A Two-Sector Approach
to Modeling U.S. NIPA Data

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Abstract

The one-sector Solow-Ramsey model is the most popular model of long-run economic growth. This paper argues that a two-sector approach, in which technological progress in the production of durable goods exceeds that in the rest of the economy, provides a far better picture of the long-run behavior of the U.S. economy. The paper shows how to use the two-sector approach to model the real chain-aggregated variables currently featured in the U.S. National Income and Product Accounts. It is shown that each of the major chain-aggregates—output, consumption, investment, and capital stock—will tend in the long-run to grow at steady, but different, rates. Implications for empirical analysis based on these data are explored.

Since the 1950s, the Solow-Ramsey model of economic growth, which treats all output in the economy as deriving from a single aggregate production function, has been the canonical model of how the macroeconomy evolves in the long run. The model has also featured prominently in the analysis of economic fluctuations: Business cycles are commonly characterized as correlated deviations from the model's long-run "balanced growth" path, which features the real aggregates for consumption, investment, output, and the capital stock, all growing together at an identical rate determined by the growth rate of the aggregate technology process.

The main purpose of this paper is to make a simple point: Despite its central role in economics textbooks and in business cycle research, the traditional one-sector model of economic growth actually provides a poor description of the long-run behavior of the U.S. economy. A simple alternative two-sector model, in which technological progress in the production of durable goods exceeds that in the rest of the economy, explains a number of crucial long-run properties of U.S. macroeconomic data that are inconsistent with the one-sector growth model, and is far better suited for modeling these data as currently constructed. As such, this two-sector approach provides a better "baseline" model for macroeconomic analysis.

In arguing the case for a two-sector approach to long-run modeling of the macroeconomy, this paper re-enforces the principal message of a recent paper by Jeremy Greenwood, Zvi Hercowitz, and Per Krusell (1997), which also employed a model in which technological progress in the production of durable goods is faster than in the rest of the economy. However, this paper differs from the work of Greenwood, Hercowitz, and Krusell in using a two-sector model to fit macroeconomic data as published in the U.S. National Income and Product Accounts (NIPAs). As a result, the approach taken here differs both in the *evidence* cited in favor of a two-sector model, and in its treatment of *aggregation*. This approach produces some new results on the properties of the real NIPA aggregates that have far-reaching implications for empirical work in macroeconomics.

Concerning the evidence for a two-sector approach, Greenwood, Hercowitz, and Krusell argued that the official price deflators for durable equipment in the NIPAs understate the relative price decline for these products, and consequently also underestimate the differential in sectoral rates of technological progress. Thus, their work was not based on NIPA data, but instead featured the equipment price in-

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dex developed by Robert Gordon (1990). More recently, much of the research on price mis-measurement has focused on non-manufacturing sectors, so this approach of adjusting only for price mis-measurement for durable equipment seems likely to overstate the true pace of relative price decline for equipment, and thus the case for a two-sector approach. However, I document that—once we include evidence from the 1990s—then a compelling case for the two-sector framework can be made using the official data from the NIPAs.

The failure of the one-sector approach to fit published NIPA data is important, because most researchers use these data for empirical work in macroeconomics, and much of this research relies on the assumption that the one-sector growth model provides an adequate approximation to the long-run behavior of these data. For example, researchers often invoke the balanced growth property to demonstrate that their empirical specifications have sound long-run properties. However, the fact that most investment spending is on durable goods, while most consumption outlays are on nondurable goods and services, implies that the balanced growth prediction is now firmly rejected, a finding that overturns an often-cited earlier result (based on a sample through 1988) of King, Plosser, Stock, and Watson (1991).

The second new aspect of this paper, the approach to aggregation, requires a little more explanation. In a one-sector world, all output has the same price. In this case, aggregate real output, as measured by weighting quantities according to a fixed set of base-year prices, should be independent of the choice of base year and should grow at a steady rate in the long-run. In reality, because the durable goods sector has faster growth in real output and a declining relative price, the growth rate of a fixed-weight measure of real GDP will depend on the choice of base year: The further back we choose the base year, the higher the current growth rate will be, so real aggregate output measured in this manner will always tend to accelerate.

There are a number of possible solutions to this problem of base-year dependence in fixed-weight measures of growth in aggregate real output. This paper follows the approach taken by the U.S. Commerce Department's Bureau of Economic Analysis (BEA) in constructing the NIPAs. The BEA abandoned the fixed-weight approach in 1996, switching to measuring all real aggregates using a "chain index" method, which uses continually updated relative price weights. I follow this procedure in modeling the major real aggregates using a chain-weighting method. The two-sector

model yields a number of important insights into the long-run properties of U.S. NIPA data. In particular, I show how each of the major chain-aggregated variables— output, consumption, investment, and capital stock—will tend in the long-run to grow at steady, but different, rates. This result has important implications for empirical analysis based on these data.

Another solution to the problem of base-year dependent real output growth is the one chosen by Greenwood, Hercowitz, and Krusell, which is to use one category as a numeraire, and define real output by deflating all nominal output by the price index for this category, which in their case was consumer nondurables and nonhousing services. Because the *nominal* output of the durable goods sector tends to grow at the same rate as other nominal output, this measure of aggregate real output can exhibit steady-state growth. Greenwood et al argue that this measure is superior to the measure of real output featured in the NIPAs. However, the position taken in this paper is that neither method produces an intrinsically "correct" measure of aggregate real output, but rather that the two approaches merely measure different concepts.

The rest of the paper is organized as follows. Section 1 uses U.S. NIPA data to document the case against the one-sector model and in favor of a two-sector approach. Section 2 presents the two-sector model. Section 3 uses the model to fit aggregate U.S. data constructed according to the current chain-aggregation procedures, and discusses the model's implications for the analysis of these data. Section 4 calculates the contribution of technological progress in the durable goods sector to growth in aggregate output and welfare. Finally, Section 5 compares our approach with that of Greenwood, Hercowitz, and Krusell.

1 Evidence

All data used in this section come from the U.S. National Income and Products Accounts.

1.1 A Look at the Data

The standard neoclassical growth model starts with an aggregate resource constraint of the form $C_t + I_t = Y_t = A_t L_t^{1-\beta} K_t^{\beta}$. Because all goods are produced using the

same technology, a decentralized market equilibrium must feature consumption and investment goods having the same price. Thus, in the standard expression of the resource constraint, the variables C, I, and Y have all been deflated by an index for this common price to express them in quantity or real terms.

The model usually assumes that a representative consumer maximizes the present discounted value of utility from consumption, subject to a law of motion for capital and a process for aggregate technology. If the technology process is of the form $\log A_t = a + \log A_{t-1} + \epsilon_t$, where ϵ_t is a stationary zero-mean series, then it is well known that the model's solution features C, I, K, and Y all growing at an average rate of $\frac{a}{1-\beta}$ in the long-run. This means that ratios of any of these variables will be stationary stochastic processes.

This hypothesis of balanced growth and stationary "great ratios" has been held as a crucial stylized fact in macroeconomics at least as far back as the well-known contribution of Kaldor (1957). More recently, King, Plosser, Stock, and Watson (1991), using data through 1988, presented evidence that the real series for investment and consumption (and consequently output) share a common stochastic trend, implying stationarity of the ratio of real investment to real consumption. Figure 1 shows, however, that including data through 2000:Q2 firmly undermines the evidence for the balanced growth hypothesis. Since 1991, investment has risen dramatically in real terms relative to consumption, growing an average of 8.4 percent per year over the period 1992-99, 4.7 percentage points per year faster than consumption. Indeed, applying a simple cyclical adjustment by comparing peaks, there is some evidence that the ratio of real investment to real consumption has exhibited an upward trend since the late 1950s, with almost every subsequent cyclical peak setting a new high.

It might be suspected that the strength of investment after 1991 was the result of some special elements in the expansion of the 1990s such as its unusual length, the surge in equity valuations, or perhaps the "crowding in" associated with the reduction of Federal budget deficits. If this were the case, then we would expect the ratio of real investment to real consumption to decline to its long-run average with the next recession. However, while the factors just cited have likely played some role in the remarkable strength in investment, a closer examination reveals that something more fundamental, and less likely to be reversed, has also been at

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work.

A standard disaggregation of the data shows that the remarkable increase in real investment in the 1990s was entirely due to outlays on producers' durable equipment, which grew 9.1 percent per year; real spending on structures grew only 2.2 percent per year.[1] Moreover, Figure 2a shows that this pattern of growth in real equipment spending outpacing growth in structures investment is a long-term one, dating back to about 1960. Importantly, this figure also documents that the same pattern is evident within the consumption bundle: Real consumer spending on durable goods has consistently grown faster than real consumption of nondurables and services. These facts are summarized jointly by Figure 2b, which shows that, over the past 40 years, total real output of durable goods has consistently grown faster than total real business output (defined as GDP excluding the output of government and nonprofit institutions).

