ECON30580 Economics of Betting Markets
14. Specials and Free Bets

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Part I

Special Bets and Bet Builders

Betting on Everything

- Before the internet, sports betting was largely about picking a team to win or perhaps beat a point spread.
- Today, you can bet on almost everything that happens in sport
  - Players to score
  - Get a yellow card
  - Be fouled at least twice or commit at least two fouls
  - The margin of victory
  - The exact score
  - The number of goals
  - The number of corners and so on.
- These **special bets** (known in the US as proposition bets) are heavily promoted by bookmakers.
- Why do they like them so much?

Why Do Bookies Like Specials?

1. **More Bets**: They expand the choice of bets and make it more likely people see something they want to bet on.
2. **Manufactured Longshots**: These bets are almost always longshots (e.g. most players don't score in most soccer matches) and we know disagreement plus profit maximisation implies higher profits on longshots.
3. **Complexity**:
   - Harder for bettors to figure out how likely they are to win. When deciding which team might win a match, there are a lot of easily availa-ble resources.
   - One sided bets: If Liverpool play Manchester United, a bookie is obliged to quote odds on all three possible outcomes of the match and you can calculate an overround. But they can post odds on Dominik Szoboszlai to score without any obligation to offer odds on Dominik not to score.
4. **Emotions**: Bettors sometimes have *parasocial* relationships with their favourite players. You like Harry Kane? You bet on Harry to score two goals and celebrate along with him when he scores.

Examples from US Sports

- **Offensive touchdown scorer bets**
  - From Odds-API.com, I collected 174,891 quotes from TheOdds-API.com covering 14 licensed US bookmakers on players to score an offensive touchdown over the 2023 to 2025 seasons
  - Quotes were matched to the NFL's official touchdown scorer statistics so we know whether they won or lost.
  - The results are ugly. On average, bets on a player to score a touchdown lost about 34% of their stakes.
  - These bets have a huge favourite-longshot bias. Bets with odds in the lowest fifth of the sample lost 11%, with this rising gradually to 55% losses for the highest fifth.
- **Over-under on NFL rushing yards and NBA points**
  - 25,655 quotes on over-under NFL rushing yard player props.
  - 146,294 quotes on NBA player points scored props.
  - Over bets do much worse than under.
  - Worst results come from betting on NBA stars to beat the over.

Loss Rates on NFL Anytime Touchdown Score Bets

Average loss rates on "anytime touchdown" NFL prop bets over 2023-2025 seasons, from 182,942 bets.

	Loss rate
All bets	33.6%
Decimal odds range	
1.14 – 3.25	11.4%
3.30 – 5.50	21.6%
5.55 – 9.50	37.9%
9.75 – 20.00	39.0%
Over 20.00	55.2%

Over-Under Bets on NFL Player Rushing Yards

Share of over wins and average loss rates on "over/under" NFL player prop rushing yards bets over 2023-2025 seasons, from 25,655 bets offered by FanDuel, DraftKings and BetMGM

Line	*Over Win Rate*	*Over Loss Rate*	*Under Loss Rate*
All	47.1%	11.1%	0.7%
Up to 12.5	46.8%	10.5%	1.9%
13.5 to 22.5	48.8%	8.2%	3.6%
23.5 to 40.5	45.5%	14.3%	-2.9%
41.5 to 58.5	47.1%	11.4%	0.3%
Over 58.5	47.4%	10.9%	0.5%

Over-Under Bets on NBA Player Points Scored

Share of over wins and average loss rates on "over/under" NBA player prop points scored bets over September 2023 to January 2026, from 146,294 bets offered by FanDuel, DraftKings and BetMGM

Line	*Over Win Rate*	*Over Loss Rate*	*Under Loss Rate*
All	48.7%	8.2%	3.5%
Up to 8.5	49.4%	7.1%	4.7%
9.5 to 11.5	50.3%	5.1%	6.5%
12.5 to 14.5	47.5%	10.3%	1.4%
15.5 to 19.5	49.2%	7.3%	4.2%
Over 19.5	46.6%	12.2%	-0.6%

Bet Builders (Same Game Parlays)

- Specials are pretty profitable for bookmakers on a standalone basis but they have another attraction.
- They feature as components in bets like *Haaland to score, City to win by 2 and the game to have over 11 corners*.
- In the UK and Ireland, these are known as "bet builders"; in the US they are "same game parlays" (SGPs).
- Unlike Accas, where the games are all independent of each other, the component bets are now related: City's chances of winning by two goals are correlated with whether Haaland scores. If they win by that many, then itâs pretty likely their star striker scores.
- For this reason, bet builders get their own unique odds rather than being a combination of separate component odds.
- Figuring out the right probability that the bet builder will win requires complex numerical methods (Monte Carlo simulation).
- What is the chance that bettors are doing these calculations correctly?

