ECON30580 Economics of Betting Markets
6. Beliefs, Betting and the Wisdom of Crowds

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From the Decision Rule to the Demand for Bets

- We have established that our rule for accepting a bet is: Accept if $pD > 1$ where
  - p is the person's belief about the probability that the bet will win.
  - D is the decimal odds (if you bet €1 and win, you get back €D)
- But where does your estimate of p come from?
- We will assume people **disagree**: Some think $pD > 1$ and take the bet and some think it is not and decline it.
- By modelling this disagreement, we can come up with predictions for how many people will accept a bet at each possible odds. This "**demand curve**" for bets will be crucial in our work on explaining how betting markets work.
- Here, we will describe a framework for modelling disagreement between bettors.
  - We start with simple numerical examples.
  - Briefly discuss the idea that people also factor in fun or excitement.
  - And then present a formal version of the demand for bets.

Part I

The Wisdom of Crowds and the Demand for Bets

How Should We Describe the Public's Beliefs?

- Suppose the true probability of a bet winning is p.
- What is a reasonable assumption about the public's beliefs about this probability?
- Figuring out probabilities is hard and most people are probably wrong in their estimates.
- But it turns out there is a lot of evidence that, on average, the public is pretty good at figuring out the right answer to questions.
- We will assume that **on average, the public is correct in their subjective assessments of p.**
- This idea is an old one. Aristotle wrote: "*The many, though not individually wise, collectively may be wiser than any individual ... For each individual among the many has a share of virtue and prudence, and when they meet together ... are better judges than a single man.*"
- In the modern era, James Surowiecki's 2004 book *The Wisdom of Crowds* promoted this idea.

A Concrete Example: Arsenal versus Chelsea

- Suppose Arsenal play Chelsea in a knock-out cup tie and you can bet on which team goes through (so there is no draw).
- Let $p = 0.7$ be Arsenal's true probability of going through.
- In line with the wisdom of crowds, assume bettors' beliefs range from $p = 0.64$ to $p = 0.76$, and that subjective probabilities are always two-digit decimal numbers.
- The median bettor believes, correctly, that Arsenal have a 70% chance of going through.
- Suppose the bookmaker sets fair odds:

$$
\begin{aligned}
D_A &= \frac{1}{p} = \frac{1}{0.70} = 1.43 \\
D_C &= \frac{1}{1-p} = \frac{1}{0.30} = 3.33
\end{aligned}
$$

- The $pD > 1$ condition implies:
  - Those who believe $p < 0.70$ bet on Chelsea.
  - Those who believe $p > 0.70$ bet on Arsenal.
  - Those with $p = 0.70$ do not bet.

Betting with Fair Odds

Betting Decisions by Belief: Fair Odds (No Margin)

Belief about probability Arsenal win	*p × 1.43 for Arsenal*	*(1-p) × 3.33 for Chelsea*
0.64	0.91 (Don't bet)	1.20 (Take bet)
0.65	0.93 (Don't bet)	1.17 (Take bet)
0.66	0.94 (Don't bet)	1.13 (Take bet)
0.67	0.96 (Don't bet)	1.10 (Take bet)
0.68	0.97 (Don't bet)	1.07 (Take bet)
0.69	0.99 (Don't bet)	1.03 (Take bet)
0.70	**1.00 (Don't bet)**	**1.00 (Don't bet)**
0.71	1.01 (Take bet)	0.97 (Don't bet)
0.72	1.03 (Take bet)	0.93 (Don't bet)
0.73	1.04 (Take bet)	0.90 (Don't bet)
0.74	1.06 (Take Bet)	0.87 (Don't bet)
0.75	1.07 (Take Bet)	0.83 (Don't bet)
0.76	1.09 (Take Bet)	0.80 (Don't bet)

Odds with a 5% Margin

- Now let's assume the odds are set by bookmakers with a 5% margin ($m = 0.05$).
- This means the bookmaker sets odds as

$$D_A = \frac{1-m}{p} = \frac{0.95}{0.70} = 1.36$$
$$D_C = \frac{1-m}{1-p} = \frac{0.95}{0.30} = 3.17$$