The faster growth in the U.S. economy's real output of durable goods reflects an ongoing decline in their relative price. Remarkably, as can be seen from Figure 3a, while the share of durable goods in *nominal* business output has bounced around over time, it has exhibited essentially no trend over the past 50 years. Similarly, the increase in real fixed investment relative to real consumption has been a function of declining prices for durable goods and the fact the such goods are a more important component of investment than consumption: Figure 3b shows that, once expressed in nominal terms, the ratio of fixed investment to consumption has also been trendless throughout the postwar period.[2]

Figure 4 illustrates the relevant relative price trends. Relative prices for durable goods, of both consumer and producer kind, have trended down since the late 1950s. The decline for equipment did not show through to the deflator for fixed investment until about 1980 because the relative price of structures, which show little trend over the sample as a whole, rose between 1960 and 1980 and fell thereafter.

Viewed from this perspective, the post-1991 increase in real investment relative to real consumption does not appear as a particularly cyclical phenomenon. Instead,

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[1]Tevlin and Whelan (2001) provide a detailed discussion of the behavior of U.S. equipment investment in the 1990s.

[2]Over 1960-1999, durable goods accounted for 13 percent of consumption expenditures and 47 percent of investment expenditures.

it reflects long-running trends. Over the long run, nominal spending on investment and consumption have tended to grow at the same rate. But the higher share of durable goods in investment, and the declining relative price of these goods, together imply that real investment tends to grow faster than real consumption.

It is instructive to reconcile this assessment that the ratio of real investment to real consumption is nonstationary with King, Plosser, Stock, and Watson's conclusion (based on a sample ending in 1988) that it was stationary. Figures 3 and 4 show that two other factors—the increase in the relative price of structures from the early 1960s until 1980, and then the decline in the nominal share of investment in the 1980s—had the effect of masking somewhat the upward trend in the ratio of real investment to real consumption until the 1990s. However, over the entire postwar period, these two variables have been stationary. Unless those patterns change going forward, the ratio of real investment to real consumption should continue to trend up.

Formal statistical tests confirm the intuition suggested by these graphs, that the real ratios are non-stationary and that the nominal ratios are stationary. An Augmented Dickey-Fuller (ADF) unit root test for the ratio of real fixed investment to real consumption (shown in Figure 1) gives a t-statistic of -0.78, meaning we cannot come close to rejecting the hypothesis that this series has a unit root.[3] The t-statistic for the corresponding nominal ratio (shown in Figure 3b) is -2.99, which rejects the unit root hypothesis at the 5 percent level. Similarly, the ADF t-statistic for the ratio of real durables output to real business output (shown in Figure 2b) is -0.05; the corresponding nominal ratio (shown in Figure 3a) has a t-statistic of -4.03, rejecting the unit root hypothesis at the 1 percent level.

1.2 Some Price-Measurement Issues

Before moving on to describe a model to fit the patterns just documented, a discussion of the data is appropriate. In previous work aimed at replacing the one-sector approach with a model containing multiple technology shocks, Greenwood, Hercowitz, and Krusell (1997) cited Robert Gordon's (1990) evidence that price

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[3]This test is based on a regression of the variable on its lagged level, an intercept, and lagged first-differences. The number of lags used (two) was chosen according to the general-to-specific procedure suggested by Campbell and Perron (1991), but the results were not particularly sensitive to this choice.

inflation for durable goods is overstated in the NIPAs because valuable quality improvements are often ignored. Their analysis replaced the official price deflator for equipment investment with the alternative index developed by Gordon, but used the NIPA price deflators for all other categories.

The preceding discussion shows that it is not necessary to apply this type of adjustment to make the case against the one-sector approach (note this is partly because we used a sample period that extended ten years beyond the sample of Greenwood et al, which ended in 1990.) However, for some of the calculations we will perform later, such as the contribution of technological progress in the production of durable goods to aggregate output growth, the accuracy of the answers will depend on having the correct series for the *relative* price of durable goods, which raises the question of whether we should apply these price adjustments.

The approach taken here will be to use the official data for all calculations. The reason for this is that much of the recent evidence on price mis-measurement, including some of the work of the Boskin Commission, has focused on prices for non-manufacturing output.[4] While durable goods are one of the areas where some of the adjustments for quality suggested by Gordon have actually been adopted by the NIPAs (most notably for computers), there are serious questions about the adequacy of the output deflators for sectors such as financial services and construction. Measured productivity growth in these sectors has been extremely weak and seems likely to have been underestimated. Given that the nominal output series are generally considered to be relatively accurate, these considerations suggest that price inflation in these non-manufacturing industries may be overstated.[5] In the absence of convincing evidence pointing one way or the other for adjusting the relative price of durable goods, I have chosen not to apply any such adjustment.

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[4]See Boskin et al (1998).

[5]See Corrado and Slifman (1999) and Gullickson and Harper (1999).

2   The Two-Sector Model

We have documented three important stylized facts about the evolution of the U.S. economy over the past 40 years:

- The real output of the durable goods sector has grown significantly faster than the real output of the rest of the economy.

- Because durable goods are a more important component of investment than consumption, real investment has grown faster than real consumption.

- These trends reflect, and are reflected in, relative price movements: The share of durable goods in nominal output has been stable, as have the shares of nominal consumption and investment.

Clearly, if we are to explain the trend in the relative price of durable goods, we need a two-sector approach that distinguishes these goods separately from other output. This section develops, and empirically calibrates, a simple two-sector model that fits the facts just described by allowing for a faster pace of technological progress in the sector producing durable goods.[6]

The model presented here shares a number of common features with that of Greenwood, Hercowitz, and Krusell (1997) in that it focuses on a two-sector economy with one sector having faster technological progress than the other. One difference worth noting is that I focus on the durable goods sector as a whole as the sector with faster technological progress, rather than limiting the focus to the production of producer durables. This is necessary if we wish to fit the facts about real expenditures on consumer durables, and this aspect of the model will be important for the welfare calculations presented later in the paper. Also, as noted above, the empirical calibration and treatment of aggregation in this paper are very different from that in the work of Greenwood et al.

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[6]There is plenty of available evidence that points towards a faster rate of technological progress in the durable goods sector. For example, the detailed Multifactor Productivity (MFP) calculations published by the Bureau of Labor Statistics reveal durable manufacturing as the sector of the economy with the fastest MFP growth. See http://stats.bls.gov/mprhome.htm.

2.1 Technology and Preferences

The model economy has two sectors. Sector 1 produces durable equipment used by both consumers and producers, while Sector 2 produces for consumption in the form of nondurables and services and for investment in the form of structures. The notation to describe this is as follows: Sector i supplies C_i units of its consumption good to households, I_{ij} units of its capital good for purchase by sector j, and keeps I_{ii} units of its capital good for itself. The production technologies in the two sectors are identical apart from the fact that technological progress advances at a different pace in each sector[7]:

$$Y_1 = C_1 + I_{11} + I_{12} = A_1 K_{11}^{\beta_1} K_{21}^{\beta_2} L_1^{1-\beta_1-\beta_2} \tag{1}$$

$$Y_2 = C_2 + I_{21} + I_{22} = A_2 K_{12}^{\beta_1} K_{22}^{\beta_2} L_2^{1-\beta_1-\beta_2} \tag{2}$$

$$\Delta \log A_{it} = a_i + \epsilon_{it} \tag{3}$$

Equipment and structures depreciate at different rates, so capital of type i used in production in sector j accumulates according to

$$\Delta K_{ij,t} = I_{ij,t} - \delta_i K_{ij,t-1} \tag{4}$$

To keep things as simple as possible, we will not explicitly model the labor-leisure allocation decision, instead assuming a fixed labor supply normalized to one

$$L_1 + L_2 = L = 1 \tag{5}$$

and that households maximize the expected present discounted value of utility

$$E_t \left[\sum_{k=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{k-t} (\alpha_1 \log D_t + \alpha_2 \log C_{2t}) \right] \tag{6}$$

Here, D is the stock of consumer durables, which evolves according to

$$\Delta D_t = C_{1t} - \delta_1 D_{t-1} \tag{7}$$

We will focus on the steady-state equilibrium growth path implied by a deterministic version of the model in which A_1 and A_2 grow at constant, but different

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[7]Importantly, I have assumed descreasing returns to accumulable factors in both sectors, which implies that growth is exogenous. As Rebelo (1991) has shown, the assumption to constant returns to accumulable factors in the capital-producing sector results in an endogenous growth rate.

rates; in a full stochastic solution, all deviations from this path will be stationary. We will define a steady-state equilibrium path of the model to have two features. First, the real output of sector i grows at a constant rate g_i. Second, we will require the nominal share of durable goods output, and of consumption and investment, to be constant, as suggested by the empirical evidence of the previous section. It is this latter requirement that has dictated our choice of log-linear preferences and technology.[8]

2.2 Calculation of Steady-State Growth Rates

We will show below that, along the steady-state growth path, the fraction of each sector's output sold to households, to Sector 1, and to Sector 2, are all constant, and that L_1 and L_2 are both fixed. This implies that investment in each type of capital good by each sector grows a fixed rate along a steady-state growth path. Now, note that if investment in a capital good grows at rate g then the capital stock for that good will also grow at rate g. Consequently, taking log-differences of (1) and (2), we get

$$g_1 = a_1 + \beta_1 g_1 + \beta_2 g_2 \tag{8}$$

$$g_2 = a_2 + \beta_1 g_1 + \beta_2 g_2 \tag{9}$$

The solutions to these equations are

$$g_1 = \frac{(1 - \beta_2) a_1 + \beta_2 a_2}{1 - \beta_1 - \beta_2} \tag{10}$$

$$g_2 = \frac{\beta_1 a_1 + (1 - \beta_1) a_2}{1 - \beta_1 - \beta_2} \tag{11}$$

so the growth rate of real output in sector 1 exceeds that in sector 2 by $a_1 - a_2$.

