Compounding an Edge?
Expected Utility and the Puzzle of Parlay Betting

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Abstract

Multi-leg parlay bets have become hugely popular in the US despite loss rates on stakes that are four times higher than regular bets. This paper shows that even if bettors believe they have an edge, maximization of subjective expected utility requires placing separate bets when per-leg win probabilities are below one-third, and combining at most two legs for probabilities between one-third and one-half. These thresholds are largely independent of risk aversion and edge size. With average leg counts of five, and component bet win probabilities typically below one-half, the popularity of parlays presents a new puzzle for expected utility theory.

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1. Introduction

A 2018 Supreme Court decision striking down the federal act prohibiting states from legalizing sports betting has unleashed a new era of online betting on sports in the US. By early 2026, betting on sports had been legalized in 39 states. One product has become particularly popular—the *parlay*.[1] These are multi-leg cumulative bets where your returns from winning each leg are placed on the next leg and you only get money back if every leg wins. At first, these bets took the form of betting on multiple different games. More recently, same-game parlays (SGPs), where people bet on individual events within the same game, have increased in popularity.

Sportsbooks build in a profit margin on their odds, so on average they come out ahead. Winning a parlay requires you to beat this margin multiple times and the chances of doing so fall as the number of legs increases. State gaming commissions report that the fraction of stakes lost for bettors is about four times higher for parlays than other bets. Table 1 reports statistics from New Jersey illustrating the high profit rates for sportsbooks on parlays and their increasing importance, accounting for 32% of all stakes in 2025 up from 23% in 2021 and 65% of sportsbook profits, up from 56%.[2]

These are clearly bad bets for most who place them. But does this mean these bettors are irrational? Most bettors are aware there is a house edge, but they may have picked their five-leg parlay based on bets they believe the sportsbooks have priced incorrectly. In this sense, it is hard to directly refute the claim that parlay bettors are behaving rationally, in Savage's (1954) sense of maximizing their own subjective expected utility. Indeed, online sports betting forums often frame parlays as a way for smart bettors to maximize their edge by earning high cumulative returns.

This paper shows, however, that bettors with CRRA utility who have profitable edges on their bets would not maximize expected utility by placing the long-legged parlays popular with bettors. For bets with less than one-third chance of winning, expected utility with an edge is maximized by placing them separately, rather than combined in a parlay. For bets with probabilities between one-third and one-half of winning, the optimal leg count is two, well short of the industry-reported average leg count of about five. Perhaps surprisingly, these results are robust across two key dimensions: the bettor's level of risk aversion and the size of their edge. More risk aversion and lower edges reduce stake size and this makes the decision on leg length almost independent of these elements.

Section 2 describes the theory of optimal staking under two-outcome gambles and how it applies to parlays, using numerical examples from CRRA utility functions to present optimal leg lengths under different conditions. Section 3 uses a second-order approximation to the utility functions to provide an analytical explanation of our findings that optimal decisions on how many legs to take are remarkably similar across people with different risk attitudes and differing edges. Section 4 describes how these results apply to typical real-world sports betting parlays and Section 5 concludes.

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[1] See for instance, this 2025 article from the Washington Post, which observed that *"America is obsessed with parlays."*

[2] The 2025 figures are available here

Table 1: New Jersey Sports Betting Gross Gaming Revenue (GGR) Profits for Sports Books and Total Stakes Placed (Handle)

(a) Total

Year	GGR ($m)	Handle ($m)	GGR/Handle
2021	770	10,899	7.1%
2022	726	10,903	6.7%
2023	995	11,978	8.3%
2024	1,082	12,812	8.4%
2025	1,163	12,246	9.5%

(b) Parlays

Year	GGR ($m)	Handle ($m)	GGR/Handle
2021	434	2,481	17.5%
2022	450	2,442	18.4%
2023	541	2,957	18.3%
2024	641	3,671	17.5%
2025	755	3,942	19.2%

(c) Non-Parlay

Year	GGR ($m)	Handle ($m)	GGR/Handle
2021	336	8,419	4.0%
2022	276	8,462	3.3%
2023	454	9,021	5.0%
2024	441	9,141	4.8%
2025	407	8,304	4.9%

2. Optimal Stakes

We begin with a model with two possible outcomes and derive the optimal stake, then illustrate optimal staking for parlay bets.

