ECON30580 Economics of Betting Markets
6. Pari-Mutuel Betting Markets and the Favourite-Longshot Bias

Karl Whelan

School of Economics, UCD

Spring 2026

Part I

Recap on Pari-Mutuel Betting

Reminder: How Pari-Mutuel Betting Works

- Suppose there is a horse race with multiple contestants.
- Suppose that \$1000 has been placed in bets and the pari-mutuel operator's "take" is 10%. This means there is \$900 available to pay out to those who picked the winner.
- The terms of the winning payments—the odds—are determined by how much money has been placed on the winning horse.
  - If a total of \$450 was placed on the winner, each winner will be paid back \$2 for every \$1 they have bet.
  - If a total of \$600 was placed on the winner, each winner will be paid back \$1.5 for every \$1 they have bet.
- The odds are inversely proportional to the betting volumes: As the fraction of money placed on the winning horse gets higher, the size of the winning payout on a \$1 bet gets lower.

Pari-Mutuel Odds and Volumes

- If you know a set of pari-mutuel odds, you can quickly figure out the fraction of the betting that was placed on each option.
- Consider an event with N possible outcomes. Let V_i be the amount of money placed on option i and let the sum of all bets be

$$V = V_1 + V_2 + ... + V_N$$

- Then the decimal odds for this bet in a pari-mutuel market will be

$$D_i = \frac{(1-t)\,V}{V_i} \Longrightarrow V_i = \frac{(1-t)\,V}{D_i}$$

where t is the track take percentage.

- We can figure out the share of bets placed on option i as

$$\frac{V_i}{V} = \frac{\frac{(1-t)V}{D_i}}{\frac{(1-t)V}{D_1} + \frac{(1-t)V}{D_2} + ... + \frac{(1-t)V}{D_N}} = \frac{\frac{1}{D_i}}{\frac{1}{D_1} + \frac{1}{D_2} + ... + \frac{1}{D_N}}$$

Efficiency of Pari-Mutuel Markets

- The share of bets placed on option i is

$$\frac{V_i}{V} = \frac{\frac{1}{D_i}}{\frac{1}{D_1} + \frac{1}{D_2} + ... + \frac{1}{D_N}}$$

- This is the same as the formula for normalised probabilities.
- Remember that strong efficiency implies the normalised probabilities equal the true probabilities.
- This gives us a condition under which a pari-mutuel betting market will satisfy strong market efficiency.
- **A pari-mutuel market will be strongly efficient if the fraction of bets placed on a contestant equals the probability that the contestant wins.**
- Is this likely to happen? It seems a bit of coincidence of two different things, so it also might not.
- We will have to check the evidence.

Part II

The Favourite-Longshot Bias

Early Research on Pari-Mutuel Markets

- For most of the 20th century, betting with bookmakers was banned in many countries.
- But betting was always likely to be a subject of interest for the emerging field of quantitative social science, so the early studies used available data from pari-mutuel betting at US horse racetracks.
- In the case of pari-mutuel markets, the equal return across bets implied by strong efficiency should be 1 minus the percentage of the funds taken by the track.
- If the "track take" is t where $0 < t < 1$, then the expected return for all bets implied by market efficiency should be $1 - t$.
- In 1949, psychologist Richard Griffith published a paper comparing the probabilities implied by the closing odds in pari-mutuel horse races to the actual probabilities that various groups of horses would win (where the horses were arranged together in groups with similar odds).
- He found the actual win probabilities were highly correlated with the betting market probabilities, so the market did well at assessing horses.

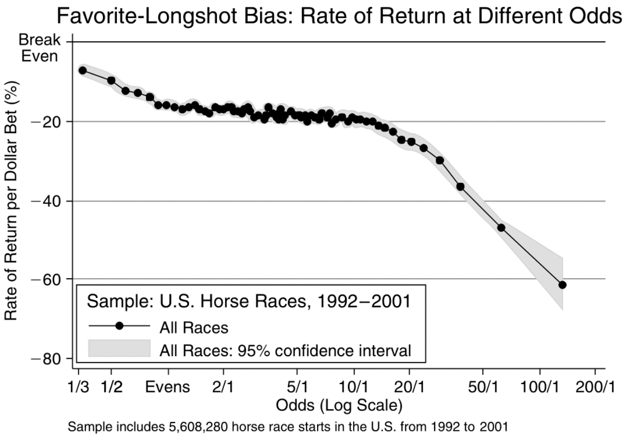
The Favourite-Longshot Bias

- Still, Griffith found that the market was not strongly efficient.
- He found that horses with short odds (favourites) won more often than the probabilities implied by the odds and the horses with long odds (longshots) won less often.
- This meant that people were losing more money betting on longshots than they were losing on favourites (none of the groups of bets actually won people money.)
- This finding was repeated in many studies across time and space.
- This has become known as the **favourite-longshot bias**.