2.3 Competitive Equilibrium

We could solve for the model's competitive equilibrium allocation by formulating a central planning problem, but our focus on fitting the facts about nominal and

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[8]Technically, by arguments that are exactly analagous to the one-sector proofs of King, Plosser, and Rebelo (1988), the first requirement (steady-state growth) requires only that technological progress in both sectors be of the labor-augmenting form, and that preferences be log-linear. However, the second requirement (constant nominal shares for each type of investment and consumption) requires both technology and preferences to be log-linear.

real ratios makes it useful to solve for the prices that generate a decentralized equilibrium. In doing so I assume the following structure. Firms in both sectors are perfectly competitive, so they take prices as given and make zero profits. Households can trade off consumption today for consumption tomorrow by earning interest income on savings. This takes the form of passing savings to arbitraging intermediaries who purchase capital goods, rent them out to firms, and then pass the return on these transactions back to households.

Relative Prices: Because relative prices, rather than the absolute price level, are what matter for the model's equilibrium, we will set sector 2's price equal to one in all periods, and denote sector 1's price by p_t. Given a wage rate, w, and rental rates for capital, c_1 and c_2, the cost function for firms in Sector i is

$$TC_i\left(w, c_1, c_2, Y_i, A_i\right) = \frac{Y_i}{A_i} \left(\frac{c_1}{\beta_1}\right)^{\beta_1} \left(\frac{c_2}{\beta_2}\right)^{\beta_2} \left(\frac{w}{1 - \beta_1 - \beta_2}\right)^{1 - \beta_1 - \beta_2}$$

Prices equal marginal cost, so the relative price of sector 1's output is

$$p = \frac{A_2}{A_1} \tag{12}$$

Combined with (10) and (11), this tells us that the ratio of sector 1's nominal output to sector 2's nominal output is constant along the steady-state growth path: The decline in the relative price of durable goods exactly offsets the faster increase in quantity. We can also show that the fraction of each sector's output devoted to consumption and investment goods is constant. Together, these results imply that the ratios of all nominal series are constant along the steady-state growth path.

So, our model is capable of fitting the stylized facts about the stability of nominal ratios, and differential growth rates for the real output of the durable goods sector and the rest of the economy. To calibrate the model empirically to match the behavior of *aggregate* real NIPA series, we will also need to derive the steady-state values of the nominal ratios of consumption, capital, investment, and output of type 1 relative to their type-2 counterparts, as functions of the model's parameters.

Consumption: Denominating household labor income, w_t, and financial wealth, Z_t, in terms of the price of sector 2's output, the dynamic programming problem

for the representative household is

$$V_t\left(Z_t, D_{t-1}\right) = \underset{C_{1t}, C_{2t}}{\text{Max}} \left[\alpha_1 \log D_t + \alpha_2 \log C_{2t} + \frac{1}{1+\rho} E_t V_{t+1}\left(Z_{t+1}, D_t\right)\right]$$

subject to the accumulation equation for the stock of durables and the condition governing the evolution of financial wealth:

$$Z_{t+1} = \left(1 + r_{t+1}\right)\left(Z_t + w_t - p_t C_{1t} - C_{2t}\right)$$

Here r is the real interest rate defined relative to the price of sector 2's output.

The first-order condition for type-2 consumption and the envelope condition for financial wealth combine to give a standard Euler equation:

$$\frac{1}{C_{2t}} = \frac{1}{1+\rho} E_t \left[\frac{1+r_{t+1}}{C_{2,t+1}}\right] \tag{13}$$

So, using a log-linear approximation, the steady-state real interest rate is

$$r = g_2 + \rho \tag{14}$$

Some additional manipulations of the first-order conditions (detailed in an appendix) produces the following equation:

$$\frac{p\left(g_1 + \delta_1 + \rho\right) D}{C_2} = \frac{\alpha_1}{\alpha_2} \tag{15}$$

Also, as noted earlier, the steady-state solution involves the stock of durables growing at rate g_1, so that $C_1 = \left(g_1 + \delta_1\right) D$ by equation (7). Substituting this expression for D into (15) implies the following condition for the ratio of nominal consumption expenditures for the two sectors:

$$\frac{pC_1}{C_2} = \frac{\alpha_1}{\alpha_2}\left(\frac{g_1 + \delta_1}{g_1 + \delta_1 + \rho}\right) \tag{16}$$

Investment: Capital goods are purchased by arbitraging intermediaries who rent them out to firms at Jorgensonian rental rates:

$$
\begin{aligned}
c_1 &= p\left(r + \delta_1 - \frac{\Delta p}{p}\right) = p\left(g_1 + \delta_1 + \rho\right) \\
c_2 &= r + \delta_2 = g_2 + \delta_2 + \rho
\end{aligned}
$$

The profit functions for the two sectors are

$$\pi_1 = pA_1 K_{11}^{\beta_1} K_{21}^{\beta_2} L_1^{1-\beta_1-\beta_2} - wL_1 - c_1 K_{11} - c_2 K_{21}$$

$$\pi_2 = A_2 K_{12}^{\beta_1} K_{22}^{\beta_2} L_2^{1-\beta_1-\beta_2} - wL_2 - c_1 K_{12} - c_2 K_{22}$$

The first-order conditions for inputs can be expressed as

$$
\begin{aligned}
L_1 &= \frac{p(1-\beta_1-\beta_2)Y_1}{w} & K_{11} &= \frac{\beta_1 Y_1}{g_1 + \delta_1 + \rho} & K_{21} &= \frac{p\beta_2 Y_1}{g_2 + \delta_2 + \rho} & (17) \\
L_2 &= \frac{(1-\beta_1-\beta_2)Y_2}{w} & K_{12} &= \frac{1}{p}\frac{\beta_1 Y_2}{g_1 + \delta_1 + \rho} & K_{22} &= \frac{\beta_2 Y_2}{g_2 + \delta_2 + \rho} & (18)
\end{aligned}
$$

These conditions imply that, for each factor, the ratio of input used in sector 1 relative to that used in sector 2 equals the ratio of nominal outputs, which is constant. One implication is that both L_1 and L_2 are fixed in steady-state, as assumed earlier.

From these equations, we can also derive the steady-state ratios for nominal investment in capital of type 1 relative to nominal investment in capital of type 2, as well as the corresponding ratio for the capital stocks. Letting

$$\mu_i = \frac{\beta_i (g_i + \delta_i)}{g_i + \delta_i + \rho} \tag{19}$$

these ratios are

$$\frac{p(I_{11} + I_{12})}{I_{21} + I_{22}} = \frac{\mu_1}{\mu_2} \tag{20}$$

$$\frac{p(K_{11} + K_{12})}{K_{21} + K_{22}} = \frac{\beta_1}{\beta_2}\frac{g_2 + \delta_2 + \rho}{g_1 + \delta_1 + \rho} \tag{21}$$

where equation (20) uses the fact that $I_{ij} = (g_i + \delta_i) K_{ij}$ in steady-state.

Output: Combining the resource constraints, (1) and (2), with the first-order conditions for capital accumulation, (17) and (18), and the fact that $I_{ij} = (g_i + \delta_i) K_{ij}$ in steady-state, we obtain the following expressions for real output:

$$
\begin{aligned}
Y_1 &= C_1 + \mu_1 Y_1 + \frac{\mu_1}{p} Y_2 \\
Y_2 &= C_2 + p\mu_2 Y_1 + \mu_2 Y_2
\end{aligned}
$$

Combined with (16) these formulae can be re-arranged to obtain the share of consumption goods in the output in each sector:

$$\frac{C_1}{Y_1} = \frac{1 - \mu_1 - \mu_2}{1 - \mu_2 + \frac{\alpha_2 \beta_1}{\alpha_1}}$$

$$\frac{C_2}{Y_2} = \frac{1 - \mu_1 - \mu_2}{1 - \mu_1 + \frac{\alpha_1 \beta_2}{\alpha_2}}$$

Note that these equations, combined with the constancy of the ratios of capital of type i used in sector 1 to capital of type i used in sector 2, implied by (17) and (18), confirm our earlier assumption (made when deriving the steady-state growth rates) that the fraction of each sector's output sold to households, to Sector 1, and to Sector 2, are all constant.