2.1. A single bet

A gamble backing an outcome of an event is available at decimal odds D. This means a stake of s yields a win of $(D-1)s$ if the outcome happens and a loss of s if it does not. An agent with starting wealth w, a concave utility function $U(w)$ and subjective belief that the outcome has a probability p of happening, solves the problem of how much to stake by maximizing subjective expected utility

$$\underset{s}{\text{Max}} \quad \left[p\,U\left(w+s(D-1)\right)+(1-p)\,U\left(w-s\right)\right] \tag{1}$$

The first-order condition is

$$p(D-1)\,U'\left(w+s(D-1)\right)=(1-p)\,U'\left(w-s\right). \tag{2}$$

The marginal utility in the case of winning is monotonically falling in s and the marginal utility in the case of losing is monotonically rising in s, so any s that solves this equation will be unique.

For CRRA utility

$$U(w)=\begin{cases}\frac{w^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma\neq 1,\\ \log w & \text{if } \sigma=1\end{cases} \tag{3}$$

the optimal betting strategy when $\sigma \neq 1$ (see Appendix A.1) is

$$s=\left(\frac{1-\left(\frac{1-p}{p(D-1)}\right)^{\frac{1}{\sigma}}}{1+(D-1)\left(\frac{1-p}{p(D-1)}\right)^{\frac{1}{\sigma}}}\right)w. \tag{4}$$

For log utility, the optimal stake is

$$s=\left(\frac{pD-1}{D-1}\right)w. \tag{5}$$

This is the Kelly (1956) criterion rule. The amount staked increases with the size of the perceived edge $pD-1$ and decreases with the odds, so higher risk bets will typically be allocated lower stakes. For non-log utility, the amount risked declines as the coefficient of relative risk aversion σ increases, so for bettors with $\sigma < 1$, their optimal strategy is to be more aggressive than the Kelly criterion while those with $\sigma > 1$ should be less aggressive. We will not examine negative stakes and restrict the analysis to the case where the bettor has an edge such that $pD > 1$.

2.2. Multi-game parlays

Consider now the option to place a parlay with component bets on separate games with independent probabilities $p_1, p_2, ..., p_N$ of success and separate decimal odds of $D_1, D_2, ...D_N$. Assume the bettor has a fixed edge such that, for all bets, the expected payout on a unit bet is

$$p_i D_i = 1 + e \tag{6}$$

where $e > 0$. This means the odds can be written as

$$D_i = \frac{1+e}{p_i} \tag{7}$$

which implies that the sportsbook has implicitly underestimated p_i when setting these odds, since they don't knowingly offer bets with negative expected profits for them.

Since the games are independent, a parlay of all N bets has probability of success $\prod_{i=1}^{N} p_i$ and the combined bet has decimal odds of

$$\prod_{i=1}^{N} D_i = \frac{(1+e)^N}{\prod_{i=1}^{N} p_i} = \left(\frac{1+e}{\left(\prod_{i=1}^{N} p_i\right)^{\frac{1}{N}}} \right)^N \tag{8}$$

so parlays with the same product of probabilities are identical for optimal staking purposes. In our analysis, we will define

$$p = \left(\prod_{i=1}^{N} p_i \right)^{\frac{1}{N}} \tag{9}$$

as the geometric mean of the probabilities of the N legs. This means the parlay fits with the two-outcome framework above, using p^N as the probability of success and decimal odds of $\left(\frac{1+e}{p}\right)^N$, using these to replace p and D in the optimal staking equations (4) and (5).

We now consider a menu of parlays available, indexed by the number of legs, each with geometric per-leg mean probabilities of success p and with the same per-leg edge. Arrow (1971, page 100) shows that a risk-averse person will always accept a positive expected value gamble once the stake is sufficiently small, so our bettors would be willing to place some positive stake on each of the options.

While in principle bettors could allocate stakes across multiple parlay lengths, in practice people typically pick one at a time, so we model the decision as a choice of a single leg length. Which parlay should they pick? We assume they will pick the parlay that delivers the highest expected utility at the optimal stake size. Table 2 illustrates the numerical results. The calculations show a monotonic relationship between the geometric mean probability p and the leg length N that gives the best possible maximized expected utility. In other words, for any values of e and σ, representing the bettor's edge and risk aversion, they add more parlay legs as p rises. We provide analytical proof of this property in the next section using a second-order approximation to the utility function. The upper panel shows the threshold probabilities at which two legs are preferred to one, three to two and so on, for a fixed edge of $e = 0.01$ and three different values of risk aversion corresponding to very low risk aversion ($\sigma = 0.2$), log utility ($\sigma = 1$) and high risk aversion ($\sigma = 4$).