- People now bet like this.
  - Those who believe $p < 0.69$ bet on Chelsea.
  - Those who believe $p > 0.73$ bet on Arsenal.
  - Those with $0.68 < p < 0.74$ do not bet.
- Note that the betting goes from balanced (equal amounts on both teams) to unbalanced (5 bets on Chelsea and 3 on Arsenal).
- This pattern is a sign of something important: **Longshot bettors are less elastic in their demand for bets.**

Betting with a 5% Margin

Betting Decisions by Belief: Adding a 5% Margin

Belief about probability Arsenal win	p × 1.36 for Arsenal	$(1-p)$ × 3.17 for Chelsea
0.64	0.87 (Don't bet)	1.14 (Take bet)
0.65	0.88 (Don't bet)	1.11 (Take bet)
0.66	0.90 (Don't bet)	1.08 (Take bet)
0.67	0.91 (Don't bet)	1.05 (Take bet)
0.68	0.92 (Don't bet)	1.01 (Take bet)
0.69	**0.94 (Don't bet)**	**0.98 (Don't bet)**
0.70	**0.95 (Don't bet)**	**0.95 (Don't bet)**
0.71	**0.96 (Don't bet)**	**0.92 (Don't bet)**
0.72	**0.98 (Don't bet)**	**0.89 (Don't bet)**
0.73	**0.99 (Don't bet)**	**0.86 (Don't bet)**
0.74	1.01 (Take bet)	0.82 (Don't bet)
0.75	1.02 (Take bet)	0.79 (Don't bet)
0.76	1.03 (Take bet)	0.76 (Don't bet)

What About Betting for Fun?

- Survey evidence shows one reason people bet is they get a buzz from placing bets on sports.
  - A UK Gambling Commission survey found that 72% of regular bettors listed "*Because it's fun*" as a reason.
  - 35% answered that they bet "*Because it's something I do with my family and friends.*"
  - 80% of respondents to a US study agreed with "*It makes watching sports more fun.*"
- So they didn't say "*I bet when I think* $pD > 1$". Well, actually ...
  - 63% in the UK Gambling Commission survey listed "*To make money*" as one of their motivations for betting.
  - A US survey showed 71% of bettors cited "*winning money*" as a major reason for betting
- And, crucially, "betting for fun" does not explain whether you pick Arsenal or Chelsea.

Adding Excitement

- Let's add excitement as a motivation.
- Assume people bet as long as $pD > 1 - e$.
- They will accept losing a bit (on average) for the fun involved.
- If they think both bets have $pD > 1 - e$ assume they pick the one with the higher value of pD.
- Let's assume $e = 0.02$ – the surveys tell us people don't want to lose much.
- People now bet like this.
    - Those who believe $p < 0.70$ bet on Chelsea.
    - Those who believe $p > 0.72$ bet on Arsenal.
    - Those with $0.69 < p < 0.73$ do not bet.
- One more person bets on Arsenal and one more on Chelsea.
- But the results are very similar and the balance of bets is still skewed.
- Excitement is definitely a factor but it doesn't help much in explaining the demand for bets, so we won't model it formally.

Betting with a 5% Margin and Excitement

Adding 0.02 Excitement from Betting

Belief about probability Arsenal win	*p × 1.36 + 0.02 for Arsenal*	*(1-p) × 3.17 + 0.02 for Chelsea*
0.64	0.89 (Don't bet)	1.16 (Take bet)
0.65	0.90 (Don't bet)	1.13 (Take bet)
0.66	0.92 (Don't bet)	1.10 (Take bet)
0.67	0.93 (Don't bet)	1.07 (Take bet)
0.68	0.94 (Don't bet)	1.03 (Take bet)
0.69	0.96 (Don't bet)	1.00 (Take bet)
0.70	**0.97 (Don't bet)**	**0.97 (Don't bet)**
0.71	**0.99 (Don't bet)**	**0.94 (Don't bet)**
0.72	**1.00 (Don't bet)**	**0.91 (Don't bet)**
0.73	1.01 (Take bet)	0.88 (Don't bet)
0.74	1.02 (Take bet)	0.86 (Don't bet)
0.75	1.04 (Take bet)	0.81 (Don't bet)
0.76	1.05 (Take bet)	0.78 (Don't bet)