We can combine these expressions with equation (16) to obtain the ratio of nominal output in sector 1 to nominal output in sector 2:

$$\frac{pY_1}{Y_2} = \frac{1 - \mu_2 + \frac{\alpha_2 \beta_1}{\alpha_1}}{1 - \mu_1 + \frac{\alpha_1 \beta_2}{\alpha_2}} \frac{pC_1}{C_2} = \left(\frac{1 - \mu_2 + \frac{\alpha_2 \beta_1}{\alpha_1}}{1 - \mu_1 + \frac{\alpha_1 \beta_2}{\alpha_2}} \right) \frac{\mu_1}{\beta_1} \frac{\alpha_1}{\alpha_2} = \theta \qquad (22)$$

Finally, in Section 5, we will find it useful to use the fact that equations (17) and (18) imply that the factor allocations to each sector can be expressed as simple functions of the economy's total allocation of each factor:

$$K_{11} = \frac{\theta}{1 + \theta} K_1 \qquad K_{12} = \frac{1}{1 + \theta} K_1 \qquad (23)$$

$$K_{12} = \frac{\theta}{1 + \theta} K_2 \qquad K_{12} = \frac{1}{1 + \theta} K_2 \qquad (24)$$

$$L_1 = \frac{\theta}{1 + \theta} L \qquad L_2 = \frac{1}{1 + \theta} L \qquad (25)$$

2.4 Calibration

To use the model for empirical applications, we will need values for the following eight parameters: a_1, a_2, β_1, β_2, δ_1, δ_2, ρ, and α_1/α_2 (only the ratio of these two parameters matters). The depreciation rates are fixed a priori at $\delta_1 = 0.13$ and $\delta_2 = 0.03$, on the basis of typical values used to construct the NIPA capital stocks for durable equipment and structures.[9] The values of the other parameters are set so that the model's steady-state growth path features important variables matching their average values over the period 1957-99, where this sample was chosen to match the start of the pattern of declining relative prices for durable goods:

- A ratio of nominal consumption of durables to nominal consumption of non-durables and services of 0.15.

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[9]See Katz and Herman (1997) for a description of these depreciation rates.

- A ratio of nominal output of durables to other nominal business sector output of 0.27.

- A growth rate of real durables output per hour worked of 3.9 percent.

- A growth rate of real other business sector output per hour worked of 1.8 percent.[10]

- An average share of income earned by capital of 0.31.

- A rate of return on capital of 6.5 percent, as suggested by King and Rebelo (2000).

In terms of the model's parameters these conditions imply the following six equations:

$$\frac{\alpha_1}{\alpha_2}\left(\frac{\frac{(1-\beta_2)a_1+\beta_2a_2}{1-\beta_1-\beta_2}+\delta_1}{\frac{(1-\beta_2)a_1+\beta_2a_2}{1-\beta_1-\beta_2}+\delta_1+\rho}\right) = 0.15 \tag{26}$$

$$\theta = 0.27 \tag{27}$$

$$\frac{(1-\beta_2)a_1+\beta_2a_2}{1-\beta_1-\beta_2} = 0.039 \tag{28}$$

$$\frac{\beta_1a_1+(1-\beta_1)a_2}{1-\beta_1-\beta_2} = 0.018 \tag{29}$$

$$\beta_1+\beta_2 = 0.31 \tag{30}$$

$$\frac{\beta_1a_1+(1-\beta_1)a_2}{1-\beta_1-\beta_2}+\rho = 0.065 \tag{31}$$

These solve to give parameter values $a_1 = 0.03$, $a_2 = 0.009$, $\beta_1 = 0.145$, $\beta_2 = 0.165$, $\rho = 0.047$, and $\frac{\alpha_1}{\alpha_2} = 0.19$. We will now use these parameter values to discuss some empirical applications of the model.

3   Fitting Aggregate NIPA Data

We have described the long-run growth path for each of our sectors, how they price their output, and how they split this output between consumption and investment goods. In this section, we consider the combined behavior of the two sectors: How do *aggregate* real variables behave? In particular, we explore the properties of the aggregate real series published in the U.S. NIPAs.

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[10] All per-hour calculations in this paper refer to total business-sector hours.

3.1 Unbalanced, Unsteady Growth?

Because the two sectors grow at steady but different rates, we have labeled our solution a steady-state growth path. However, it is clearly not a *balanced* growth path in the traditional sense. Indeed, according to the usual theoretical definition of aggregate real output as the sum of the real output in the two sectors, aggregate growth also appears to be highly *unsteady*. The growth rate will tend to increase each period with sector 1 tending to become "almost all" of real output. Similarly, we should expect to see the growth rates for the aggregates for consumption and investment also increasing over time, as the durable goods components become ever larger relative to the rest.

To illustrate this pattern, consider what we usually mean when we say that aggregate real output is the sum of the real output of the two sectors. If our economy produces four apples and three oranges, we don't mean that real GDP is seven! Rather, we start with a set of prices from a base year and use these prices to weight the quantities. Recalling our assumption that the price of output in sector 2 always equals one, we can construct such a measure by choosing a base year, b, from which to use a price for sector 1's output:

$$
\begin{aligned}
Y_t^{fw} &= p_b Y_{1t} + Y_{2t} \\
&= \left(p_0 e^{(g_2 - g_1)b} \right) \left(Y_{10} e^{g_1 t} \right) + Y_{20} e^{g_2 t} \\
&= p_0 Y_{10} e^{g_2 b} e^{g_1 (t-b)} + Y_{20} e^{g_2 t}
\end{aligned}
\tag{32}
$$

The growth rate of this so-called "fixed-weight" measure of real output will tend to increase every period, asymptoting towards g_1. And the time path of this acceleration will depend on the choice of base year: The further back in time we choose b, the faster Y_t^{fw} will grow. Intuitively, this occurs because the fixed-weight series has the interpretation "how much period t's output would have cost had all prices remained at their year-b level" and the percentage change in this measure must depend on the choice of base year, b. Because the fastest growing parts of the output bundle (durable goods) were relatively more expensive in 1960 than in 1990, the cost of the this bundle in 1960 prices must grow faster than the cost in 1990 prices.

This problem of unsteady and base-year-dependent growth is not just a prediction of our model; it is an important pattern to which national income accountants

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have had to pay a lot of attention. In the past, the BEA dealt with this problem by moving the base year forward every five years. This re-basing meant that current-period measures of real GDP growth could always be interpreted in terms of a recent set of relative price weights. However, this approach also had problems. For example, while 1996 price weights may be useful for interpreting 1999's growth rate, they are hardly a relevant set of weights for interpreting 1950's growth rate, given the very different relative price structure prevailing then. So, re-basing improves recent measures of growth at the expense of worsening the measures for earlier periods. In addition, periodic re-basing leads to a pattern of predictable downward revisions to recent estimates of real GDP growth.

3.2   Chain-Aggregates

Because of the problems with fixed-weight measures, the BEA abandoned this approach in 1996. Instead, it now uses a so-called chain index method to construct all real aggregates in the U.S. NIPAs, including real GDP. Rather than using a fixed set of price weights, chain indexes continually update the relative prices used to calculate the growth rate of the aggregate. Since the growth rate for each period is calculated using relative price weights prevailing close to that period, chain-weight measures of real GDP growth do not suffer from the interpretational problems of fixed-weight series, and do not need to re-calculated every few years.

It is important to note that the *levels* of real chain-aggregated series, as published by the BEA, have a very different interpretation to their fixed-weight counterparts. The chain-aggregated series are equated with their nominal counterparts in some arbitrary "base year," and then "chained" forward and backward from there using the index. Importantly, though, the choice of base year used to equate the nominal and real series has no affect on the growth rates of the series. Chain-aggregated levels need to interpreted carefully. For example, because the level of chain-aggregated real GDP is not the arithmetic sum of real consumption, real investment, and so on, one cannot interpret the ratio of real investment to real GDP as "the share of investment in real GDP" because this ratio is not a share: The sum of these ratios for all categories of GDP does not equal one.[11] Thus, when using these chain-

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[11]See Whelan (2000) for more discussion of the implications for empirical analysis of the non-additivity property of chain-aggregated data.

aggregated levels, it is best to simply think of them as index numbers with the reference value set to some value other than one.

