# ECON30580 Economics of Betting Markets
# 4. Risk Aversion and Betting Decisions

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# Part I

## How We Will Model Decisions of Bettors

# A Simple Rule: $pD > 1$

- We will model the decision that bettors take on whether to make a bet or not using a simple rule.
- Each bettor will use their own subjective assessment of the bet's chance of winning $p$.

### The Rule We Assume Bettors Follow

When offered decimal odds of $D$, accept the bet if

$$pD > 1$$

- So they will accept a bet if the expected payout from a €1 bet exceeds €1.
- In other words: **People place a bet when they believe that, on average, taking bets of this sort will make a profit.**
- We will also keep the rule about **how much** to bet simple for now – anyone who places a bet will risk the same unit-sized bet (assumed to be small).

# Questions About This Rule

- The rule is simple but it raises a lot of questions.
  1. **What About Risk?**
     - ★ Microeconomic theory usually assumes people are risk averse. Shouldn't the rule factor in that taking the bet involves risk?
  2. **Do Bettors Love Risk?**
     - ★ Bookmakers earn profits. People know that – maybe bettors know that $pD < 1$ but take the bet because they like risk?
  3. **What About Fun?**
     - ★ Don't some people place bets because they find gambling to be fun?
- We will discuss the first two questions in this lecture.
- I will argue that people who bet are actually risk averse, but they place small bets and $pD > 1$ works pretty well to describe betting behaviour.
- We will return to the "fun" element in the next lecture.