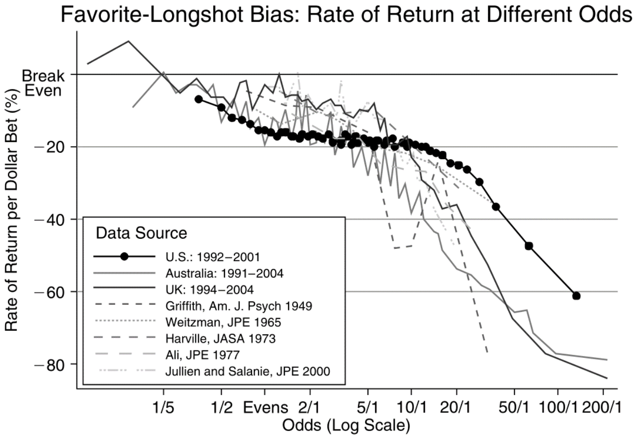
The Shape of the Favourite-Longshot Bias

- A good study summarising the evidence is "Examining Explanations of a Market Anomaly: Preferences or Perceptions?" by Erik Snowberg and Justin Wolfers.
  - They collected a dataset of pari-mutuel odds and outcomes for 5,608,208 different US horse races.
  - They knew the results of the races, so they knew which bets won and which lost and could figure out average returns for different kinds of bets.
  - The figure on the next page breaks all the bets into subgroups organising them by their odds and shows the average rate of return per dollar staked for each of these groups.
  - The page after shows the same calculations from other studies as well as their calculations for Australian pari-mutuel betting and UK fixed-odds betting.
  - There is a striking consistency in the evidence: Average loss rates are higher for longshots and the pattern is highly nonlinear. As the odds get higher, the loss rate curves turn sharply downwards.
- A theory to explain the favourite-longshot bias would ideally predict this shape.

The Favourite-Longshot Bias for US Pari-Mutuel Betting



The Favourite-Longshot Bias Around the World



Richard Thaler and Anomalies

- Richard Thaler was a key figure in the development of behavioural economics.
- He won the 2017 Nobel Prize in Economics for his contributions to behavioural economics.
- He is also famous for his best-selling book *Nudge*, co-written with Cass Sunstein.
- When the American Economic Association founded the *Journal of Economic Perspectives* in the 1980s as a way of promoting research findings to a more general audience, Thaler was commissioned to write a column called "Anomalies" which detailed various ways in which standard models in economics seemed to fail.
- One of the earliest columns (co-authored with William Ziemba) covered the favourite-longshot bias in pari-mutuel betting.
- It summarised various explanations that had been put forward for the bias.

Convex Utility as an Explanation

- In a 1986 paper in the prestigious *Quarterly Journal of Economics* called "Betting and Equilibrium" Richard Quandt explained the favourite-longshot bias in pari-mutuel markets as follows.
- Betting loses money on average. Quandt reasoned that people knew this and so, if they placed bets, it was because they had convex utility functions and thus were risk lovers.
- Remember that, for two bets with the same expected payout, the payouts on bets with higher odds have a higher variance.
- Quandt argued that when faced with two bets with the same expected payout pD, these convex utility risk-loving bettors would prefer to all take the one with the lowest value of p and thus the high variance.
- But suppose everyone decided to bet on the same horse.
- At that point, even an infinitesimally small bet on one of the other horses would win the whole post-track-take pool if that horse won despite betting only a tiny amount. These huge odds would make the betting on the other horses very attractive.
- This showed everyone preferring the same horse could not be an equilibrium.

A Compensating Differentials Explanation

- Quandt argued that the equilibrium outcome had to have each bet being equally attractive.
- If everyone viewed each of the bets as equally attractive, then there was no incentive for people to switch their preference from betting on one horse to another.
- This meant equilibrium could not involve each bet having the same pD because the risk-loving bettors would find the low-variance high-p bets to be less attractive than the high-variance low-p bets.
- To make bettors indifferent between all the bets, those low-variance bets with high values of p needed to have higher expected returns.
- The favourite-longshot bias emerges as an example of what economists call a **compensating differential**. A product that has one unattractive feature has to compensate by having another feature that is attractive or vice versa (e.g. jobs featuring pleasant tasks will sometimes have relatively lower wages).