The specific chain aggregation method used by the NIPAs is the "ideal chain index" pioneered by Irving Fisher (1922). Technically, the Fisher index is constructed by taking a geometric average of the gross growth rates of two separate fixed-weight indexes, one a Paasche index (using period t prices as weights) and the other a Laspeyres index (using period $t-1$ prices as weights). In practice, however, the Fisher approach is well approximated by a Divisia index, which weights the growth rate of each category by its current share in the corresponding nominal aggregate. Returning to our two-sector model, note that from (16), (20), (21), and (22), we can derive the share of durable goods in each of the nominal aggregates as functions of our model's parameters. Thus, we can use the Divisia approximation to the Fisher formula to derive steady-state growth rates as nominal share weighted averages of the growth rates of our two sectors. Specifically, our model predicts the following steady-state growth rates for the chain-aggregates for per capita output, consumption, investment, and the capital stock:

$$g_Y = \frac{\theta}{1+\theta}g_1 + \frac{1}{1+\theta}g_2 \tag{33}$$

$$g_C = \frac{\alpha_1\left(\frac{g_1+\delta_1}{g_1+\delta_1+\rho}\right)}{\alpha_1\left(\frac{g_1+\delta_1}{g_1+\delta_1+\rho}\right)+\alpha_2}g_1 + \frac{\alpha_2}{\alpha_1\left(\frac{g_1+\delta_1}{g_1+\delta_1+\rho}\right)+\alpha_2}g_2 \tag{34}$$

$$g_I = \frac{\frac{\beta_1(g_1+\delta_1)}{g_1+\delta_1+\rho}}{\frac{\beta_1}{g_1+\delta_1+\rho}+\frac{\beta_2}{g_2+\delta_2+\rho}}g_1 + \frac{\frac{\beta_2(g_2+\delta_2)}{g_2+\delta_2+\rho}}{\frac{\beta_1}{g_1+\delta_1+\rho}+\frac{\beta_2}{g_2+\delta_2+\rho}}g_2 \tag{35}$$

$$g_K = \frac{\frac{\beta_1}{g_1+\delta_1+\rho}}{\frac{\beta_1}{g_1+\delta_1+\rho}+\frac{\beta_2}{g_2+\delta_2+\rho}}g_1 + \frac{\frac{\beta_2}{g_2+\delta_2+\rho}}{\frac{\beta_1}{g_1+\delta_1+\rho}+\frac{\beta_2}{g_2+\delta_2+\rho}}g_2 \tag{36}$$

The stability of nominal shares, documented earlier, implies that unlike fixed-weight series, chain-aggregates do not place ever-higher weights on the faster growing components. In fact, our model predicts that chain-aggregates will grow at a steady rate over the long run, even if one of the component series continually has a higher growth rate than the other.

Table 1 provides an illustration of the stability properties of chain-weight series, and the accelerating property of fixed-weight series. It shows the growth rates of the chain-weight and 1992-based fixed-weight aggregates for GDP, investment, and

consumption over the period 1992-98.[12] It shows that, for every year after 1992, each of the fixed-weight series grows faster than the corresponding chained series, with these differences increasing over time. This is most apparent for investment because durable equipment is a larger component of that series, so the relative price shift between equipment and structures is more important. For 1998, six years after the base year, the chain-aggregate for investment grows 11.4 percent while the 1992-based fixed-weight series grows 22.5 percent. The difference between fixed- and chain-weight measures of real GDP, as we move away from the base year for the fixed-weight calculation, are also quite notable. For 1997 and 1998, the fixed-weight measure of GDP grows 5.2 percent and 6.6 percent, while the corresponding chain series grows at a steady 3.9 percent pace. These calculations show that the switch to a chain-aggregation methodology is not simply a technical issue, but rather has important implications for the analysis of U.S. macroeconomic data.

3.3 Calibrated Steady-State Growth Rates

Just as important as the prediction that chain-aggregates should tend to grow at steady rates, illustrated in Table 1, is the fact that our model also predicts that the real NIPA aggregates for output, consumption, investment, and the capital stock should each tend to grow at *different* rates in the long run. As equations, (33) to (36) make clear, this pattern of differential steady-state growth rates will reflect the different mixes of durable goods and other output within each category.

Table 2 uses our calibrated parameters to illustrate the model's prediction of different steady-state growth rates for each of the major chain-aggregates, and compares these predictions with the empirical average values for their NIPA counterparts. The model's steady-state growth rate for output per hour exactly matches the published average of 2.3 percent, and the predicted values for consumption and investment are only one-tenth different from the empirical averages of 2.2 percent and 3.1 percent respectively. Of course, it should not be too surprising that the

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[12]The fixed-weight figures shown in this table are unpublished estimates obtained from the Department of Commerce's STAT-USA website. Earlier estimates, going through 1997, were published as Table 8.27 of Department of Commerce (1998). In this table, I have used 1992-based data, which pre-date the 1999 comprehensive revision to the NIPAs, to better illustrate how the chain- and fixed-weight series differ as we move away from the base year. However, all other data used in the paper are current as of March 2001 and use a base year of 1996.

model matches these values closely, because the parameters have been calibrated to match the average share of durable goods in nominal consumption and output, and also the average real growth rates of our two sectors.[13]

The model matches the empirical average growth rate of the capital stock less well: The steady-state growth rate for the chain-aggregated capital stock (per hour) is 2.3 percent, compared with the empirical average value of 1.7 percent. However, this disparity should not be too surprising. Given that capital stocks adjusts slowly, empirical averages for this variable may be less likely to correspond to the steady-state growth path described by the model. Also, using a slightly longer sample starting in 1948, the empirical average value increases to 2.0 percent, closer to our predicted steady-state value.

One interesting aspect of Table 2 is the fact that the model's calibrated steady-state values for g_K and g_Y are both 2.3 percent. This prediction of a constant long-run capital-output ratio may seem familiar from the Solow-Ramsey model but, in this case, it is simply due to a coincidence of coefficients taking the right values rather than being a fundamental feature of the model's steady-state growth path. Any change in parameters that resulted in the nominal share of durable goods in the capital stock rising relative to their share in nominal output, will result in an increasing capital-output ratio. One example of such a shock is a shift in consumer preferences away from durable goods and towards nondurables and services.

3.4   Implications for Analysis of NIPA Data

Our model's predictions about the steady-state behavior of the NIPA chain-aggregated series have far-reaching implications for how these series should be treated in empirical analysis. In particular, much empirical analysis in macroeconomics relies implicitly on the assumption that the one-sector growth model provides a good de-

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[13]One potential problem that could exist with comparing the model's steady-state growth rates with those published in the NIPAs is that the differences in construction (we use a Divisia index, BEA use Fisher; we are using two components, BEA use over one thousand) could lead to a failure to match the published data simply because we are explaining a different theoretical construct. However, this turns out not to be a problem: Time series of two-category Divisia aggregates actually do a remarkably good job of tracking the published series. For each of the four categories in Table 2, the growth rates of the time series based on the Divisia approximations have correlations with the published NIPA growth rates of over 0.98.

scription of the long-run properties of these data, and this assumption can lead to misleading conclusions. I will point out three simple examples.

1. Calibration of Business Cycle Models: The long-run growth paths for most general equilibrium business cycle models have the balanced growth feature of the Ramsey model. Indeed, the usual approach in empirical applications of these models is to calibrate the preference and technology parameters to ensure that, over the long-run, the models feature the traditional "great ratios," such as the real consumption-output ratio or the real capital-output ratio, conforming to their sample averages.[14] Our results imply that this approach is flawed. Because our model predicts that the real NIPA aggregates for output, consumption, investment, and the capital stock each tend to grow at different rates in the long run—reflecting their different mixes of durable goods and other output—it implies that ratios of these variables are unlikely to be stationary. Consequently, model parameters calibrated to match these empirical averages (and hence the empirical results from these models) will depend arbitrarily on the sample used.

2. Cointegration of Consumption and Income: That consumption and income should move together in the long-run is one of the most basic ideas in macroeconomics. However, our model implies that, when using real NIPA series, one needs to be careful about exactly what definition of real income is being used. In general, it should not be presumed that all commonly-used aggregate measures of real income, such as real GDP, will grow at the same rate as real consumption in the long run. For example, real aggregate consumption will tend to grow slower than real GDP as currently constructed. Empirical work on consumption tends to focus on real outlays on nondurables and services, and this series will exhibit slower long-run growth than both aggregate real consumption and real GDP.