Intuitively, you might expect that more risk aversion implies a preference for lower-variance smaller-leg parlays and this result holds, but the magnitude of the differences are very small. For example, the threshold probability at which the low risk aversion agents will prefer two legs to one ($p = 0.327$) is less than one percentage point below the threshold for the same comparison for the highly risk averse agent ($p = 0.336$). The results show that agents with $\sigma = 0.2$, $\sigma = 1$, and $\sigma = 4$ choose the same optimal parlay length over most of the relevant probability range. Despite substantial differences in risk aversion, across the interval $p \in [0, 0.85]$, the three agents select the same preferred parlay length for approximately 81.5% of the probability space, and the difference in the preferred leg lengths of the different agents is almost always at most one.[3]

The bottom panel shows the threshold probabilities for a fixed level of risk aversion (log utility, $\sigma = 1$) for three different values of the gambler's edge $e = 0.005$, $e = 0.01$ and $e = 0.05$. The latter is included as an extreme example. Most recreational bettors have no edge but even professional gamblers tend to have a limited edge over the long-term. For example, Hill (2019) profiles Gadoon Kyrollos, the professional sports bettor known in gambling circles as Spanky. Kyrollos estimated that his long-run profit rate per bet was between 1 percent and 2 percent of the amount placed.

Again, intuition suggests people with a greater edge should want to add more legs and again this is the case, but the magnitudes of the differences in preferences are even smaller. The thresholds in Table 2 imply that these three agents with widely differing edges choose the same optimal parlay length for approximately 93.8% of the interval $p \in [0, 0.85]$.

Figure 1 provides a visual illustration of how the threshold structure is determined. It shows maximized utility for values of p up to 0.53 for standalone bets and parlays with two and three legs. We will explain below how this is the most relevant range for the probabilities of the component bets in parlays. As the number of legs increases, the maximized expected utility curve becomes more convex in the per-leg win probability p. Longer parlays therefore have relatively low value when p is small, but their value increases more rapidly as p rises. This generates the pattern seen in the table in which the optimal parlay length rises monotonically with p.

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[3] I use $p = 0.85$ as a cutoff because you rarely see bets with winning probabilities higher than this. For example, using odds quotes on team moneylines (backing a team to win) from licensed US bookmakers in recent years from www.Odds-API.com, the heaviest favorites generally have win probabilities below 85%: the bottom decile of NFL favorites (as measured by odds) wins 83% of the time. The same figures for the NBA and Major League Baseball are 82% and 68%. The number of legs is capped at 10 because evidence discussed later shows that fewer than 10% of parlays have 10 legs or more.

Table 2: Threshold geometric means of probabilities (p) for optimal parlay length choice

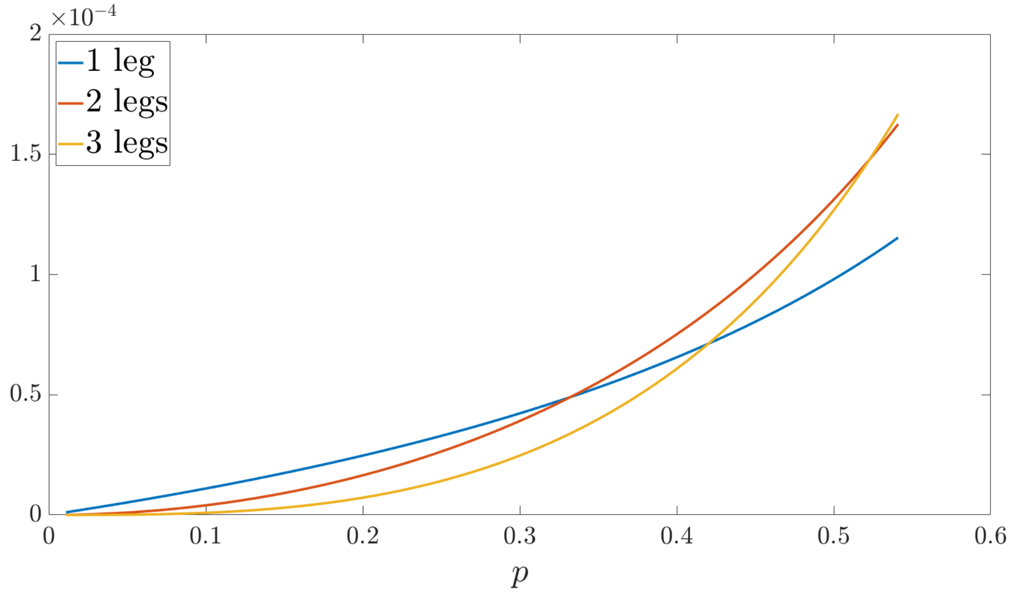
Section 1: Fixed Edge $e = 0.01$, Varying σ