Why Do People Place These Bets?

We know from earlier that these multi-leg bets have terrible returns. Why do people place them? Some theories.

1. **Misunderstanding Probabilities**: Failure to understand how unlikely you are to win and misunderstanding "near wins" as signs you might win next time.
2. **Lottery Tickets**: Small probabilities of winning big make people treat them a bit like lottery tickets.
3. **Small Stakes**: Most people place only a small bet on these. If you win a few legs, you can have a lot at risk on the final leg, but people feel like they really only ever lose the initial amount they placed.
4. **Loss Aversion**: People could cash out if the multi-leg bet is going well but they feel like they could miss out on the potential big win and would feel bad if it came through. Losing €5 on the other hand, doesn't trigger a lot of loss aversion.

Part II

Free Bets

Free Bets

- Bookmakers frequently offer **free bets**.
- The typical structure:
  - Stake is provided by the bookmaker
  - If the bet wins: you keep the **profit only**
  - If the bet loses: you get nothing
- So a one unit free bet at decimal odds D pays

$$D - 1$$

  if it wins.
- If it has probability p of winning the expected profit from the free bet is $p(D-1)$
- Assuming (as always) that $D = \frac{1-m}{p}$, we get the value of free bet is

$$E\left[\Pi_A^F\right] = p\left(\frac{1-m}{p} - 1\right) = 1 - m - p$$

Free Bets Are Best Placed on Longshots

$$E\left[\Pi_A^F\right] = p\left(\frac{1-m}{p} - 1\right) = 1 - m - p$$

- This is an interesting result: For a fixed value of the bookmaker's margin m, **the expected profit is maximized by placing the bet on the biggest possible longshot**.
- For regular bets, we had shown returns for longshot bets are the worst, because they have high values for m.
- But free bets are best placed on outcomes with low p (albeit m features also in the formula and if m rises faster than p falls, then you might not want to use it on an extreme longshot).
- Intuition: Suppose you place your free bet on an extreme favourite with odds of $D = 1.01$. It will probably win but you will only get 1c back for every euro you placed. So placing the bet on the longshot is the best option.

An Alternative: Hedging

- Placing your free bet on a longshot is the best way to maximize its expected profit but it is also a very risky bet. You probably don't get many free bets of this sort and this strategy will result in you most likely not winning any money.
- An alternative is to **hedge** the free bet by betting on the alternative outcome to produce a **guaranteed return**. This is known as **matched betting**
- It works like this. A contest has two possible outcomes, A and B
  - Place a one unit free bet on A at odds D_A
  - Place a hedge bet of H on B at odds D_B
  - Profit if A wins is
    $$(D_A - 1) - H$$
  - Profit if B wins is
    $$H(D_B - 1)$$
  - Set H so
    $$(D_A - 1) - H = H(D_B - 1)$$
    so you get a guaranteed profit.

Hedge Size and the Guaranteed Profit

- Calculate the optimal hedge size from

$$(D_A - 1) - H = H(D_B - 1) \Longrightarrow H = \frac{D_A - 1}{D_B}$$

- The guaranteed profit is

$$\Pi = (D_A - 1) - \frac{D_A - 1}{D_B} = (D_A - 1)\left(1 - \frac{1}{D_B}\right)$$

- Using bookmaker pricing

$$D_A = \frac{1 - m}{p} \qquad D_B = \frac{1 - m}{1 - p}$$

- The guaranteed profit becomes

$$\Pi = \left(\frac{1 - m - p}{p}\right)\left(1 - \frac{1 - p}{1 - m}\right) = \frac{(1 - m - p)(p - m)}{p(1 - m)}$$

About the Guaranteed Return

$$\Pi = \frac{(1-m-p)(p-m)}{p(1-m)}$$

- If $m = 0$, then $\Pi = 1 - m$, so you can replicate the expected value of the free bet via hedging.
  - Consider a coin-flip event so $p = 0.5$.
  - With $m = 0$, decimal odds are $D_A = D_B = 2$.
  - Use your free one unit bet on A and bet $50c$ on B and whatever happens you are 50c better off. Either A happens, so you win a dollar but lose 50c on your bet on B.
  - Or B happens, so the free bet doesn't matter but you win 50c from the bet on B.
- But suppose $m = 0.05$, so the odds on both options are now $D_A = D_B = 1.9$.
  - If A wins, your free bet gets you a 90c profit.
  - The hedging formula now says to place 47c on B and the guaranteed return falls to 43c.
  - Both your free bet and your hedging bet are being placed at less than their fair value and this eats into the risk-free return.