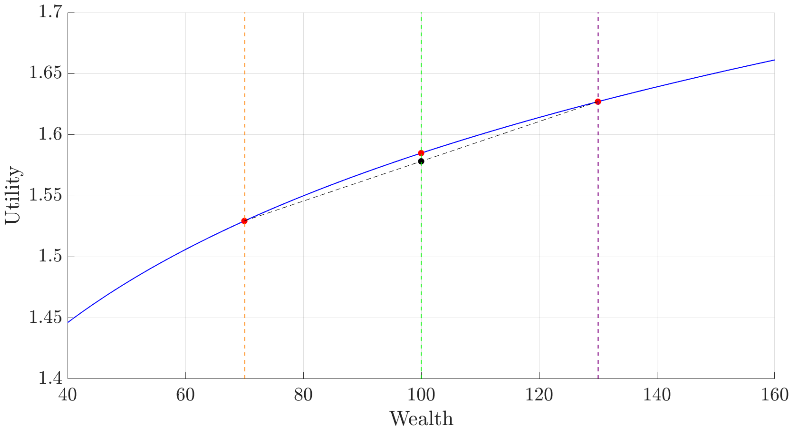
# Part II

## Concave Utility and Risk Aversion

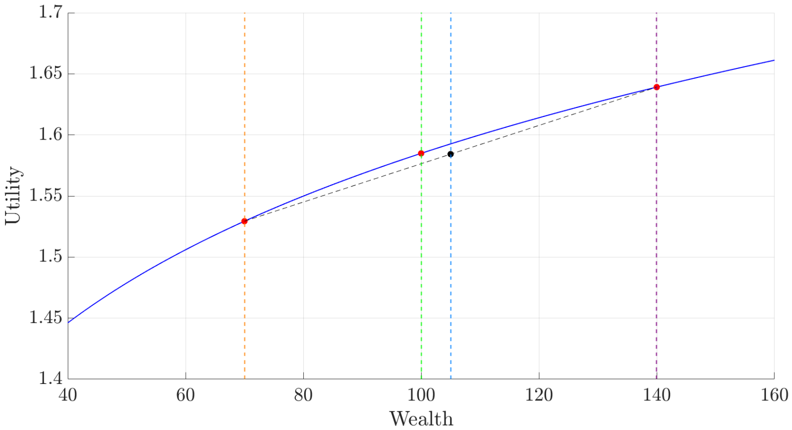
# Concave Utility and Risk Aversion

- Economists usually assume people have **concave utility**, meaning they have diminishing marginal utility. It implies that people are **risk averse**.
- Consider a "fair gamble" with a 50/50 chance of having either winning or losing $X$.
- With concave utility, the potential gain in utility $U(W+X) - U(W)$ is less than the potential loss of utility $U(W) - U(W-X)$. So people prefer the certainty of having wealth of $W$ to accepting the fair gamble.
- See the graph on the next page: The average of $U(70)$ and $U(130)$ (the black dot) is less than $U(100)$, so someone with wealth of $W = 100$ turns down a 50/50 gamble with equal chances of winning or losing 30.
- Concave utility means people may also turn down some offers that are expected to be profitable. For example, they may turn down offers with a 50/50 chance of having either $W + X_1$ or $W - X_2$ where $X_1 > X_2$.
- The graph two pages down shows a person with wealth of $W = 100$ turning down a 50/50 gamble with equal chances of winning 40 or losing 30. This has an expected profit of $+5$ but is still rejected.

# Concave Utility Implies Turning Down a Fair Gamble



# Turning Down a Gamble with Positive Expected Value



# Implications of Risk Aversion

Risk aversion explains various aspects of human behaviour.

- **Financial Assets**: Assets that have stable, predictable returns (e.g. short-term government securities) tend to have lower average returns than assets with volatile and uncertain returns (e.g. stocks). People are willing to trade off a lower return in exchange for less variance in their wealth.
- **Insurance**: People are willing to pay small fees to rule out the possibility of losing lots of money due to risks such as sickness or major damage to your house. Insurance companies make profits on this business. This means people are willing to pay more than the actual expected loss from the risk to make their wealth stable rather than risky.
- **Career Choices**: Many people are willing to trade off a higher but more uncertain earnings path by accepting a stable job with lower average pay.
- **Business Investment**: Established firms often prefer to invest in safer projects rather than investing in high-risk, potential high-reward innovations.

# A Taylor Approximation to the Utility Function

- A famous result from calculus states that if $h$ is small then we can approximate the values of a function $F(x+h)$ as

$$F(x+h) = F(x) + F'(x)h + \frac{F''(x)}{2}h^2$$

- Here $F'(x)$ is the first derivative of the function and $F''(x)$ is the second derivative.
- Explanation of the formula:
  - As $h$ increases that changes the function and $F'(x)$ is an estimate of how much the function changes in the region of the value $x$.
  - The second term with $F''(x)$ takes into account that the first derivative of the function will change as we move away from $x$.
- We will use this formula to show some interesting results about decisions involving risk.

# Concave Utility: High Average Good, High Variance Bad

- Consider someone with wealth of $W$ offered a gamble such that after the gamble is accepted, their wealth will be $W+X$ where $X$ is a random variable with mean $E(X)=\mu$ and variance $E\left[(X-\mu)^2\right]=\sigma^2$.
- We can write their utility after the outcome of the gamble as

$$U(W+X)=U(W+\mu+X-\mu)$$

- We can use the Taylor approximation to write this as

$$U(W+X)=U(W+\mu)+U'(W+\mu)(X-\mu)+\frac{U''(W+\mu)}{2}(X-\mu)^2$$

- Now we can calculate the expected utility across all values of $X$ as

$$E[U(W+X)]=U(W+\mu)+\frac{U''(W+\mu)\sigma^2}{2}$$

- If two gambles have equal mean, people with concave utility (negative second derivative) will prefer the one with the lower variance.

# How to Measure Risk Aversion

- It will be helpful for us to work with a measure of how risk averse people are.
- An initial idea might be to use the second derivative, $\frac{d^2U}{dW^2} = U''(W)$. The more negative the second derivative is, then the more risk averse people are.
- But what is more helpful is understanding the ratio of the second derivative to the first derivative: This gives us the ratio of the "risk aversion" element of the utility function to the "likes more wealth" element.
- The **Arrow-Pratt measure of absolute risk aversion** is

$$RA(W) = -\frac{U''(W)}{U'(W)}$$

- For a given gamble, we can show that those with $RA(W)$ above a certain level will not take the gamble and those with $RA(W)$ below it will take the gamble.
- Usually, it is more useful to frame gambles relative to the size of wealth. In this case, we look at a person's **relative risk aversion**, defined as