Transition	$\sigma = 0.2$	$\sigma = 1$	$\sigma = 4$
Prefer 2 to 1	0.327	0.335	0.336
Prefer 3 to 2	0.515	0.526	0.529
Prefer 4 to 3	0.620	0.634	0.637
Prefer 5 to 4	0.687	0.703	0.706
Prefer 6 to 5	0.733	0.750	0.753
Prefer 7 to 6	0.767	0.784	0.788
Prefer 8 to 7	0.793	0.811	0.814
Prefer 9 to 8	0.813	0.831	0.835
Prefer 10 to 9	0.829	0.848	0.851

Section 2: Fixed $\sigma = 1$, Varying Edge e

Transition	$e = 0.005$	$e = 0.01$	$e = 0.05$
Prefer 2 to 1	0.334	0.335	0.337
Prefer 3 to 2	0.526	0.526	0.531
Prefer 4 to 3	0.634	0.634	0.640
Prefer 5 to 4	0.702	0.703	0.709
Prefer 6 to 5	0.749	0.750	0.756
Prefer 7 to 6	0.784	0.784	0.791
Prefer 8 to 7	0.810	0.811	0.817
Prefer 9 to 8	0.830	0.831	0.838
Prefer 10 to 9	0.847	0.848	0.854

Figure 1: Maximized expected utility for 1-, 2-, and 3-leg parlays as a function of the geometric average per-leg win probability p.



2.3. Same-game parlays (SGPs)

SGPs are growing in popularity. The component bets in these parlays typically do not conform to the independence assumption: If Josh Allen throws three touchdowns, then it is more likely that the Bills will win by 10 points, so these are not independent events. Calculating the correct success probabilities for these SGPs is complex and sportsbooks are well aware of this complexity. They typically use Monte Carlo simulation to do these calculations – firms like Huddle and Genius, which provide odds for sportsbooks have documented publicly that their models are Monte-Carlo-based.[4]

One approach to modeling bettors having edges on SGPs would be to build a general model of correlated outcomes but what matters for the optimal stake and leg-length problem is the offered odds and subjective joint success probability; the micro-foundations of how that joint probability is determined or calculated by sportsbooks are not needed for our question. The key elements of the bettor believing they have an edge on each leg and the chance of winning declining as you add more legs can be modeled independently of the correlated outcomes question.

SGPs are also typically priced as a bundle, so their payout when successful is not the product of available decimal odds on individual component bets. So SGPs are best modeled as a simple two-outcome bet with combined odds of

$$D_N^{SGP} = \frac{(1+e)^N}{q} \tag{10}$$

so that the true probability of winning is q and the edge of $(1+e)^N$ is calibrated to be interpreted in the same units as the edge when bets are independent, measured correctly against the full correlation-consistent probability. In this case, the sportsbook is assumed to be correctly controlling for the cross-outcome correlations but systematically under-estimating the probability of the bundle of bets all winning. In this case, setting

$$p = q^{\frac{1}{N}} \tag{11}$$

we get the same results as in the table above. SGPs have a probability p^N of winning and the decimal odds are $\left(\frac{1+e}{p}\right)^N$, which is the same formulation of the problem as with independent bets.

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[4] See here for Huddle's description and here for Genius's.

3. Analytics: A Second-Order Approximation

The analytics underlying these results can be explained using a simple second-order approximation to the utility function. Here, we show how the optimal choice of parlay length is approximately independent of the level of risk aversion, present a formula determining the thresholds at which people prefer a higher leg length to a lower one and illustrate why the size of the edge also has little influence on the optimal parlay length choice.