For and Against Risk-Loving Preferences as an Explanation

- In favour of people having risk-loving preferences as an explanation:
  - It's an elegant and simple theory.
  - It may explain the shape of the favourite-longshot bias because the variance of payoffs depends on $\frac{1}{p}$ (we derived this before) and this explodes when p gets small. Maybe people are willing to tolerate really bad returns for extreme bets because they have huge variance.
- Against this theory (recapping our earlier discussion of convex utility):
  - If people who bet like risk so much, why do they buy insurance and require premium returns for risky investments?
  - You have to argue that people are risk-averse for big risks but risk-loving for small risks. This seems a bit ad hoc.
  - Why don't groups of people bet on coin tosses amongst themselves all the time? This would increase the expected utility of risk lovers.
  - The loss rates for extreme longshots are really big. Is it really credible that people know this and say "I'm probably going to lose a big fraction of my money, but hey I'm loving the riskiness"?

Other Explanations

- Thaler and Ziemba put forward various other explanations.
  1. **People are bad at judging small probabilities**: Psychologists often argue that people are poor judges of the probabilities of unlikely events and tended to over-weight them when making decisions. In the case of sports, perhaps people imagine the longshots are more likely to win than they are, which leads to too many people betting on longshots.
  2. **Utility value**: It may be more fun to bet on a longshot since, if they win, you get more "bragging rights."
  3. **Random choice**: Some people may pick a horse because they like the name, or the colour of the team's jersey or because they like backing the team they support. These random bets may split evenly between the contestants and don't depend on the odds that are quoted. This may generate more bets on the longshot than if everyone was rationally assessing the probabilities.
- I like some of these theories more than others but my **preferred explanation (and remember I grade the exam)** is a simpler one: People disagree about probabilities and this leads to favourite-longshot bias in pari-mutuel betting markets. We will discuss this explanation next.

Part III

How Disagreement Explains the Pari-Mutuel Favourite-Longshot Bias

Simultaneous Odds and Volumes

- Odds in pari-mutuel markets are determined by the amount of money placed on each competitor.
- But, in our approach, the amount of money that will be placed is determined by the odds.
- We are going to assume the betting volumes and the odds are **simultaneously determined**.
- In other words, we figure out the odds and volumes such that the odds are consistent with the volumes and the volumes are consistent with the odds.
- This is just like when we simultaneously figure out prices and quantities that are consistent with both demand and supply functions.
- In practice, gamblers do not know what the final odds will be in a pari-mutuel market but pari-mutuel betting operators display indicative odds based on betting volumes and update these figures frequently, often as quickly as every 30 or 45 seconds.
- While a flood of late bets can change the odds from the indicative ones that were displayed, it is probably closer to true than false to say that bettors in pari-mutuel markets have a good idea of their odds.

Ali (1977)

- In a 1977 paper in the prestigious *Journal of Political Economy*, Mukhtar Ali explained how people disagreeing with each other could result in favourite-longshot bias in pari-mutuel betting. His idea can be explained as follows.
- Consider a match in which team F (the favourite) has probability $p > 0.5$ of winning and team L (the longshot) has probability $1 - p$ of winning.
- Assume the median value of beliefs about the probability of the favourite winning among potential bettors $\tilde{p}$ is equal to the correct true value of p: On average the crowd is "wise".
- Let's set the pari-mutuel "take percentage" equal to zero.
- In this case, the strongly efficient market outcome with equal returns on both bets is the fair odds

$$D_F = \frac{1}{p} \qquad D_L = \frac{1}{1-p}$$

- Could these odds emerge from pari-mutuel betting?
- Those with $\tilde{p} > p$ would take the bet on the favourite and those with $\tilde{p} < p$ would take the bet on the longshot.

Why Fair Odds are Not the Outcome

- However, with some inspection, we can see why these efficient market odds can't be the outcome of the pari-mutuel betting process.
- The fair odds would produce the same amount of betting on the favourite as on the longshot because p is assumed to be the median of the distribution of beliefs.
- At these odds, the number of people with backing the favourite (those with $\tilde{p} > p$) is equal to the number backing the longshot (those with $\tilde{p} < p$).
- But pari-mutuel odds are set in inverse proportion to the betting volumes. If betting amounts are equal on the two teams, then their odds would be equal, which in turn means they wouldn't equal the efficient odds.
- And if everyone was offered equal odds on the two teams, most of the public (who on average are correct about p) will bet on the favourite, which should imply lower odds for the favourite.
- These contradictions mean that the fair odds of $D_F = \frac{1}{p}$ and $D_L = \frac{1}{1-p}$ cannot be the outcome of the betting.