$$RRA(W) = -\frac{WU''(W)}{U'(W)}$$

# CRRA Utility Functions

- The most common utility functions used in economics are those of the **Constant Relative Risk Aversion** class, defined as

$$U(W) = \begin{cases} \frac{W^{1-\gamma}-1}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log W & \text{if } \gamma = 1 \end{cases}$$

  where $\gamma$ (pronounced gamma) is a positive number. Taking derivatives, we can see that if $\gamma \neq 1$

$$\begin{aligned} U'(W) &= W^{-\gamma} > 0 \\ U''(W) &= -\gamma W^{-\gamma-1} < 0 \end{aligned}$$

- If $\gamma = 1$

$$\begin{aligned} U'(W) &= \frac{1}{W} > 0 \\ U''(W) &= -\frac{1}{W^2} < 0 \end{aligned}$$

- These functions have negative second derivatives and so they are concave.

# CRRA Utility: Calculating Risk Aversion

- If $\gamma \neq 1$, we can calculate the absolute and relative risk aversion as follows

$$RA(W) = -\frac{U''(W)}{U'(W)} = \frac{-\gamma W^{-\gamma-1}}{W^{-\gamma}} = \frac{\gamma}{W}$$

$$RRA(W) = -\frac{WU''(W)}{U'(W)} = \gamma$$

- If $\gamma = 1$, then

$$RA(W) = -\frac{U''(W)}{U'(W)} = \frac{-\frac{1}{W^2}}{\frac{1}{W}} = \frac{1}{W}$$

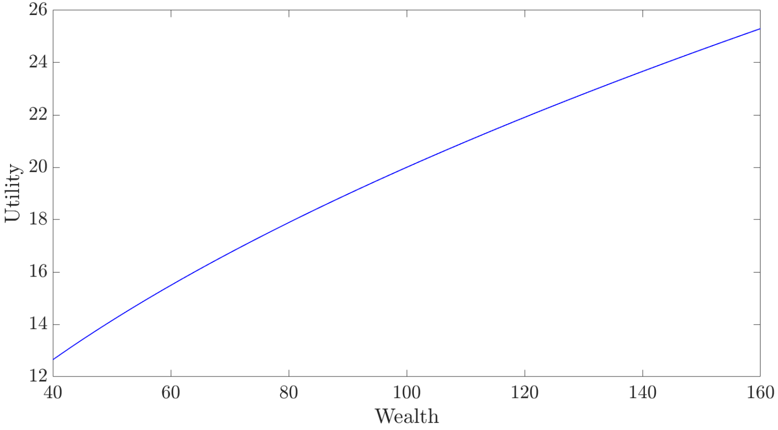
$$RRA(W) = -\frac{WU''(W)}{U'(W)} = 1$$

- The separate definition for $\gamma = 1$ is required because $\frac{W^{1-\gamma}}{1-\gamma}$ is not defined when $\gamma = 1$ but the function tends towards the natural log function as $\gamma$ goes towards one.
- As you would expect from the name, these functions have constant relative risk aversion and the relative risk aversion equals $\gamma$. The higher is $\gamma$, the more risk averse people are.