3.1. Approximate neutrality of risk aversion

Let

$$R_N = \begin{cases} \left(\frac{1+e}{p}\right)^N - 1 & \text{with probability } p^N, \\ -1 & \text{with probability } 1 - p^N. \end{cases} \tag{12}$$

For CRRA preferences, expected utility is

$$E\left[U_N\right] = \frac{w^{1-\sigma} E[(1+sR_N)^{1-\sigma}] - 1}{1-\sigma}. \tag{13}$$

A second-order Taylor expansion of $(1+sR_N)^{1-\sigma}$ around $s = 0$ gives

$$(1+sR_N)^{1-\sigma} \approx 1 + (1-\sigma)sR_N - \frac{\sigma(1-\sigma)}{2} s^2 R_N^2. \tag{14}$$

Taking expectations, we get

$$E[(1+sR_N)^{1-\sigma}] \approx 1 + (1-\sigma)sE\left[R_N\right] - \frac{\sigma(1-\sigma)}{2} s^2 E\left[R_N^2\right], \tag{15}$$

Substituting this into expected utility yields

$$E\left[U_N\right] \approx \frac{w^{1-\sigma}}{1-\sigma}\left[1 + (1-\sigma)sE\left[R_N\right] - \frac{\sigma(1-\sigma)}{2} s^2 E\left[R_N^2\right]\right] - \frac{1}{1-\sigma}. \tag{16}$$

Terms that do not depend on s do not affect the choice of stake. Dropping these constants and the positive factor $w^{1-\sigma}$, maximizing expected utility is therefore equivalent to maximizing

$$\tilde{U} = sE\left[R_N\right] - \frac{\sigma s^2 E\left[R_N^2\right]}{2} \tag{17}$$

The optimal stake is thus

$$s = \frac{E\left[R_N\right]}{\sigma E\left[R_N^2\right]} \tag{18}$$

Inserting this, the maximized object becomes

$$\tilde{U} = \frac{1}{2\sigma} \frac{E\left[R_N\right]^2}{E\left[R_N^2\right]} \tag{19}$$

Since $1/(2\sigma)$ is positive for all $\sigma > 0$, the ranking of different bets in this approximation is governed by the ratio of the square of the first moment of R_N to its second moment. This does not depend on σ so while the level of risk aversion affects the optimal stake, it does not affect the ranking of parlays with differing lengths.

3.2. Thresholds

Let

$$r = 1 + e \tag{20}$$

denote the bettor's expected gross return per leg. For an N-leg parlay the net return per unit stake is

$$R_N = \begin{cases} \left(\frac{r}{p}\right)^N - 1 & \text{with probability } p^N, \\ -1 & \text{with probability } 1 - p^N. \end{cases} \tag{21}$$

The expected return is therefore

$$E[R_N] = p^N \left[\left(\frac{r}{p}\right)^N - 1\right] + (1 - p^N)(-1) = r^N - 1 \tag{22}$$

The second moment is

$$E[R_N^2] = p^N \left[\left(\frac{r}{p}\right)^N - 1\right]^2 + (1 - p^N) = \frac{r^{2N}}{p^N} - 2r^N + 1. \tag{23}$$

This means people will be indifferent between taking the optimal stake with N legs and with $N+1$ legs if

$$\frac{(r^N - 1)^2}{\frac{r^{2N}}{p^N} - 2r^N + 1} = \frac{(r^{N+1} - 1)^2}{\frac{r^{2N+2}}{p^{N+1}} - 2r^{N+1} + 1} \tag{24}$$

Appendix section A.2 shows that agents prefer the smaller value of N below this indifference level of p and the bigger one above it, so the second-order approximation also shows the property reported in Table 2 that the optimal decisions imply taking more legs as p rises.

For the case of $N = 1$, this complicated equation reduces to

$$(2r + 1)p^2 - (r + 1)^2 p + r^2 = 0. \tag{25}$$

One root is $p = 1$, corresponding to the trivial certainty case. The other, economically relevant, root is

$$p = \frac{r^2}{2r + 1}. \tag{26}$$

For credible possible edges so that r is relatively close to one, this equals about one-third, which explains why we find agents preferring two legs to one at about a one-third probability average per leg for each level of risk aversion and each of the assumed edges above.

3.3. Higher Leg Thresholds

Appendix A.3 shows that for relatively small edges, equation (24) reduces to the N-th order polynomial

$$(2N + 1)p^{N+1} - (N + 1)^2 p + N^2 = 0 \tag{27}$$

which does not feature the size of the edge. This explains why the table showed very similar parlay length choice across widely different values for e.

This polynomial always has the trivial root $p = 1$. Factoring gives

$$(p - 1)\left((2N + 1)\sum_{k=1}^{N} p^k - N^2\right) = 0 \tag{28}$$

The second factor is strictly increasing for $p > 0$, since its derivative is

$$(2N + 1)\sum_{k=1}^{N} kp^{k-1} > 0$$

Because it is negative at $p = 0$ and positive at $p = 1$, it has a unique root in $(0, 1)$. This is the only nontrivial positive root and hence the unique threshold relevant for the parlay problem.