Ali's Explanation for Favourite-Longshot Bias

- The equilibrium must have consistent volumes and odds.
- We can see that $D_F = \frac{1}{p}$ and $D_L = \frac{1}{1-p}$ generates equal amounts of betting on the favourite and the longshot, which can't be the outcome.
- However (keeping the assumption of zero takeout) if the pari-mutuel odds on the favourite are improved a bit relative to fair odds, this generates more betting on the favourite.
- And if the odds on the longshot were worsened, then we would get more bets on the favourite and fewer bets on the longshot.
- The right upward adjustment of odds for favourites and downward adjustment of odds for longshots can make the odds consistent with betting volumes.
- But these adjustments away from fair odds mean the ratio of the odds can't equal the ratio of the probabilities so the market is not efficient.
- The upward movement in odds on the favourites and downward movement in odds on longshots mean people will do better betting on favourites than longshots.

Demand for Bets with Our "Wisdom of Crowds" Beliefs

- We can use our uniformly-distributed beliefs model of disagreement to provide a formal demonstration of the favourite-longshot bias in pari-mutuel betting markets.
- Remember that we assume beliefs about p are from a uniform distribution on $[L, H]$.
- This gave the following formula for the demand for bets

$$B(D) = \frac{H - \frac{1}{D}}{H - L}$$

- For an event where the true probability of success is p, we are assuming $L = p - \sigma$ and $H = p + \sigma$.
- Plugging in H and L, the fraction of people who will take a bet at odds of D (where $\frac{1}{H} < D < \frac{1}{L}$) will be

$$B(D) = \frac{p + \sigma - \frac{1}{D}}{2\sigma}$$

Demand for Bets in a Two Team Event

- Let's stick with the two team event with beliefs distributed uniformly on $[p - \sigma, p + \sigma]$.
- Let D_F and D_L be the pari-mutuel odds on the teams and let $B(D_F)$ and $B(D_L)$ be the volume of bets placed on the two teams.
- Let τ be the pari-mutuel operator's percentage take.
- The pari-mutuel odds have to be inversely related to the share of betting placed on each competitor,

$$D_F = \frac{(1-\tau)(B(D_F) + B(D_L))}{B(D_F)}$$

$$D_L = \frac{(1-\tau)(B(D_F) + B(D_L))}{B(D_L)}$$

Betting Volumes

- The demand functions for the two bets are

$$B(D_F) = \frac{p + \sigma - \frac{1}{D_F}}{2\sigma}$$
$$B(D_L) = \frac{1}{2\sigma}\left[1 - \frac{1}{D_L} - p + \sigma\right]$$

- The first equation here we have derived already. It comes from only people with $\tilde{p} > \frac{1}{D_F}$ placing bets on the favourite.
- The second equation just does the same thing for the longshot.
- Only people with $1 - \tilde{p} > \frac{1}{D_L}$ will bet on the longshot, meaning only those with $\tilde{p} < 1 - \frac{1}{D_L}$.

Betting Volumes

- We can insert the formulas for the odds as functions of the betting volumes into these formulas to get two complicated-looking equations

$$
\begin{aligned}
B(D_F) &= \frac{1}{2\sigma}\left[p+\sigma-\frac{B(D_F)}{(1-\tau)(B(D_F)+B(D_L))}\right] \\
B(D_L) &= \frac{1}{2\sigma}\left[\frac{B(D_L)}{(1-\tau)(B(D_F)+B(D_L))}-p+\sigma\right]
\end{aligned}
$$

- These look very complicated but if we add up the demand for the two bets, lots of terms cancel and we get a simple expression for total betting volume

$$
B(D_F)+B(D_L)=1-\frac{\tau}{2\sigma(1-\tau)}
$$

- Betting volume depends positively on the amount of disagreement (as measured by σ) and negatively on the operator's take, τ. If $\tau=0$, everyone bets but as τ gets bigger, fewer people place bets.
- This raises the question of what the profit-maximising τ would be: I've done calculations with realistic levels of beliefs and the answer is the profit-maximising τ would be high (over 10%). This is consistent with the high track takes at old pari-mutuel racecourses.