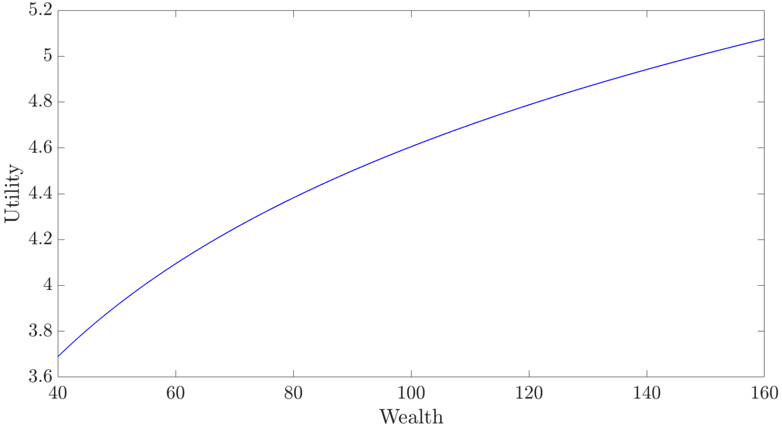
# Evidence on Relative Risk Aversion

- A lot of studies have attempted to measure relative risk aversion.
  - Some use experiments asking people about their preferences over different possible gambles.
  - Others use surveys that relate people's happiness to their financial situation.
  - Some use data on prices for different types of financial assets.
- Obviously everyone has different levels of tolerance for risk.
- But my assessment is that a relative risk aversion of $\gamma = 1$ is about as low as can be justified as a "typical" level based on these studies.
- It also seems unlikely that many people have $\gamma$ greater than about 5.
- We will use this evidence to consider how most people will behave when offered gambles that are small relative to their wealth.
- The next 3 pages show three different CRRA utility functions with different values of the relative risk aversion parameter $\gamma$.

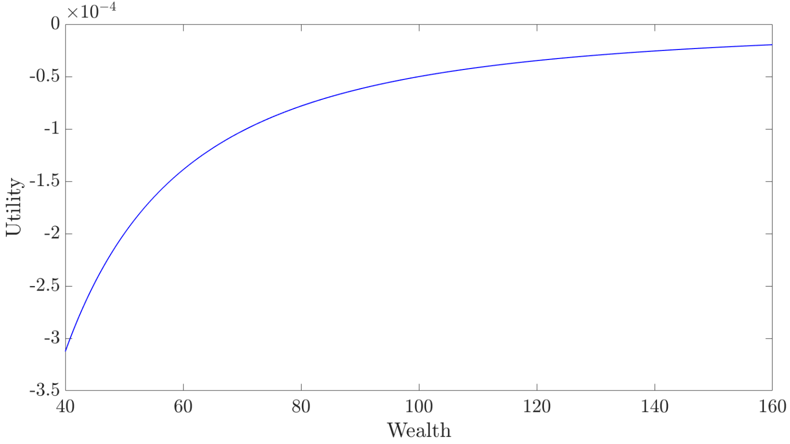
# $\gamma = 0.5$: Low Risk Aversion



# $\gamma = 1$: Moderate Risk Aversion



# $\gamma = 3$: High Risk Aversion



# Calculating Expected Utility with CRRA Utility Functions

- Suppose you have a CRRA utility function

$$U(W) = \frac{W^{1-\gamma} - 1}{1-\gamma}$$

  and you are offered a gamble with a 50 percent chance of winning $G$ and a 50 percent chance of losing $L$.

- If your current wealth level is $W$, your expected utility from accepting the gamble would be

$$E[U(W)] = \frac{1}{2}\left[\frac{(W+G)^{1-\gamma} - 1}{1-\gamma} + \frac{(W-L)^{1-\gamma} - 1}{1-\gamma}\right]$$

- In the assignment, I will ask you to do expected utility calculations for various gambles using a formula like this.
- You can do these calculations however you want but probably the easiest way to do it is using Excel.
- I will provide a video showing how to do expected utility calculations with Excel.