Our model does predict that the ratio of *nominal* consumption to nominal income should be stable in the long run. So, clearly, if one divides nominal consumption and nominal income by a *common* deflator, then the ratios of the resulting two series will be stable. This implies that the way to construct a real income se-

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[14]See Cooley and Prescott (1995) and King and Rebelo (2000) for two standard examples of how these models are calibrated.

ries that is cointegrated with chain-aggregated consumption is to deflate nominal income by the aggregate consumption deflator (i.e. the ratio of nominal consumption to chain-aggregated consumption.) This insight of obtaining a cointegrating vector for real income and consumption by using a common deflator also extends to the analysis of consumption of nondurables and services. Our model predicts that real consumption of nondurables and services will only be cointegrated with the real income series defined by deflating nominal income by the deflator for nondurables and services.[15] It is interesting to note that this runs counter to a common practice in empirical consumption studies, which tend to relate real consumption of nondurables and services to a measure of real income defined by deflating nominal income by an aggregate consumption price index.[16]

3. Aggregate Investment Regressions: Most macroeconomic regression specifications for investment are derived from the assumption that there is one type of capital, which depreciates at a constant rate. In this case, the ratio of aggregate real investment to the lagged aggregate real capital stock, $\frac{I_t}{K_{t-1}}$, summarizes the growth rate of the capital stock. For this reason, the aggregate $\frac{I_t}{K_{t-1}}$ ratio has been the most commonly used dependent variable in macroeconomic investment regressions.[17] Our model implies that, if applied to current NIPA data, such regressions would be mis-specified. The problem, of course, is that not all types of capital are identical. In particular, our model captures two crucial differences between equipment and structures: The stock of equipment grows faster than the stock of structures, and equipment also depreciates significantly faster.

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[15]This argument appears to contradict the results of Cochrane (1994), who found that the ratio of real consumption of nondurables and services to real GDP was stationary, and used this finding to explore the "error-correction" properties of this ratio for forecasting output growth. However, Cochrane's finding of stationarity derived from his use of total real GDP, which includes government purchases, a category that has grown slower in real terms than private output. Our model does not incorporate government output, and so it should be interpreted as referring to private GDP. Once private real GDP is used, the results are as predicted by our model, with a downward drift in the ratio of real consumpion of nondurables and services to real private GDP evident from the late 1950s onward.

[16]See Blinder and Deaton (1985), Campbell (1987), and Carroll, Fuhrer, and Wilcox (1994) for three papers that take this approach.

[17]See, for example, Blanchard, Rhee, and Summers (1994), Hayashi (1982), and Oliner, Rudebusch, and Sichel (1995).

From equations (19), (20) and (21), our model predicts that, because $g_1 + \delta_1$ is greater than $g_2 + \delta_2$, equipment should make up a higher share of total nominal investment than of the total nominal capital stock. This is confirmed by the data: Over 1957-99, equipment accounted for 47 percent of nominal investment, and 19 percent of the nominal capital stock. Because it places a higher weight on the fast-growing asset (equipment), the growth rate of chain-aggregated investment will be higher than the growth rate of the chain-aggregated capital stock. Thus, the aggregate series for $\frac{I_t}{K_{t-1}}$ will tend to grow without bound, making it a very poor proxy for g_K (which is constant along the steady-state growth path). Remarkably, over the period 1948-98, the correlation between $\frac{I_t}{K_{t-1}}$ and g_K is -0.05.

For this reason, investment regressions using this dependent variable will be mis-specified. In contrast, though, one can easily re-arrange the first-order conditions for profit maximization to obtain separate specifications for equipment and structures, in which $\frac{I_t}{K_{t-1}}$ is a function of chain-aggregated output growth and the growth rate of the asset-specific real cost of capital.

4 Technological Progress, Growth, and Welfare

Our calibrated parameters can be used to calculate the contributions of the two types of technological progress to long-run real aggregate output growth as measured in the NIPAs. Inserting (10) and (11) into (33) we get

$$
\begin{aligned}
g_Y &= \frac{\theta}{1+\theta}\left[\frac{(1-\beta_2)\,a_1 + \beta_2 a_2}{1-\beta_1-\beta_2}\right] + \frac{1}{1+\theta}\left[\frac{\beta_1 a_1 + (1-\beta_1)\,a_2}{1-\beta_1-\beta_2}\right] \\
&= \frac{1}{1-\beta_1-\beta_2}\left[\frac{\beta_1 + (1-\beta_2)\,\theta}{1+\theta}a_1 + \frac{\beta_2\theta + (1-\beta_1)}{1+\theta}a_2\right] \quad (37)
\end{aligned}
$$

This equation has a simple interpretation along the lines of the equilibrium growth rate of the one-sector model, which is $\frac{a}{1-\beta}$, where a is the growth rate of the model's unique aggregate technology process and $\frac{1}{1-\beta}$ is a scaling factor representing the additional output growth caused by induced capital accumulation. In the two-sector case, the picture is a little more complex—with two types of technological progress, there are two separate direct effects on output, and with two capital goods used in two sectors, there are four separate induced accumulation effects—but the logic is the same.

Plugging in our parameter values, we find that 58 percent of the long-run growth in business sector output per hour as currently measured can be attributed to technological progress in the production of durable goods (a_1), with 35 percentage points of this coming from the direct effect on the production of durables, and the other 23 points representing the induced effect of this technological progress on the production of other goods and services. In contrast, of the 42 percent of long-run growth due to technological progress in the production of nondurables, services, and structures (a_2), only 2 percentage points represents induced effects on the production of durable goods.

Recalling the representative agent's utility function from equation (6), we can also calculate the contribution of each type of technological progress to the growth in welfare. Specifically, along the steady-state growth path, the change in the utility of the representative consumer is

$$\begin{aligned}
\Delta U\left(D_t, C_{2t}\right) &= \alpha_1 g_1 + \alpha_2 g_2 \\
&= \alpha_1 \left[\frac{(1 - \beta_2)\, a_1 + \beta_2 a_2}{1 - \beta_1 - \beta_2}\right] + \alpha_2 \left[\frac{\beta_1 a_1 + (1 - \beta_1)\, a_2}{1 - \beta_1 - \beta_2}\right] \\
&= \frac{\alpha_1 (1 - \beta_2) + \alpha_2 \beta_1}{1 - \beta_1 - \beta_2} a_1 + \frac{\alpha_1 \beta_2 + \alpha_2 (1 - \beta_1)}{1 - \beta_1 - \beta_2} a_2
\end{aligned} \qquad (38)$$

Our parameter values imply that technological progress in the production of durable goods accounts for 53 percent of the change in the utility of the representative agent. This is lower than the 58 percent calculated for g_Y because durable goods receive a higher weighting in steady-state nominal output than in the utility function ($\theta > \frac{\alpha_1}{\alpha_2}$).

5   Comparison with Greenwood et al (1997)

As noted earlier, the model presented in this paper shares some important similarities with that of Greenwood, Hercowitz, and Krusell (1997). This section compares the approach and results of this paper with their work.

5.1   An Alternative Representation

There are two main differences between the model presented in this paper, and the work of Greenwood, Hercowitz, and Krusell (henceforth, GHK). The first is

that the model in this paper acknowledges that technological progress has been faster for all durable goods, including consumer durables. GHK's model did not include a separate treatment of consumer durables, instead featuring two production technologies, one for consumption goods and structures, and the other for producers' durable equipment. I will return to this issue below.

The second difference is a presentational one. To explain this difference, note that (23), (24), (25) together imply that the sectoral resource constraints can be written as

$$Y_1 = \frac{\theta}{1+\theta} A_1 K_1^{\beta_1} K_2^{\beta_2} L^{1-\beta_1-\beta_2} \tag{39}$$

$$Y_2 = \frac{1}{1+\theta} A_2 K_1^{\beta_1} K_2^{\beta_2} L^{1-\beta_1-\beta_2} \tag{40}$$

Thus, the real output of each sector can be expressed in terms of the sector's share in nominal output, its technology parameter, and the aggregate factor allocations.

GHK did not emphasize their model's two-sector interpretation. Instead of working with a model in this form, they summarized these two production technologies in a single equation of the form

$$\frac{Y_1}{q} + Y_2 = z K_1^{\beta_1} K_2^{\beta_2} L^{1-\beta_1-\beta_2} \tag{41}$$

Note that this equation is consistent with the following two-sector representation:

$$Y_1 = \frac{\theta}{1+\theta} z q K_1^{\beta_1} K_2^{\beta_2} L^{1-\beta_1-\beta_2} \tag{42}$$

$$Y_2 = \frac{1}{1+\theta} z K_1^{\beta_1} K_2^{\beta_2} L^{1-\beta_1-\beta_2} \tag{43}$$

Thus, within our framework, z can be thought of as A_2, and q can be thought of as $\frac{A_1}{A_2}$. Clearly, though, the two systems—equations (39) and (40) on the one hand, and (42) and (43) on the other—represent two different ways of summarizing the same model.

