offer stake-not-returned free bets. If you want to turn these into guaranteed risk-free money, we have described the optimal strategy under a simple model of how odds are priced. At typical margins, this involves placing the free bet at decimal odds of about 4.5 and hedging the position with a different sportsbook. Bets of this type on tennis are widely available and evidence from thousands of tennis odds quoted by FanDuel and DraftKings confirms that the theory performs well in practice.

## References

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