Solving for the unique root in $(0, 1)$, this polynomial for successive values of N yields the following approximate thresholds in Table 3. These values match the numerical solutions of the exact indifference condition closely, showing that both the second-order approximation and the small-edge approximation provide good characterizations of optimal parlay length.

Table 3: Threshold geometric means of probabilities (p) based on equation (27)

Comparison	p^*
Prefer 2 to 1	0.333
Prefer 3 to 2	0.525
Prefer 4 to 3	0.633
Prefer 5 to 4	0.701
Prefer 6 to 5	0.748
Prefer 7 to 6	0.782
Prefer 8 to 7	0.808
Prefer 9 to 8	0.829
Prefer 10 to 9	0.846

4. Real-World Parlays

Multi-game parlays combine either point spread bets (will a team outperform their predicted margin of victory or loss?) or moneyline bets (will a team win?) Point spread bets are typically set with lines that mean both available bets have equal chances of winning, meaning $p = 0.5$ in our terminology. Moneyline bets have an average probability of winning of about one half, since one team wins and the other loses (though there can be ties in some cases), so $p = 0.5$ is again a reasonable assumption for their average probability of winning. It is possible in most major sports to find a set of games with very strong favorites, with probabilities of winning of about 80%, but it seems unlikely that many of the multi-game parlays placed by bettors are designed to feature only these heavy favorites.

Same-game parlays (SGPs) typically involve one bet on the result of the game along with other game- or player-specific proposition bets. Most of these proposition bets are of the over/under style with a points line. These are bets like — Will more than 50 points be scored? Will Josh Allen rush for more than 40 yards? They are typically priced with about equal chances of the over and under bets winning, so again $p = 0.5$ is appropriate.

SGPs sometimes also include “Yes only” bets like “Will Derrick Henry score a touchdown?” To estimate the likelihood of success of these bets, I gathered 174,891 quotes over the 2023 to 2025 NFL seasons, covering 14 licensed US sportsbooks, taken from TheOdds-API.com, on players to score an offensive touchdown, meaning they either rush the ball into the endzone or make a catch there. These quotes were matched to the NFL’s official touchdown scorer statistics to establish whether they won or lost. Dividing the odds quotes into deciles by the value of the decimal odds, as expected, you find that win rates increase as the odds decline. But even for bets in the decile with the lowest odds (average decimal odds of 2.01), win rates were only 45%. Similarly, for the same group of sportsbooks, across 55,351 quotes on players to score in the English Premier League in the 2024-25 season, bets with odds in the lowest decile won only 29% of the time.

I conclude from this that the geometric average probability of winning for the component bets in parlays is generally equal to or below one half, with the average for SGPs likely to be lower. Even if people had an edge on these bets, they should place at most a two-leg parlay and perhaps not combine bets at all.

The evidence, however, shows the median parlay has five legs, which is only consistent with maximizing expected utility if the geometric average winning probability of the components is 70% or over.

A 2024 report by Citizens Bank reported that the average number of legs per parlay in the US rose from 4.2 in January 2022 to 5.0 in January 2024. They used data from Juice Reel, a freely available bet-tracking application that aggregates data from users' connected sportsbook accounts. Because it draws on actual account data rather than self-reported behavior, its statistics on betting patterns are more reliable than survey-based evidence.[5] The application's user base skews toward engaged, experienced bettors rather than casual participants, which if anything suggests the average leg count it reports understates the true population average, since casual bettors attracted by advertising are more likely to place the long-legged parlays promoted by sportsbooks. Citizens also reported that 10% of all parlays in 2024 had 10 legs or more, up from 8% in 2022.[6] Investor relations meetings with the big sportsbook firms have also reported higher leg counts as a key driver behind increased profitability in recent years.[7]

5. Conclusions

In considering the decision to take a combination of bets that each has a positive expected value, our topic has echoes of Paul Samuelson's (1963) famous puzzle set for colleagues as to whether they would take multiple positive expected value bets that they would turn down on a standalone basis. While many people have argued the decision to take multiple such bets is sensible, Samuelson argued that those who had agreed to take N of these bets should turn down an additional standalone bet, and on this basis, induction meant they should not take any of them. Later, Pratt and Zeckhauser (1987) showed that Samuelson was correct, in the sense that a wide range of utility functions (including all CRRA) imply rejecting a combination of gambles that would each be turned down on a standalone

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[5] Here

[6] Here

[7] See, for example, this transcript of a DraftKings investor relations call from 2024.

basis.