GHK constructed an empirical q series as the price of consumer nondurables and services relative to the price of producers durable equipment, as in equation (12) of this paper. Note now that, because q is the deflator for sector 2's output (consumer nondurables and services) divided by the deflator for sector 1's output (durable equipment), then $z K_1^{\beta_1} K_2^{\beta_2} L^{1-\beta_1-\beta_2}$ in equation (41) can be calculated as aggregate nominal output divided by sector 2's price. Thus z can be calculated as

a Solow residual for this measure of real output. GHK used their empirical series for z and q to calculate that 58 percent of the growth in $zK_1^{\beta_1}K_2^{\beta_2}L^{1-\beta_1-\beta_2}$ was due to the q variable, with the rest due to the z series.

This 58 percent figure should not be confused with our calculation in the previous section—based on equation (37)—that 58 percent of the growth in real chain-aggregated business output (g_Y) was due to technological progress in the production of durable goods. In our model, the direct equivalent to GHK's $zK_1^{\beta_1}K_2^{\beta_2}L^{1-\beta_1-\beta_2}$ series (total nominal output deflated by sector 2's price) can be calculated as

$$pY_1 + Y_2 = (1 + \theta) Y_2 \tag{44}$$

(Recall that we normalized sector 2's price to equal 1). Obviously, this series will have a steady-state growth rate of g_2. Thus, the comparable calculation is not our previous figure of 58 percent, but rather the boost to g_2 coming from a_1 growing faster than a_2, which is only 25 percent.[18] Our calculation of the effect of $\frac{A_1}{A_2}$ (or q) on g_2 is lower primarily because we used a different measure of the relative price of durable goods. As noted above, GHK's procedure—using Gordon's alternative price deflator for durable equipment and the NIPA deflator for consumer nondurables and services—likely overstates the relative price decline for durable goods, and thus the impact of technological progress in this sector.

5.2   Which Measure of Real Output is Correct?

Beyond the presentation of the two-sector model and its calibration, a more substantive difference between GHK's approach and that in this paper concerns the measurement of *aggregate* real output. Specifically, GHK use $zK_1^{\beta_1}K_2^{\beta_2}L^{1-\beta_1-\beta_2}$ as defined in equation (41)—nominal output divided by the deflator for consumer nondurables and services—as their measure of aggregate real output, and argue that this is superior to the real output measure in the NIPAs. It turns out, though, that their characterization of the NIPA measure is somewhat misleading, for two reasons.

First, GHK interpret the measure of aggregate real output in the NIPAs as being of the form $Y_1 + Y_2$, summing the real output of the two sectors.[19] However, while

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[18]This figure was calculated as $\frac{\beta_1(a_1-a_2)}{\beta_1 a_1 + (1-\beta_1)a_2}$.

[19]This can be seen from equation 24 of their paper.

this was true for the real NIPA data used by GHK in their study, it is no longer an accurate description of how U.S. real GDP is measured. As we have described, since 1996, a chain-weighting method has been employed.

Second, GHK argue that the NIPA measure of real output is inconsistent with the fact that durable goods prices have been falling relative to prices for other goods and services. Specifically, they associate this measure of real output with an aggregate production function of the form

$$Y_1 + Y_2 = zK_1^{\beta_1} K_2^{\beta_2} L^{1-\beta_1-\beta_2} \tag{45}$$

and point out that if type-1 and type-2 output are perfect substitutes in production (as this equation implies) then, counterfactually, they must have the same price.[20] This argument, however, appears to confuse a measurement methodology with a theory, specifically a one-sector theory. Even if aggregate real output was measured as in the left-hand-side of equation (45), the use of such a measure wouldn't necessarily imply any judgement on the properties of the "aggregate production function" or on the economy's ability to substitute production of type-1 output for type-2 output. Indeed, if one adopts the multisector approach used in this paper, then there is no aggregate production function. In this case, the choice between different measures of real output depends only on what exactly it is that one wishes to use the measures for.

These considerations suggest that neither the NIPA measure of real output nor GHK's measure are intrinsically the "correct" one. Rather, they measure different concepts. However, as a summary statistic for *aggregate* real output, the NIPA measure—which weights the growth rates of the two sectors according to their shares in nominal output—is more representative of the growth occurring in the economy as a whole. As can be seen from (44), in the long run, GHK's measure will only reflect the growth in production of consumer nondurables, services, and structures.

One argument for a consumption-unit measure of real output is that it may be a better proxy for welfare. However, GHK's measure does not account for the welfare gains due to rising consumption of durable goods. In fact, for this reason, our model suggests that the NIPA measure of real output is the superior statistic

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[20]Hercowitz (1998) makes this argument in more detail.

for welfare. In steady-state, the change in utility of our representative consumer is $\alpha_1 g_1 + \alpha_2 g_2$. While the weights on g_1 and g_2 determining the steady-state growth rate of real chain-aggregated output are not exactly the same as that for the change in utility—g_Y places 21 percent of its weight on g_1, compared with 16 percent for $\Delta U(D_t, C_{2t})$—this series comes a lot closer than GHK's measure, which grows at rate g_2 in the long-run.

6 Conclusions

This paper has argued that, despite its central role in economics textbooks and in business cycle research, the one-sector model of economic growth provides a poor description of the long-run evolution of the U.S. economy. In particular, the model's central prediction, that the real aggregates for consumption, investment, output, and the capital stock, should all grow at the same rate over the long run, is firmly rejected by postwar U.S. NIPA data.

The reason for the failure of the balanced growth hypothesis is the model's inability to distinguish the behavior of the durable goods sector from that of the rest of the economy. The real output of the durable goods sector has consistently grown faster than the rest of the economy, and most investment spending is on durable goods while most consumption spending is on nondurables and services. As a result, real investment has tended to grow faster than real consumption, a pattern that has been particularly evident since 1991. These patterns are easily modeled using the simple two-sector approach of this paper.

While the case for a multi-sector approach to long-run modeling of the macroeconomy has also recently been made by Greenwood, Hercowitz, and Krusell (1997), this paper makes two new and important contributions. First, I show that the one-sector model can be strongly rejected based on published NIPA data. This is important because most empirical work in macroeconomics uses these data, and often relies on the one-sector model's balanced growth predictions. The message here for practitioners is that consideration of a two-sector approach to long-run modelling should not be limited to datasets that have adjusted the NIPA equipment prices for measurement error. In any case, given the recent research on price measurement error in the finance and construction industries, one could argue that the evidence presented here for the two-sector approach as a model of the underlying

reality is more convincing than that cited by Greenwood et al, because it does not rely on adjustments to published data that ignore potential price mis-measurements outside the durable goods sector.

The paper's second main contribution is the use of the two-sector framework to establish the long-run properties of the real NIPA aggregates constructed according to the current chain index method. Specifically, it has been shown that each of the major chain-aggregated variables—output, consumption, investment, and capital stock—will tend in the long-run to grow at steady, but different, rates. This result has far-reaching implications for empirical practice in macroeconomics. In particular, our examples have shown how treating these data as if they are fixed-weight aggregates generated by a one-sector model can often result in incorrect conclusions.

The results in this paper suggest that there may be large gains to future empirical research aimed at understanding the apparently large gap between technological progress in the durable goods sector and in the rest of the economy. In particular, an exploration of whether this pattern can be explained by differential rates of R&D activity, or other "spillovers" stressed in the endogenous growth models of Romer (1990) and Jones and Williams (1998), would appear to be particularly worthwhile, and could yield important policy implications.

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A   Derivation of Equation (15)

The first-order conditions for consumption expenditures are

$$\frac{\alpha_1}{D_t} + \frac{1}{1+\rho} E_t \left[\frac{\partial V_{t+1}}{\partial D_t} \right] = \frac{1}{1+\rho} E_t \left[p_t \left(1 + r_{t+1} \right) \frac{\partial V_{t+1}}{\partial Z_{t+1}} \right] \tag{46}$$

$$\frac{\alpha_2}{C_{2t}} = \frac{1}{1+\rho} E_t \left[\left(1 + r_{t+1} \right) \frac{\partial V_{t+1}}{\partial Z_{t+1}} \right] \tag{47}$$

while the envelope conditions for the value function are

$$\frac{\partial V_t}{\partial D_{t-1}} = \left(1 - \delta_1 \right) \frac{\alpha_1}{D_t} + \frac{1 - \delta_1}{1+\rho} E_t \left[\frac{\partial V_{t+1}}{\partial D_t} \right] \tag{48}$$

$$\frac{\partial V_t}{\partial Z_t} = \frac{1}{1+\rho} E_t \left[\left(1 + r_{t+1} \right) \frac{\partial V_{t+1}}{\partial Z_{t+1}} \right] \tag{49}$$

From (46) and (47), we know that

$$\frac{\alpha_1}{D_t} + \frac{1}{1+\rho} E_t \left[\frac{\partial V_{t+1}}{\partial D_t} \right] = \frac{p_t \alpha_2}{C_{2t}} \tag{50}$$