Forces Affecting the Guaranteed Return

$$\Pi = \frac{(1-m-p)(p-m)}{p(1-m)}$$

- The guaranteed profit can be written as the product of the expected risky profit from the free bet $(1-m-p)$ and a multiplicative factor that describes the discount due to the bookmaker's margin.
- The discount gets bigger as m increases but it also depends negatively on p because the size of hedging bets get bigger as p falls, with the discount falling particularly fast as p reaches low levels.
- In fact, with $p = m$, the guaranteed return is zero and it is negative for $p < m$.
- For these low values of p, the size of the hedging bet gets so large, it wipes out the guaranteed profits.
- Taking these points together, it seems like there is going to be an "interior solution" for the best value of p.
- Let's confirm that.

Simplifying the Profit Function

$$\Pi = \left(\frac{1-m-p}{p}\right)\left(1-\frac{1-p}{1-m}\right) = \frac{(1-m-p)(p-m)}{p(1-m)}$$

- This looks complicated. We can simplify it to allow us ask which p gives the highest profits.
- Expand the numerator

$$\begin{aligned}(1-m-p)(p-m) &= (1-m)p-(1-m)m-p^2+pm \\ &= p-p^2-m+m^2 \\ &= p-p^2-m(1-m)\end{aligned}$$

- So

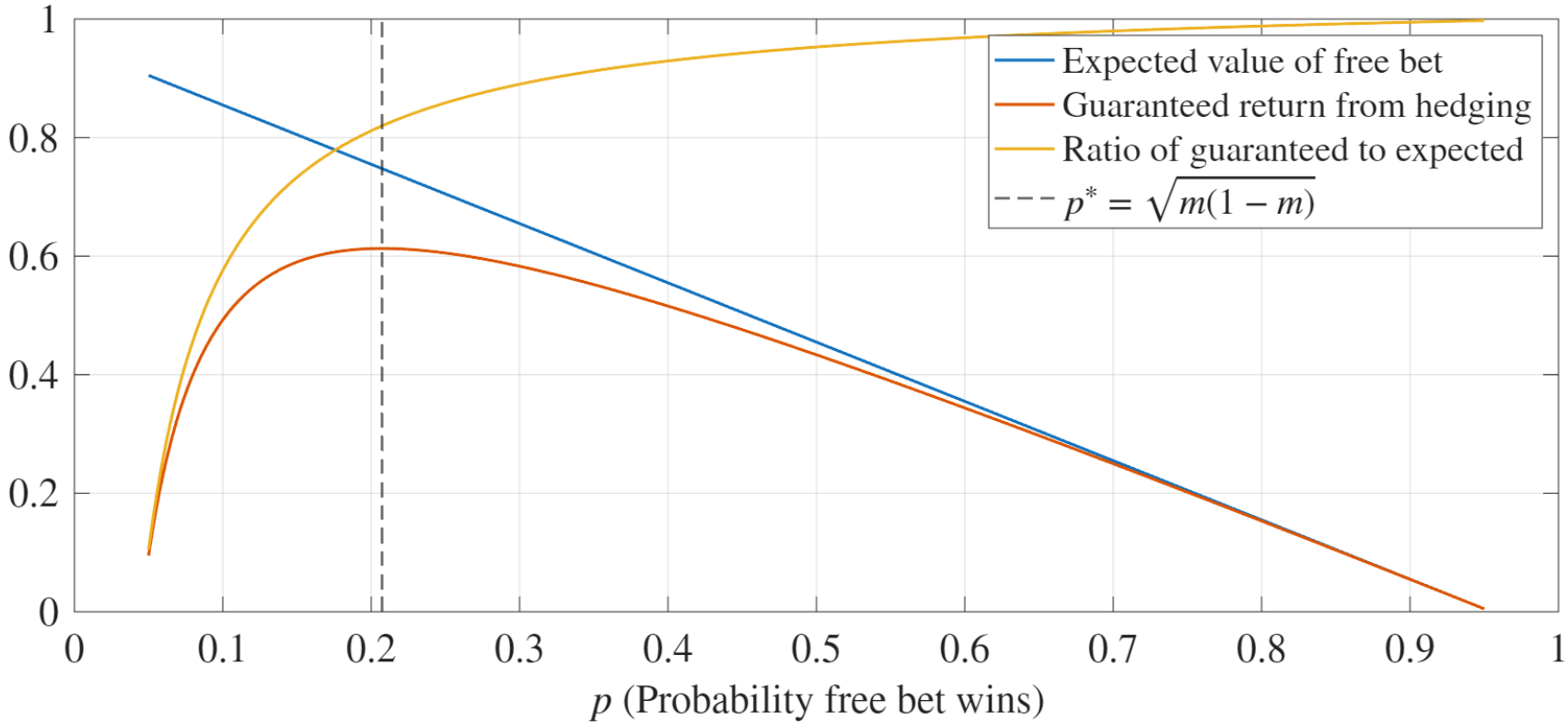
$$\Pi(p) = \frac{p-p^2}{p(1-m)} - \frac{m(1-m)}{p(1-m)} = \frac{1-p}{1-m} - \frac{m}{p}$$