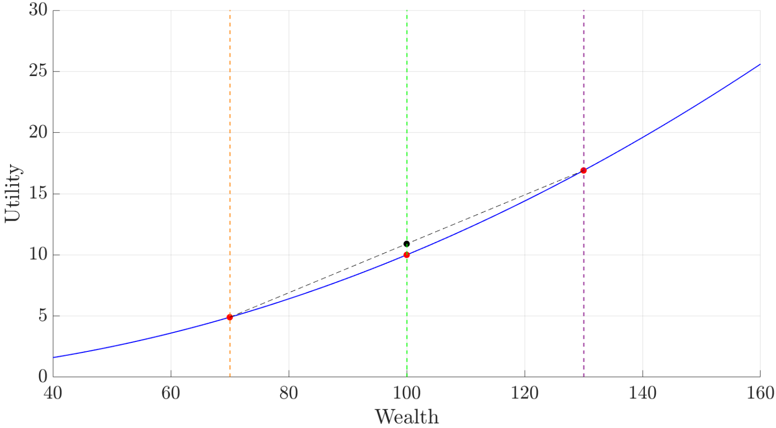
# Part III

## Do Bettors Just Love Taking Risks?

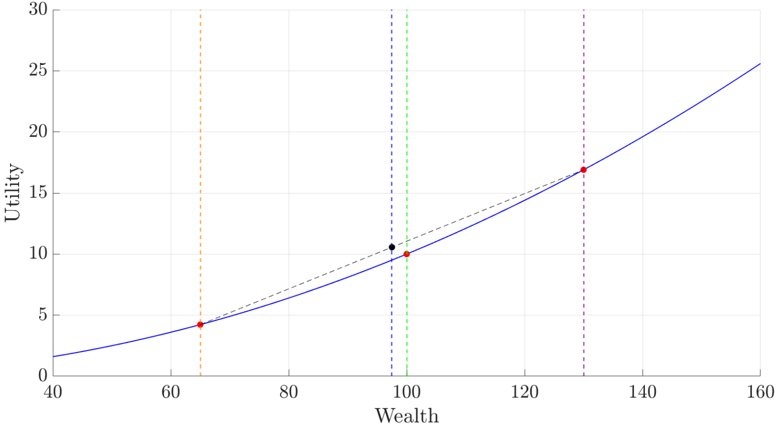
# Convex Utility

- Most people don't place bets. Maybe those who do place bets just have different kinds of utility functions?
- If someone has a **convex utility function** (one with a positive second derivative), then the earlier results from the charts about accepting gambles are reversed.
- The next page illustrates how someone with a convex utility function always accepts a 50/50 fair gamble (equal chance of winning or losing the same amount).
- The page after shows how someone with convex utility may accept some unfair gambles with negative expected values. This chart shows someone with convex utility accepting a gamble with an equal chance of winning 30 and losing 35.
- Maybe convex utility functions for gambles explain why they accept bets that have a negative expected value?

# Convex Utility Implies Accepting a Fair Gamble



# Accepting a Gamble with Negative Expected Value



# Is This Really How Bettors Behave?

- Surveys show one in ten UK adults had placed a bet on sports over the previous four weeks. What is the chance that one in ten people are enthusiastic about taking risks purely for their own sake?
- Most people who bet on sports are perfectly normal. They routinely make decisions that reflect risk aversion.
- They buy insurance. They choose their investments carefully. They avoid unnecessary risks in everyday life.
- The idea that sports bettors have increasing marginal utility of wealth does not fit this behaviour.
- Someone who truly enjoyed risk for its own sake would not need a bookmaker. You would expect to see them **adding risk everywhere, for fun**.
- If bettors had convex utility functions, you'd see these people getting together to enjoy games of tossing coins to win or lose money just because they like the uncertainty. But you don't see this.
- There must be more going on than a simple love of risk.

# Small Stakes and Risk Preferences

- Across countries and platforms, the **typical bet size is modest**.
- Studies show that the median bet size is typically about €5 to €10.
- This is hard to square with genuinely risk-loving preferences.
- If you like a €5 bet because you enjoy risk, you should like a €10 bet even more: it is the same gamble, just with bigger swings.
- But most people do not scale up like this. They keep their stakes small.
- The simplest interpretation is that most bettors are risk averse in the usual sense: they want the hoped-for profit (and perhaps the entertainment) without risking serious losses.
- We won't be assuming bettors have convex risk-loving preferences.

# Part IV

## Why Risk Averse People Take Small Gambles

# Arrow and Small-Stakes Risk Taking

- **Kenneth Arrow** was one of the most important economists ever. He won the Nobel Prize for economics in 1972.
- He pioneered modern general equilibrium theory and the economics of decision-making under risk and uncertainty (and lots more ...)
- One of Arrow's key insights is surprisingly simple.

**Arrow's small-stakes result**

A risk-averse person will accept any gamble with positive expected value, provided the stakes are small enough.