Our problem is different, both because it involves individual bets that people *will* take and because the combination method involves geometric accumulation. And while Samuelson's conjecture was essentially a hypothetical one, millions of people are placing parlay bets every day. But our result again highlights a puzzle in which people are perhaps overstating the benefits that come from combining multiple gambles. Even with an edge, people with CRRA utility functions should not combine typical sports bets into parlays with more than 2 legs. This means that two pieces of "expert" advice common on sports betting websites—to use parlays to compound an edge and to use the Kelly criterion to size bets—are contradictory.

The result that the optimal length of parlays is almost independent of the size of an edge or the level of risk aversion is perhaps surprising. For a fixed stake size, those with a greater edge and with lower risk aversion will take longer-legged parlays. But once the stake can be adjusted, this relationship barely exists. Those with higher risk aversion or a lower edge place smaller stakes and this means their decisions about how many legs to take are essentially the same as those who tolerate more risk or have a bigger edge.

Of course, in most cases, the correct argument for why parlays are bad bets is that the gambler does not have any edge, so they are compounding their disadvantage. But it can be hard to convince people who have carefully selected their component bets that this argument applies to their choice. At the same time, it is perhaps unlikely they would be more convinced by our demonstration that their decision violates expected utility theory, even if they have an edge. This perhaps recalls Ellsberg's (1961) attempts to explain to people who preferred the less ambiguous red-black urn that they were violating Savage's axioms. But just as Ellsberg's results prompted research to explain ambiguity aversion, our findings suggest models other than rational subjective expected utility theory are likely required to explain why people place parlays.

It is also worth noting that some bettors and investors have advocated for the Kelly criterion not because of its association with log utility, but for its well-known long-run growth properties (see, for example, Thorp, 1971). Kelly (1956) showed that maximizing expected log returns corresponds to maximizing the asymptotic rate of capital growth, while Breiman (1961) showed that this strategy maximizes the almost-sure exponential growth rate of wealth and eventually dominates any essentially different strategy in terms of accumulated wealth in repeated favorable gambles. Our log utility results show, however, that long parlays do not maximize long-run growth. So, leaving expected utility theory aside and focusing only on long-run payoffs, parlays with a high number of legs appear to be bad bets, even if you have an edge.

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A Derivations

This contains the derivations for the results in the paper.

A.1 Optimal stake derivation

For the utility function $U(w) = \frac{w^{1-\sigma}-1}{1-\sigma}$, marginal utility is $U'(w) = w^{-\sigma}$. Substituting this into the first-order condition, we get

$$\frac{(w + x(D-1))^{-\sigma}}{(w-x)^{-\sigma}} = \frac{1-p}{p(D-1)} \tag{A.1}$$

Raising both sides of the equation to the power of $-1/\sigma$ and cross-multiplying yields:

$$(w + x(D-1)) \left(\frac{1-p}{p(D-1)}\right)^{1/\sigma} = w - x \tag{A.2}$$

Grouping all terms containing the stake x on the left side and all terms containing initial wealth w on the right side:

$$x + x(D-1)\left(\frac{1-p}{p(D-1)}\right)^{1/\sigma} = w - w\left(\frac{1-p}{p(D-1)}\right)^{1/\sigma} \tag{A.3}$$

and solving for x, we get

$$x = \left(\frac{1 - \left(\frac{1-p}{p(D-1)}\right)^{1/\sigma}}{1 + (D-1)\left(\frac{1-p}{p(D-1)}\right)^{1/\sigma}}\right) w \tag{A.4}$$

Setting $\sigma = 1$, this reduces to the Kelly Criterion:

$$x = \left(\frac{pD-1}{D-1}\right) w \tag{A.5}$$

which can also be derived from starting from a log utility function.

A.2 Ranking above and below thresholds

Let

$$B_N(p) \equiv \frac{(r^N-1)^2}{\frac{r^{2N}}{p^N} - 2r^N + 1} = \frac{p^N(r^N-1)^2}{r^{2N} - (2r^N-1)p^N} \tag{A.6}$$

denote the approximate attractiveness of an N-leg parlay. To compare N and $N+1$ legs, note that

$$B_N(p) - B_{N+1}(p) \tag{A.7}$$

has the same sign as

$$\begin{aligned}\Psi_N(p) &= (r^N-1)^2\left[r^{2N+2}-(2r^{N+1}-1)p^{N+1}\right]-p(r^{N+1}-1)^2\left[r^{2N}-(2r^N-1)p^N\right] \\ &= r^{2N}\left[r^2(r^N-1)^2-p(r^{N+1}-1)^2+p^{N+1}(r-1)(2r^{N+1}-r-1)\right]. \end{aligned} \tag{A.8}$$