- We calculate the best value of p by differentiating this with respect p and setting it equal to zero.

The Square Root Rule

- Profits are
$$\Pi(p) = \frac{1-p}{1-m} - \frac{m}{p}$$
- Differentiate
$$\Pi'(p) = \frac{-1}{1-m} + \frac{m}{p^2}$$
- Setting this equal to zero gives
$$\frac{1}{1-m} = \frac{m}{p^2}$$
- Multiplying both sides by $p^2(1-m)$, we get
$$p^2 = m(1-m)$$
- The highest possible guaranteed value is obtained when
$$p^* = \sqrt{m(1-m)}$$

The Guaranteed Return As a Function of p with $m = 0.05$



Implications of the Square Root Rule

- The highest possible guaranteed value is obtained when

$$p^* = \sqrt{m(1-m)}$$

- At a typical margin of about 5% so $m = 0.05$, this gives

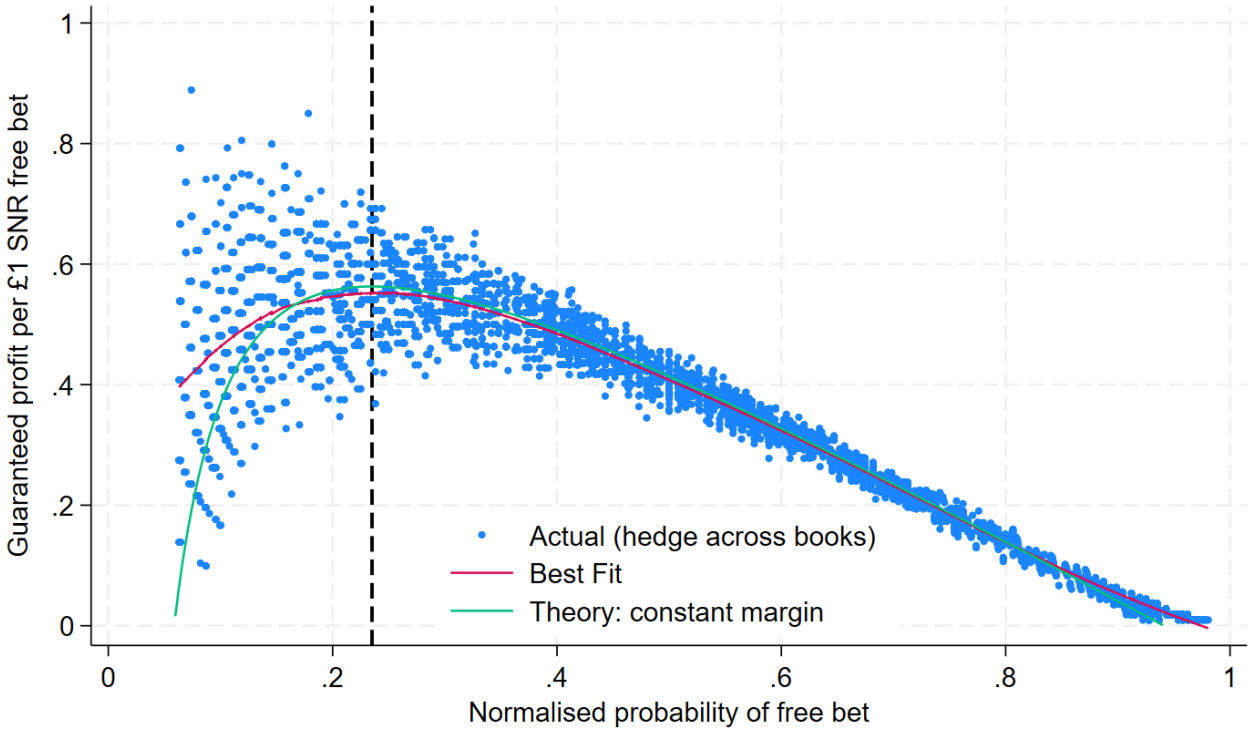
$$p^* = \sqrt{(0.05)(0.95)} = 0.218$$

- So the guaranteed profit from hedging is maximised by placing the free bet on outcomes that are unlikely but not huge longshots.
- This gives $\Pi^* = 0.594$.
- If you place your bet on an outcome with a 21.8% chance of winning, you can turn a €10 free bet into €5.94 of guaranteed money.
- But this is theoretical and relies on a very specific assumption of how bookmakers set odds.
- Let's see if it works in practice.

Empirical Evidence: Betting on Tennis in the UK

- From Odds-API.com, I got odds quotes on about 7,000 tennis matches where odds were available from both Ladbrokes and Paddy Power.
- In theory you could hedge with the same bookmaker you got the free bet from but bookmakers view this as an "abuse" of promotions and might cancel your remaining free bets.
- So, for each quote, I figured out the guaranteed money from placing the free bet on that quote and hedging with the other bookmaker.
- This meant 4 different guaranteed returns for each match, so about 28,000 different returns.
- The average margin was $m = 0.058$, so the predicted optimal probability was $p^* \approx 0.23$
- The data match the theory closely until p falls below the optimal value.
- Once you go below $p = 0.23$ the real-world guaranteed returns are a bit higher than the theory. The odds on the hedge bets on favourites are a bit better than predicted by the theory because the theory does not account for favourite-longshot bias.

Guaranteed Returns from Tennis Bets with UK Bookmakers



Caveats