Lotteries? Buying a Dream

- Many people buy lottery tickets.
- And it is well known that lotteries make big profits. Expected payouts (pD) for lottery tickets are way below one, often as low as a half.
- Disagreement doesn't explain ticket purchases. The winning numbers are random and there is no reason to think the numbers you have picked are especially likely to win.
- So, we can't use people thinking $pD > 1 - e$ with small e to explain why they buy lottery tickets.
- But there is evidence that the higher is the lottery payout, the more tickets are sold, so pD does seem to matter.
- Lotteries seem to fit with a big e from the possibility of winning a life-changing amount of money—the ticket purchase is "buying a dream."
- This idea may help to explain why more exotic sports bets that offer a small chance of a big win get bought despite objectively awful values of pD—perhaps people view them as mini-lotteries that are exciting to think about winning.

Part II

A Formal Version

A Uniform Distribution of Beliefs

- The kind of numerical calculations we just did are good for intuition but not much use if we want to do things like figure out profit-maximising odds for bookmakers.
- For that we need a formal model.
- Let's say each person has a subjective belief denoted as $\tilde{p}$ (the squiggle is known as a tilde) and that there is a continuum of people with beliefs ranging from $\tilde{p} = L$ to $\tilde{p} = H$ with equal amounts of people holding each possible belief.
- In other words, there is a **uniform distribution** of beliefs.
- We will need an expression for the cumulative distribution function (CDF), $F(x)$ for this distribution. This tells us the fraction of people with beliefs less than the value x.
- The general formula for the CDF of a uniform distribution on $[L, H]$ is

$$F(x) = \begin{cases} 0 & \text{if } x < L \\ \frac{x-L}{H-L} & \text{if } L \leq x \leq H \\ 1 & \text{if } x > H \end{cases}$$

Examples

- Suppose beliefs are uniform $[0.1, 0.5]$ so the median and mean are both 0.3.
- Clearly $F(0.3) = 0.5$, meaning half the people have beliefs below and half above. See how this works with the formula.

$$F(0.3) = \frac{x - L}{H - L} = \frac{0.3 - 0.1}{0.5 - 0.1} = 0.5$$

- Similarly, a quarter of the people have beliefs below 0.2.

$$F(0.2) = \frac{x - L}{H - L} = \frac{0.2 - 0.1}{0.5 - 0.1} = 0.25$$

- So the formula may look intimidating but the logic of it is pretty clear — we are measuring the size of a subset of the beliefs relative to the full range.
- We will often want to know the fraction of people with beliefs **above** a certain value. This is given by

$$1 - F(x) = \begin{cases} 1 & \text{if } x < L \\ \frac{H-x}{H-L} & \text{if } L \leq x \leq H \\ 0 & \text{if } x > H \end{cases}$$

What Fraction Accept a Bet?

- Given odds of D, only people who have a belief such that $\tilde{p}D > 1$ accept the bet.
- In other words, only those with $\tilde{p} > \frac{1}{D}$ accept the bet.
- The cumulative distribution function $F(x)$ tells us the fraction of people with beliefs below x.
- So $1 - F(x)$ tells us the fraction of people with beliefs above x.
- At odds of D (where $\frac{1}{H} < D < \frac{1}{L}$) the fraction of people that will take the bet (the fraction of people with $\tilde{p}$ above $\frac{1}{D}$) will be

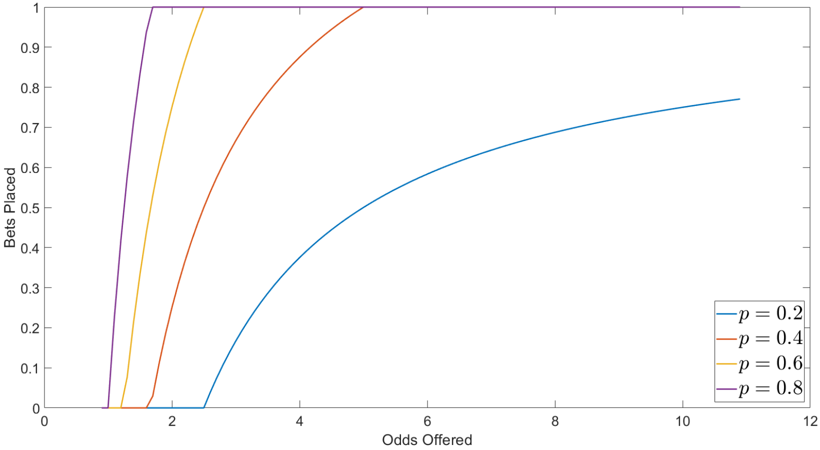
$$B(D) = \frac{H - \frac{1}{D}}{H - L}$$