Equilibrium Volumes

- With the formula for total volume just derived, the equations for demand for bets for the two options can be simplified a little to be

$$B(D_F) = \frac{1}{2\sigma}\left[p + \sigma - \frac{B(D_F)}{\left(1 - \tau - \frac{\tau}{2\sigma}\right)}\right]$$

$$B(D_L) = \frac{1}{2\sigma}\left[\frac{B(D_L)}{\left(1 - \tau - \frac{\tau}{2\sigma}\right)} - p + \sigma\right]$$

- These are complicated looking but even complicated-looking equations can be solved and the solution is

$$B(D_F) = \frac{1}{1 + \frac{1}{2\sigma\left(1-\tau-\frac{\tau}{2\sigma}\right)}}\left(\frac{1}{2} + \frac{p}{2\sigma}\right)$$

$$B(D_L) = \frac{1}{1 + \frac{1}{2\sigma\left(1-\tau-\frac{\tau}{2\sigma}\right)}}\left(\frac{1}{2} + \frac{1-p}{2\sigma}\right)$$

- I won't ask you to derive these solutions in the exam.

Equilibrium Odds Relative Returns on Bets

- Inserting the solutions for the equilibrium volumes into the equations linking the volumes with the odds, you can show that the equilibrium odds are

$$D_F = \frac{(1-\tau)(1+2\sigma)}{p+\sigma}$$
$$D_L = \frac{(1-\tau)(1+2\sigma)}{1-p+\sigma}$$

- As the probability of a bet increases, the odds fall.
- But not in a way that keeps the expected return on the bet constant.

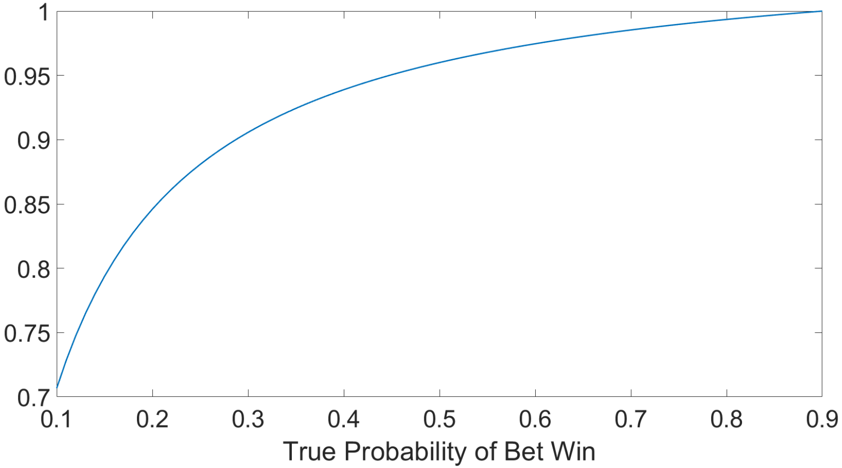
Expected Payouts Get Worse As p Falls

- We have called p the probability that the favourite wins but actually p can take any value.
- What is expected payout on a bet with probability p of winning? It is

$$pD = (1-\tau)(1+2\sigma)\left(\frac{p}{p+\sigma}\right)$$

- The term $(1-\tau)(1+2\sigma)$ is the same for all the bets.
- But the term $\frac{p}{p+\sigma}$ gets bigger as p gets bigger
  - If $p = 0.1$ and $\sigma = 0.1$, then $\frac{p}{p+\sigma} = \frac{0.1}{0.2} = 0.5$
  - If $p = 0.9$ and $\sigma = 0.1$, then $\frac{p}{p+\sigma} = \frac{0.9}{1.0} = 0.9$
- On the next page, we see the pattern for expected returns on bets predicted by the model with $\tau = 0.05$ and $\sigma = 0.05$. It matches the nonlinear pattern in the data, with rapidly falling average payouts for bets with the lowest chances of success.

Expected Payouts on a $1 Bet from the Disagreement Model ($\tau = 0.05$, $\sigma = 0.05$)



Reasons I Like This Explanation

- It doesn't rely on assumptions that people place irrational bets or fail to understand small probabilities or have risk-loving preferences.
- Those things might be true—and they may help to explain the favourite-longshot bias in pari-mutuel markets—but a good approach to economics is to start with a model where people are rationally acting in their own best interests (albeit in this case, people who act on imperfect information).
- If models of that sort fail, then we probably need to rely on the more "behavioural" explanations.
- The model also predicts the shape of the favourite-longshot bias documented in studies of pari-mutuel betting, with average losses rising more sharply as the probability of winning the bet falls.
- Another reason I like it is we can also use this model to explain how odds are set by bookmakers.

Supplementary Material in the Draft Book

- Chapter 9: Solving an Old Puzzle: Bias in Pari-Mutuel Odds