- This result is useful because it clarifies what economists mean by "risk aversion".
- Risk aversion does not mean refusing all gambles. It just means trading off the expected return against the potential risk in an appropriate way.
- We will prove Arrow's result using the same Taylor-approximation logic you've already seen.

# Arrow's Result: The Small-Stakes Argument

- Utility $U(W)$ satisfies $U' > 0$ and $U'' < 0$.
- Consider a gamble with return $X$ taken at scale $t > 0$: final wealth is $W(1+tX)$. Assume $E[X] > 0$.
- Second-order Taylor approximation around $W$ gives

$$E[U(W(1+tX))] - U(W) \approx U'(W)Wt\,E[X] + \frac{1}{2}U''(W)W^2 t^2 E[X^2]$$

- Using $RRA(W) \equiv -\frac{WU''(W)}{U'(W)}$, this becomes

$$E[U(W(1+tX))] - U(W) \approx U'(W)Wt\left(E[X] - \frac{1}{2}RRA(W)\,t\,E[X^2]\right)$$

- The gamble will be accepted if the bracket is positive:

$$E[X] - \frac{1}{2}RRA(W)\,t\,E[X^2] > 0 \quad \Rightarrow \quad t < \frac{2E[X]}{RRA(W)\,E[X^2]}$$

# Understanding Arrow's Small-Stakes Condition

## The small-stakes condition

Take the bet of scale $t$ as long as

$$t < \frac{2E[X]}{RRA(W)\, E[X^2]}$$

- Let's take a step back and interpret this condition.
  - $E[X]$ captures how attractive the gamble is on average. A higher expected profit makes the gamble easier to accept.
  - $RRA(W)$ captures how strongly the person dislikes risk. More risk-averse people require smaller stakes.
  - $E[X^2]$ captures how risky the gamble is. More volatile gambles must be taken at smaller scale.
- The key economic message is that even a very risk-averse person will accept a gamble they believe is profitable, as long as it is small enough relative to their wealth.
- Risk aversion limits how big a bet you are willing to take, not whether you are willing to bet at all.

# Example: 50/50 Gain/Loss Returns

- Special case: with probability $1/2$, $X = g$, with probability $1/2$, $X = -\ell$. So payoff is $+Wtg$ or $-Wt\ell$.
- Compute moments:

$$E[X] = \frac{g-\ell}{2}, \qquad E[X^2] = \frac{g^2+\ell^2}{2}.$$

- Plug into the stake threshold condition

$$t < \frac{2(g-\ell)}{RRA(W)\,(g^2+\ell^2)}.$$

- Or, for a given stake share $t$, the (approximate) condition on relative risk aversion is

$$RRA(W) < \frac{2(g-\ell)}{t\,(g^2+\ell^2)}.$$

- **Interpretation:** The smaller $t$ gets, the more risk averse people have to be to turn down a positive expected value gamble.

# Examples

- Consider a gamble with an equal chance of losing 1% of wealth ($\ell = 0.01$) or gaining 1.5% of wealth ($g = 0.015$).
- Expected utility theory predicts people will accept the gamble if

$$RRA(W) < \frac{2(0.005)}{0.01^2 + 0.015^2} = 30.77$$

- This is a very high level of relative risk aversion, so we should expect almost everybody to accept this offer.
- Even if we make the potential gain a lot smaller, most people should accept. Suppose the gain is 1.1% of wealth. The same calculation gives

$$RRA(W) < \frac{2(0.001)}{0.01^2 + 0.011^2} = 9.05$$

  That's still a really high level of risk aversion so again pretty much everyone should accept.

# The Bottom Line

- Lots of maths, I know.
- But there was a point.
- We have shown something useful.

For small-stakes gambles, risk averse people should generally accept gambles they believe have a positive expected value.

- In other words, as long as the stakes are small (and they usually are in sports betting) then

$$pD > 1$$

  is a perfectly good condition to describe whether people will bet.
- As long as people believe a bet has positive expected value, it is rational for them to place at least some amount of money on it.
- But where do these beliefs come from? That's our next topic.

# Supplementary Material in the Draft Book

- Chapter 7: Risk, Beliefs and a Rule for Betting