- The higher D is (the more generous the odds) the more people will take the bet.
- Note that if $D = \frac{1}{H}$, then $B(D) = 0$. Nobody takes the bet.
- And if $D = \frac{1}{L}$, then $B(D) = 1$. Everybody takes the bet.

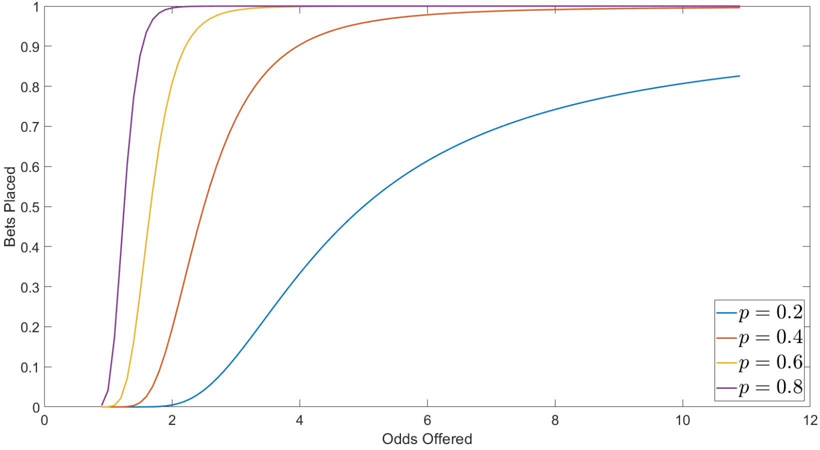
The Demand for Bets

- We have figured out what fraction of people would accept a bet at odds of D.
- If we add the assumption that everybody takes the same (presumably small) sized bet, then $B(D)$ becomes a **demand function for bets**. It tells us the amount of money that will be placed on the bet.
- It tells us at the price offered (the odds D), how many bets will be placed.
- The next page shows the demand for bets for different values of p where beliefs are uniform on $[p-0.2, p+0.2]$ as a function of the odds.
- Note how the $p = 0.2$ demand curve is much less steep: Longshot/underdog demand is less sensitive to the odds even when everyone is using the same $\tilde{p}D > 1$ rule to bet.
- The final page shows the same demand curve when beliefs are $N(p, 0.115)$, which implies the same variance. of beliefs as the uniform distribution with a maximum gap of 0.4 between the top and bottom of beliefs.
- The Normal distribution case needs to be solved numerically but gives essentially the same demand curves.

Demand for Bets with Uniform Beliefs on $[p - 0.2, p + 0.2]$



Demand for Bets with Normally Distributed Beliefs, $N(p, 0.115)$



How Demand for Longshots is Less Sensitive to Odds

- Consider an event with two competitors where the probability of favourite winning is $p = 0.9$.
- Bettors have beliefs ranging from $\tilde{p} = 0.8$ to $\tilde{p} = 1$.
- Let D_F and D_L be the decimal odds for the favourite and longshot.
- The fair odds at which bets break even on average are

$$
\begin{aligned}
D_F &= \frac{1}{0.9} = 1.11 \\
D_L &= \frac{1}{0.1} = 10
\end{aligned}
$$

- The most optimistic about the favourite's prospects (those with $\tilde{p} = 1$) would accept odds of $D_F = 1$, which is 11% lower than fair odds.
- The most optimistic about the longshot (those with $\tilde{p} = 0.8$) would accept odds of $D_L = \frac{1}{0.2} = 5$ which is 50% lower than fair odds.
- This illustrates that the longshot enthusiasts are less sensitive to the odds than the favourite enthusiasts.

Supplementary Material in the Draft Book

- Chapter 7: Risk, Beliefs and a Rule for Betting
- Chapter 8: Betting and the Wisdom of Crowds