Why have I been holding back on this guarantee of free money? You can open accounts with a bunch of bookmakers and use the promo free bets to make money.

- **Free But Not Hassle-Free**: Opening accounts with multiple bookmakers requires going through Know Your Customer (KYC) checks a bunch of times. We know from evidence that most people don't want to do this, with the average bettor having two accounts.
- **Placing the Bets**: This takes time. You have to find the right options to place the free bet on and then go place the hedging bet, hoping you've done the calculations right and haven't made a mistake and backed the same team twice.
- **Limited**: If you only appear to be placing bets during the initial promo-heavy periods, bookmakers will identify you as a low-value client and you won't get more free bets.

Matched betting might be a way to raise a modest-sized fixed sum for a specified purpose, without leaving your own home. But it's not clear that the hourly rate for all this effort beats delivering pizza.

Why Do Bookmakers Give Away Free Bets?

- **Customer Acquisition**: New customers are valuable. It is worth giving away some money to get some of them to hang around to become long-term losers.
- **Customer Retention**: Keeping long-term heavy regular losers coming back is a key part of a profit-maximising strategy. Free bets and other perks help those people feel appreciated and they stay loyal.
- **Promoting Longshots**: Promotional bets push people towards longshots. Bookmakers will hope customers develop a taste for longshots and keep placing those bets even when they go back to using their own money.
- **Promoting High Loss Rate Products**: Promotional offers are sometimes tied to the products with the highest expected losses, like accumulators, bet builders or virtual casino games. If the free bets encourage you to keep picking these options after the free bets are finished, then the bookmakers will likely win back more than they have given away.
- **House Money**: Customers may feel they are betting with "house money" and just bet more over the period after the free bets than they would have otherwise. If I fill my account with €300 every two months, maybe an extra €100 just means I keep betting until I've lost €400 (including the free bets).

Supplementary Material in the Draft Book

- Chapter 21: The Complexity Trap: Specials and Bet Builders
- Chapter 22: Free Bets