Thus, (48) can be re-expressed as

$$\frac{\partial V_t}{\partial D_{t-1}} = \left(1 - \delta_1 \right) \left(\frac{p_t \alpha_2}{C_{2t}} \right)$$

Shifting this equation forward one period and taking the expectation we get

$$E_t \left[\frac{\partial V_{t+1}}{\partial D_t} \right] = \left(1 - \delta_1 \right) E_t \left[\frac{p_{t+1} \alpha_2}{C_{2,t+1}} \right]$$

Inserting this into (50) and re-arranging we get

$$\frac{\alpha_1}{D_t} = \frac{p_t \alpha_2}{C_{2t}} - \frac{1 - \delta_1}{1+\rho} E_t \left[\frac{p_{t+1} \alpha_2}{C_{2,t+1}} \right]$$

which becomes

$$\frac{\alpha_1}{\alpha_2} \frac{C_{2t}}{p_t D_t} = 1 - \frac{1 - \delta_1}{1+\rho} E_t \left[\frac{C_{2t}}{C_{2,t+1}} \frac{p_{t+1}}{p_t} \right]$$

Using our formulae for steady-state growth rates of consumption and relative prices, this becomes

$$\frac{\alpha_1}{\alpha_2} \frac{C_{2t}}{p_t D_t} = 1 - \frac{\left(1 - \delta_1 \right) \left(1 + g_2 - g_1 \right)}{\left(1 + \rho \right) \left(1 + g_2 \right)} \approx g_1 + \delta_1 + \rho$$

as required.

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	GDP		Consumption		Investment	
	Chain-	Fixed-	Chain-	Fixed-	Chain-	Fixed-
1992	2.7	2.7	2.8	2.7	5.7	5.5
1993	2.3	2.4	2.9	3.0	7.6	7.7
1994	3.5	3.6	3.3	3.4	8.6	9.0
1995	2.3	2.7	2.7	2.9	5.5	7.2
1996	3.4	4.1	3.2	3.6	8.8	11.2
1997	3.9	5.2	3.4	4.1	8.3	12.7
1998	3.9	6.6	4.9	6.1	11.4	22.5

Table 1: 1992-Dollar Chain- and Fixed-Weight Growth Rates for Major Aggregates

Source: Department of Commerce STAT-USA Website (www.stat-usa.gov)

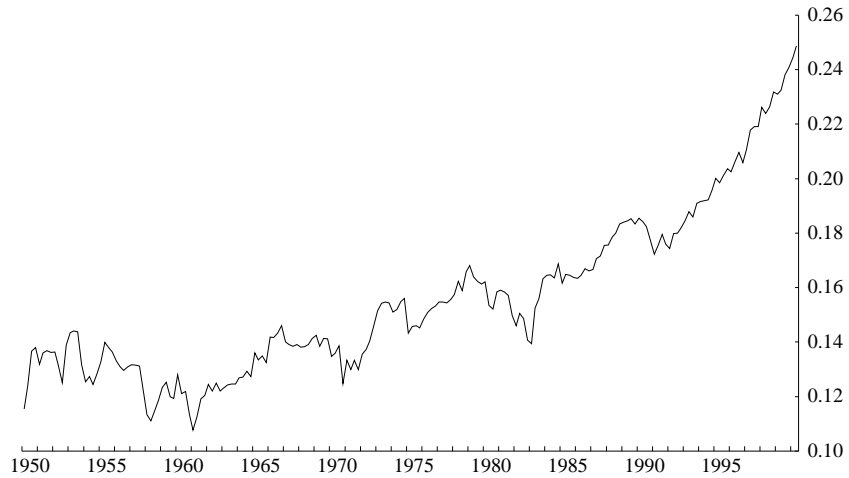|	Steady-State Values	Average, 1957-99
Output Per Hour	2.3	2.3
Consumption Per Hour	2.1	2.2
Investment Per Hour	3.0	3.1
Capital Stock Per Hour	2.3	1.7

Table 2: Growth Rates for Chain-Aggregates

Figure 1
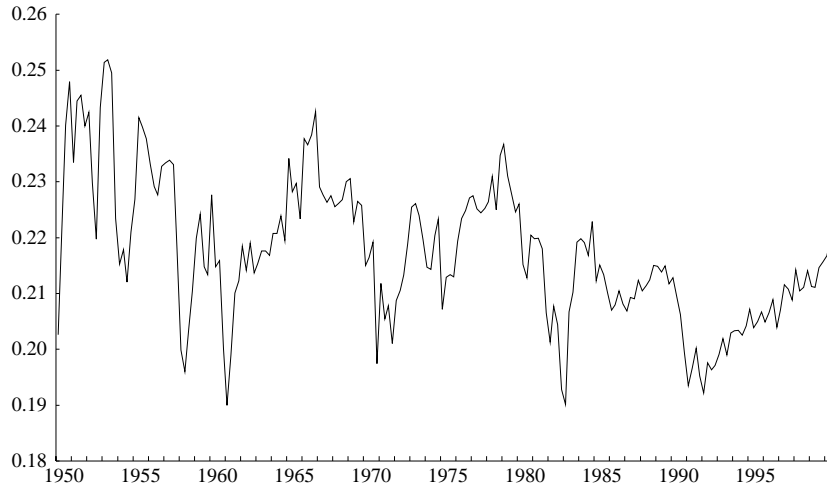**Ratio of Real Private Fixed Investment to Real Consumption**

Figure 2a
Ratios of Major Components of Investment and Consumption

Real Equipment Investment / Real Structures Investment →
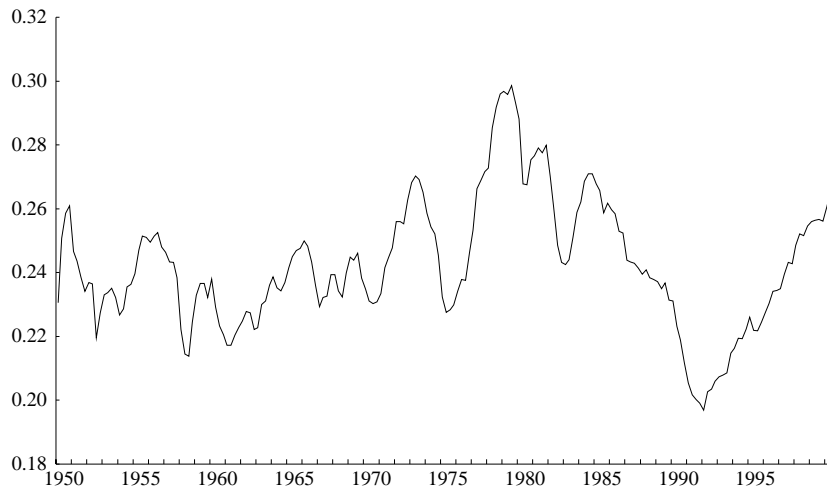← Real Durable Goods Consumption / Real Nondurables and Services Consumption
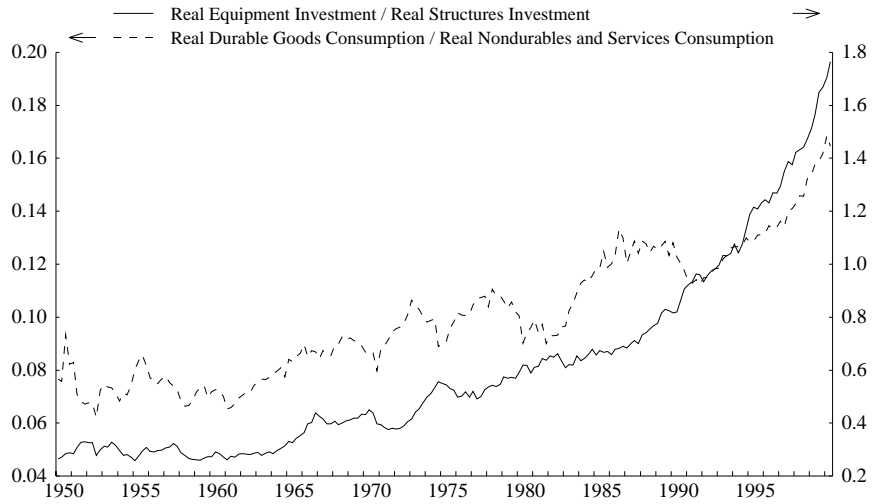
Figure 2b
Ratio of Total Real Durables Output to Real Business Output

Figure 3a
Ratio of Total Nominal Durables Output to Nominal Business Output

Figure 3b
Ratio of Nominal Private Fixed Investment to Nominal Consumption

Figure 4
Relative Prices of Durables and Fixed Investment
(In Logs, All Prices Relative to Price of Consumer Nondurables and Services)

All Durable Goods

Producers' Durable Equipment

Fixed Investment

Structures

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