Since $r^{2N}>0$, the sign of $B_N(p)-B_{N+1}(p)$ is the sign of

$$K_N(p)\equiv r^2(r^N-1)^2-p(r^{N+1}-1)^2+p^{N+1}(r-1)(2r^{N+1}-r-1). \tag{A.9}$$

Now use the identity

$$(r^{N+1}-1)^2-r^2(r^N-1)^2=(r-1)(2r^{N+1}-r-1) \tag{A.10}$$

to rewrite this as

$$K_N(p)=(1-p)r^2(r^N-1)^2-p(1-p^N)(r-1)(2r^{N+1}-r-1). \tag{A.11}$$

Since

$$1-p^N=(1-p)(1+p+\cdots+p^{N-1}), \tag{A.12}$$

we obtain

$$K_N(p)=(1-p)\,J_N(p), \qquad \text{where} \tag{A.13}$$

$$J_N(p)\equiv r^2(r^N-1)^2-(r-1)(2r^{N+1}-r-1)\,p(1+p+\cdots+p^{N-1}). \tag{A.14}$$

For $0<p<1$, we have $1-p>0$, so the sign of $B_N(p)-B_{N+1}(p)$ is the sign of $J_N(p)$. But $J_N(p)$ is strictly decreasing on $(0,1)$, because

$$(r-1)(2r^{N+1}-r-1)>0 \tag{A.15}$$

and

$$p(1+p+\cdots+p^{N-1}) \tag{A.16}$$

is strictly increasing on $(0,1)$. Hence $J_N(p)=0$ has at most one solution in $(0,1)$.

Thus, if an interior cutoff p_N^* exists such that $B_N(p_N^*)=B_{N+1}(p_N^*)$, then it is unique, and

$$p<p_N^* \quad\Rightarrow\quad B_N(p)>B_{N+1}(p), \qquad p>p_N^* \quad\Rightarrow\quad B_N(p)<B_{N+1}(p). \tag{A.17}$$

So below the threshold the bettor prefers the shorter parlay, while above it the bettor prefers the longer parlay.

A.3 Higher leg threshold polynomial

To derive approximate thresholds for longer parlays and to see why the cutoff depends only weakly on the edge, start by flipping the fractions in equation (24)

$$\frac{\frac{r^{2N}}{p^N} - 2r^N + 1}{(r^N - 1)^2} = \frac{\frac{r^{2N+2}}{p^{N+1}} - 2r^{N+1} + 1}{(r^{N+1} - 1)^2} \tag{A.18}$$

Using

$$\frac{r^{2N}}{p^N} - 2r^N + 1 = (r^N - 1)^2 + r^{2N}\left(\frac{1}{p^N} - 1\right) \tag{A.19}$$

this can be written as

$$\frac{r^{2N}\left(\frac{1}{p^N} - 1\right)}{(r^N - 1)^2} = \frac{r^{2N+2}\left(\frac{1}{p^{N+1}} - 1\right)}{(r^{N+1} - 1)^2} \tag{A.20}$$

For a small edge,

$$r^N - 1 = (1+e)^N - 1 \approx Ne \qquad r^{N+1} - 1 \approx (N+1)e \tag{A.21}$$

so

$$\frac{r^{2N}\left(\frac{1}{p^N} - 1\right)}{N^2 e^2} \approx \frac{r^{2N+2}\left(\frac{1}{p^{N+1}} - 1\right)}{(N+1)^2 e^2}. \tag{A.22}$$

Cancelling e^2 gives

$$\frac{r^{2N}\left(\frac{1}{p^N} - 1\right)}{N^2} \approx \frac{r^{2N+2}\left(\frac{1}{p^{N+1}} - 1\right)}{(N+1)^2} \tag{A.23}$$

Finally, since

$$r^{2N+2} = r^{2N}(1+e)^2 \tag{A.24}$$

and $(1+e)^2 \approx 1$ for small e, we have $r^{2N+2} \approx r^{2N}$, so

$$\frac{N^2}{\frac{1}{p^N} - 1} \approx \frac{(N+1)^2}{\frac{1}{p^{N+1}} - 1} \tag{A.25}$$

Simple algebra then yields

$$(2N+1)p^{N+1} - (N+1)^2 p + N^2 = 0 \tag{A.26}$$